Computer algebra independent integration tests

1_Algebraic_functions/1.1_Binomial_products/1.1.2Quadratic/1.1.2.5(a+bx^2)^p(c+dx^2

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Contents

1 Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from Albert Rich Rubi web site.

1.1 Listing of CAS systems tested

The following systems were tested at this time.

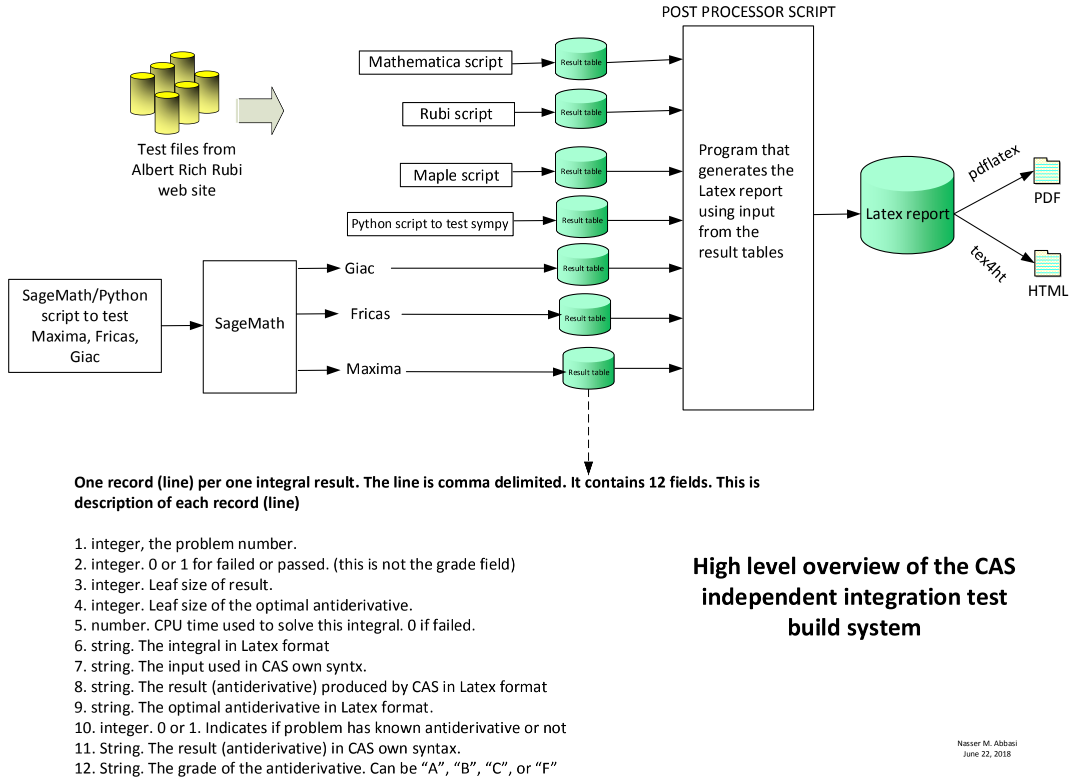
1. Mathematica 11.3 (64 bit).
2. Rubi 4.15.2 in Mathematica 11.3.
3. Rubi in Sympy (Version 1.3) under Python 3.7.0 using Anaconda distribution.
4. Maple 2018.1 (64 bit).
5. Maxima 5.41 Using Lisp ECL 16.1.2.
6. Fricas 1.3.4.
7. Sympy 1.3 under Python 3.7.0 using Anaconda distribution.
8. Giac/Xcas 1.4.9.

Maxima, Fricas and Giac/Xcas were called from inside SageMath version 8.3. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python. Rubi in Sympy was also called directly using sympy 1.3 in python.

1.2 Design of the test system

The following diagram gives a high level view of the current test build system.



1.3 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.4 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.5 Important notes about some of the results

Important note about Maxima results Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS and Giac/XCAS results There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

Important note about finding leaf size of antiderivative For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-express

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems implement a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.6 Grading of results

The table below summarizes the grading of each CAS system.

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (115)	% 0. (0)
Rubi in Sympy	% 67.83 (78)	% 32.17 (37)
Mathematica	% 93.91 (108)	% 6.09 (7)
Maple	% 91.3 (105)	% 8.7 (10)
Maxima	% 13.04 (15)	% 86.96 (100)
Fricas	% 28.7 (33)	% 71.3 (82)
Sympy	% 20.87 (24)	% 79.13 (91)
Giac	% 30.43 (35)	% 69.57 (80)

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

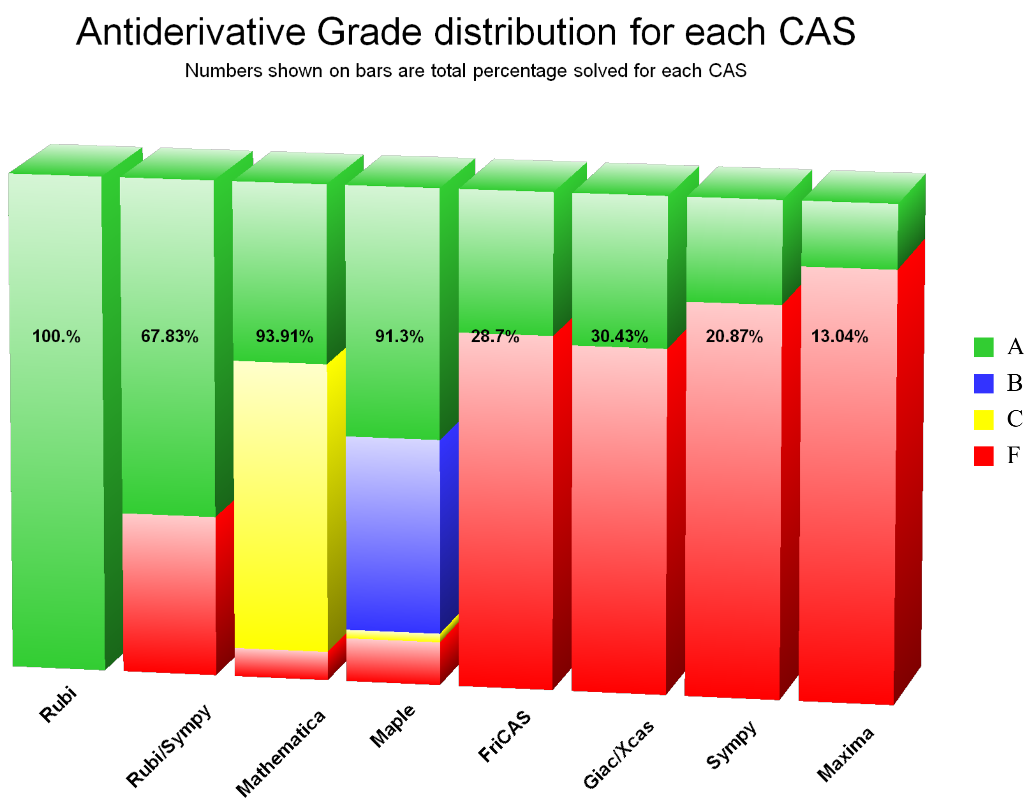
grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Grading is currently implemented only for for Mathematica, Rubi and Maple results. For all other CAS systems (Maxima, Fricas, Sympy, Giac, Rubi in sympy), the grading function is not yet implemented. For these systems, a grade of A is assigned if the integrate command completes successfully and a grade of F otherwise.

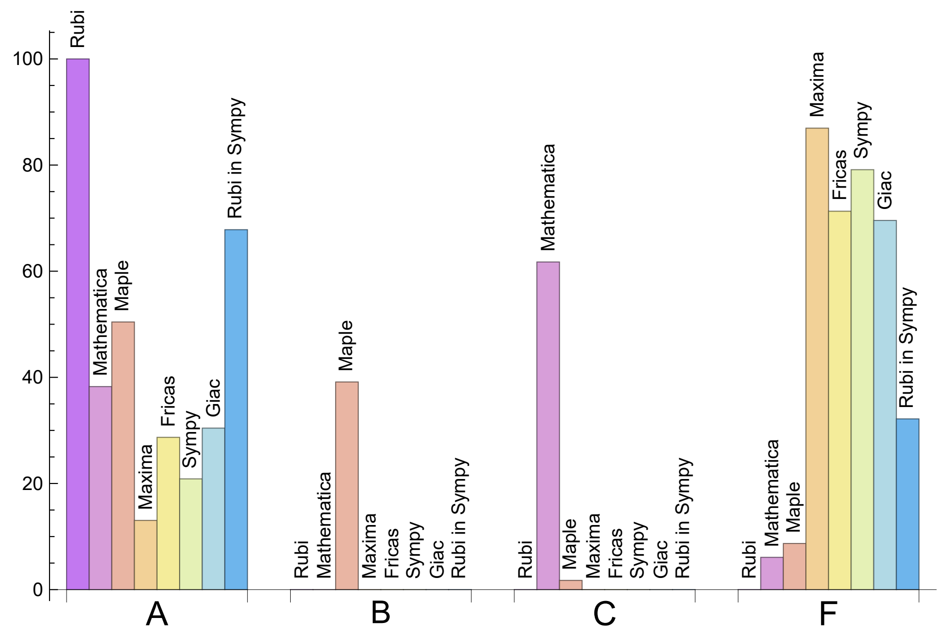
Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Rubi in Sympy	67.83	0.	0.	32.17
Mathematica	38.26	0.	61.74	6.09
Maple	50.43	39.13	1.74	8.7
Maxima	13.04	0.	0.	86.96
Fricas	28.7	0.	0.	71.3
Sympy	20.87	0.	0.	79.13
Giac	30.43	0.	0.	69.57

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.7 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.82	283.78	0.96	242.	1.
Rubi in Sympy	76.52	229.03	0.99	211.5	0.89
Mathematica	1.44	275.72	0.93	210.	0.9
Maple	0.04	867.3	2.73	397.	1.78
Maxima	0.9	150.27	0.93	135.	1.37
Fricas	3.3	0.88	0.01	1.	0.01
Sympy	11.5	284.21	1.6	241.	1.37
Giac	0.48	245.66	1.41	234.	1.67

1.8 list of integrals that has no closed form antiderivative

{103, 107, 110, 112, 115}

1.9 list of integrals not solved by each system

Not solved by Rubi {}

Not solved by Rubi in Sympy {1, 2, 3, 4, 9, 10, 11, 16, 17, 18, 23, 28, 29, 34, 35, 41, 42, 47, 56, 64, 70, 71, 76, 82, 83, 87, 88, 101, 103, 104, 107, 108, 110, 111, 112, 113, 115}

Not solved by Mathematica {104, 105, 106, 108, 109, 111, 113}

Not solved by Maple {55, 56, 104, 105, 106, 108, 109, 111, 113, 114}

Not solved by Maxima {5, 6, 7, 8, 12, 13, 14, 15, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 108, 109, 111, 113, 114}

Not solved by Fricas {23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 104, 105, 106, 108, 109, 111, 112, 113, 114}

Not solved by Sympy {22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 113, 114}

Not solved by Giac {23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 104, 105, 106, 108, 109, 111, 113, 114}

1.10 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Rubi in Sympy {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Rubi in Sympy Verification phase not implemented yet.

2 detailed summary tables of results

2.1 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	172	176	236	1	236	284	0
normalized size	1	1.	1.	1.02	1.37	0.01	1.37	1.65	0.
time (sec)	N/A	0.541	0.185	0.002	1.355	0.183	0.112	0.229	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	130	135	181	1	173	217	0
normalized size	1	1.	1.	1.04	1.39	0.01	1.33	1.67	0.
time (sec)	N/A	0.381	0.12	0.002	1.347	0.184	0.087	0.229	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	96	94	126	1	121	154	0
normalized size	1	1.	1.02	1.	1.34	0.01	1.29	1.64	0.
time (sec)	N/A	0.249	0.07	0.002	1.351	0.182	0.074	0.226	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	53	70	1	63	89	0
normalized size	1	1.	1.	0.95	1.25	0.02	1.12	1.59	0.
time (sec)	N/A	0.116	0.025	0.001	1.349	0.184	0.056	0.227	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	72	119	0	1	206	108	76
normalized size	1	1.	0.89	1.47	0.	0.01	2.54	1.33	0.94
time (sec)	N/A	0.207	0.107	0.009	0.	0.217	1.621	0.227	25.974

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	95	163	0	1	190	128	92
normalized size	1	1.	0.88	1.51	0.	0.01	1.76	1.19	0.85
time (sec)	N/A	0.244	0.127	0.013	0.	0.218	3.173	0.229	27.511

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	130	175	0	1	246	182	117
normalized size	1	1.	1.	1.35	0.	0.01	1.89	1.4	0.9
time (sec)	N/A	0.313	0.153	0.011	0.	0.218	7.201	0.228	29.802

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	171	210	0	1	313	248	156
normalized size	1	1.	1.	1.23	0.	0.01	1.83	1.45	0.91
time (sec)	N/A	0.479	0.195	0.014	0.	0.222	14.924	0.232	34.675

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	226	237	319	1	304	382	0
normalized size	1	1.	1.	1.05	1.41	0.	1.35	1.69	0.
time (sec)	N/A	0.626	0.165	0.001	1.355	0.184	0.119	0.227	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	158	169	227	1	216	273	0
normalized size	1	1.	1.	1.07	1.44	0.01	1.37	1.73	0.
time (sec)	N/A	0.496	0.114	0.002	1.348	0.183	0.099	0.228	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	96	101	135	1	121	162	0
normalized size	1	1.	1.02	1.07	1.44	0.01	1.29	1.72	0.
time (sec)	N/A	0.251	0.058	0.001	1.357	0.182	0.075	0.227	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	115	243	0	1	343	240	153
normalized size	1	1.	0.81	1.71	0.	0.01	2.42	1.69	1.08
time (sec)	N/A	0.534	0.12	0.007	0.	0.216	2.512	0.228	60.738

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	134	299	0	1	479	263	168
normalized size	1	1.	0.82	1.82	0.	0.01	2.92	1.6	1.02
time (sec)	N/A	0.61	0.172	0.014	0.	0.221	6.531	0.23	58.195

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	183	397	0	1	400	321	196
normalized size	1	1.	0.88	1.92	0.	0.	1.93	1.55	0.95
time (sec)	N/A	0.644	0.238	0.016	0.	0.226	29.016	0.229	63.162

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	242	360	0	1	486	420	236
normalized size	1	1.	1.01	1.5	0.	0.	2.02	1.75	0.98
time (sec)	N/A	0.754	0.313	0.015	0.	0.224	120.765	0.232	66.166

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	310	339	440	1	423	541	0
normalized size	1	1.	1.	1.09	1.42	0.	1.36	1.75	0.
time (sec)	N/A	0.9	0.242	0.002	1.361	0.183	0.147	0.227	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	226	244	323	1	304	390	0
normalized size	1	1.	1.	1.08	1.43	0.	1.35	1.73	0.
time (sec)	N/A	0.617	0.169	0.002	1.346	0.184	0.117	0.223	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	130	149	197	1	173	234	0
normalized size	1	1.	1.	1.15	1.52	0.01	1.33	1.8	0.
time (sec)	N/A	0.38	0.097	0.	1.367	0.183	0.087	0.225	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	179	401	0	1	508	414	279
normalized size	1	1.	0.79	1.77	0.	0.	2.24	1.82	1.23
time (sec)	N/A	0.918	0.177	0.007	0.	0.218	3.7	0.229	120.378

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	176	475	0	1	654	433	275
normalized size	1	1.	0.73	1.96	0.	0.	2.7	1.79	1.14
time (sec)	N/A	1.033	0.24	0.017	0.	0.228	11.501	0.23	128.165

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	219	589	0	1	862	501	332
normalized size	1	1.	0.75	2.02	0.	0.	2.96	1.72	1.14
time (sec)	N/A	1.059	0.317	0.018	0.	0.227	74.199	0.228	129.74

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	295	735	0	1	0	603	359
normalized size	1	1.	0.85	2.11	0.	0.	0.	1.73	1.03
time (sec)	N/A	1.178	0.427	0.021	0.	0.229	0.	0.224	148.443

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	544	544	373	1332	0	0	0	0	0
normalized size	1	1.	0.69	2.45	0.	0.	0.	0.	0.
time (sec)	N/A	1.896	2.065	0.06	0.	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	267	865	0	0	0	0	374
normalized size	1	1.	0.7	2.27	0.	0.	0.	0.	0.98
time (sec)	N/A	1.102	1.322	0.023	0.	0.	0.	0.	115.453

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	212	394	0	0	0	0	252
normalized size	1	1.	0.75	1.39	0.	0.	0.	0.	0.89
time (sec)	N/A	0.611	0.638	0.029	0.	0.	0.	0.	69.583

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	192	328	0	0	0	0	224
normalized size	1	1.	0.71	1.21	0.	0.	0.	0.	0.83
time (sec)	N/A	0.589	0.562	0.058	0.	0.	0.	0.	70.4

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	297	1236	0	0	0	0	231
normalized size	1	1.	1.08	4.51	0.	0.	0.	0.	0.84
time (sec)	N/A	0.619	1.739	0.067	0.	0.	0.	0.	66.203

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	379	2856	0	0	0	0	0
normalized size	1	1.	0.98	7.42	0.	0.	0.	0.	0.
time (sec)	N/A	1.121	2.306	0.08	0.	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	543	543	372	1332	0	0	0	0	0
normalized size	1	1.	0.69	2.45	0.	0.	0.	0.	0.
time (sec)	N/A	1.714	2.121	0.028	0.	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	400	275	870	0	0	0	0	389
normalized size	1	1.	0.69	2.17	0.	0.	0.	0.	0.97
time (sec)	N/A	1.216	1.518	0.027	0.	0.	0.	0.	114.798

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	248	671	0	0	0	0	345
normalized size	1	1.	0.67	1.82	0.	0.	0.	0.	0.93
time (sec)	N/A	1.124	1.197	0.038	0.	0.	0.	0.	114.324

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	373	373	296	1225	0	0	0	0	342
normalized size	1	1.	0.79	3.28	0.	0.	0.	0.	0.92
time (sec)	N/A	1.144	1.811	0.043	0.	0.	0.	0.	118.492

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	382	2860	0	0	0	0	333
normalized size	1	1.	1.02	7.61	0.	0.	0.	0.	0.89
time (sec)	N/A	1.137	2.353	0.055	0.	0.	0.	0.	108.873

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	531	531	545	5113	0	0	0	0	0
normalized size	1	1.	1.03	9.63	0.	0.	0.	0.	0.
time (sec)	N/A	1.725	3.278	0.1	0.	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	551	551	386	1386	0	0	0	0	0
normalized size	1	1.	0.7	2.52	0.	0.	0.	0.	0.
time (sec)	N/A	1.796	2.193	0.037	0.	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	279	924	0	0	0	0	389
normalized size	1	1.	0.7	2.33	0.	0.	0.	0.	0.98
time (sec)	N/A	1.287	1.484	0.028	0.	0.	0.	0.	114.943

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	215	501	0	0	0	0	250
normalized size	1	1.	0.76	1.78	0.	0.	0.	0.	0.89
time (sec)	N/A	0.601	0.7	0.024	0.	0.	0.	0.	68.772

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	131	158	0	0	0	0	177
normalized size	1	1.	0.64	0.77	0.	0.	0.	0.	0.86
time (sec)	N/A	0.371	0.193	0.03	0.	0.	0.	0.	44.518

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	206	334	0	0	0	0	173
normalized size	1	1.	0.99	1.6	0.	0.	0.	0.	0.83
time (sec)	N/A	0.314	1.049	0.04	0.	0.	0.	0.	35.322

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	302	1352	0	0	0	0	257
normalized size	1	1.	1.06	4.76	0.	0.	0.	0.	0.9
time (sec)	N/A	0.63	2.138	0.049	0.	0.	0.	0.	90.132

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	401	401	393	3039	0	0	0	0	0
normalized size	1	1.	0.98	7.58	0.	0.	0.	0.	0.
time (sec)	N/A	1.182	2.286	0.059	0.	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	501	501	369	1169	0	0	0	0	0
normalized size	1	1.	0.74	2.33	0.	0.	0.	0.	0.
time (sec)	N/A	1.696	2.058	0.069	0.	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	260	750	0	0	0	0	345
normalized size	1	1.	0.73	2.09	0.	0.	0.	0.	0.96
time (sec)	N/A	1.096	1.302	0.038	0.	0.	0.	0.	113.906

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	208	393	0	0	0	0	230
normalized size	1	1.	0.81	1.52	0.	0.	0.	0.	0.89
time (sec)	N/A	0.56	0.594	0.035	0.	0.	0.	0.	73.774

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	212	349	0	0	0	0	173
normalized size	1	1.	1.01	1.67	0.	0.	0.	0.	0.83
time (sec)	N/A	0.312	0.608	0.044	0.	0.	0.	0.	37.087

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	262	581	0	0	0	0	236
normalized size	1	1.	0.96	2.14	0.	0.	0.	0.	0.87
time (sec)	N/A	0.641	1.251	0.043	0.	0.	0.	0.	71.83

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	428	1742	0	0	0	0	0
normalized size	1	1.	1.14	4.65	0.	0.	0.	0.	0.
time (sec)	N/A	1.12	3.827	0.056	0.	0.	0.	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	212	349	0	0	0	0	173
normalized size	1	1.	1.01	1.67	0.	0.	0.	0.	0.83
time (sec)	N/A	0.306	0.648	0.042	0.	0.	0.	0.	35.635

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	220	359	0	0	0	0	209
normalized size	1	1.	0.89	1.45	0.	0.	0.	0.	0.85
time (sec)	N/A	0.747	1.234	0.073	0.	0.	0.	0.	113.146

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	213	345	0	0	0	0	204
normalized size	1	1.	0.9	1.46	0.	0.	0.	0.	0.86
time (sec)	N/A	0.712	0.687	0.06	0.	0.	0.	0.	112.439

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	221	354	0	0	0	0	202
normalized size	1	1.	0.91	1.46	0.	0.	0.	0.	0.83
time (sec)	N/A	0.756	0.731	0.063	0.	0.	0.	0.	147.743

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	81	105	0	0	0	0	180
normalized size	1	1.	0.42	0.55	0.	0.	0.	0.	0.94
time (sec)	N/A	0.343	0.139	0.051	0.	0.	0.	0.	37.869

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	142	367	0	0	0	0	245
normalized size	1	1.	0.54	1.4	0.	0.	0.	0.	0.94
time (sec)	N/A	0.548	0.283	0.028	0.	0.	0.	0.	59.893

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	186	775	0	0	0	0	350
normalized size	1	1.	0.52	2.18	0.	0.	0.	0.	0.98
time (sec)	N/A	0.926	0.533	0.033	0.	0.	0.	0.	101.835

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	104	0	0	0	0	0	100
normalized size	1	1.	0.92	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.488	0.661	0.29	0.	0.	0.	0.	47.707

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	526	526	203	0	0	0	0	0	0
normalized size	1	1.	0.39	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.863	0.699	0.2	0.	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	124	1942	0	1	0	227	117
normalized size	1	1.	0.97	15.17	0.	0.01	0.	1.77	0.91
time (sec)	N/A	0.406	0.419	0.056	0.	1.326	0.	0.283	47.283

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	194	1541	0	1	0	392	280
normalized size	1	1.	0.64	5.07	0.	0.	0.	1.29	0.92
time (sec)	N/A	0.909	0.423	0.062	0.	5.953	0.	0.273	100.828

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	133	1052	0	1	0	248	148
normalized size	1	1.	0.8	6.34	0.	0.01	0.	1.49	0.89
time (sec)	N/A	0.419	0.249	0.023	0.	1.669	0.	0.272	54.89

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	93	646	0	1	0	159	78
normalized size	1	1.	1.02	7.1	0.	0.01	0.	1.75	0.86
time (sec)	N/A	0.198	0.148	0.017	0.	1.074	0.	0.27	26.641

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	306	0	1	0	100	42
normalized size	1	1.	1.	6.24	0.	0.02	0.	2.04	0.86
time (sec)	N/A	0.08	0.039	0.015	0.	0.264	0.	0.251	11.519

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	113	782	0	1	0	234	104
normalized size	1	1.	0.93	6.41	0.	0.01	0.	1.92	0.85
time (sec)	N/A	0.345	0.277	0.056	0.	92.534	0.	0.267	41.852

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	203	1865	0	0	0	647	182
normalized size	1	1.	1.	9.19	0.	0.	0.	3.19	0.9
time (sec)	N/A	0.752	0.987	0.082	0.	0.	0.	8.455	97.726

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	608	776	456	1891	0	0	0	0	0
normalized size	1	1.28	0.75	3.11	0.	0.	0.	0.	0.
time (sec)	N/A	2.23	4.74	0.056	0.	0.	0.	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	400	346	1059	0	0	0	0	355
normalized size	1	1.	0.86	2.65	0.	0.	0.	0.	0.89
time (sec)	N/A	0.968	2.58	0.028	0.	0.	0.	0.	130.427

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	184	340	0	0	0	0	269
normalized size	1	1.	0.57	1.06	0.	0.	0.	0.	0.84
time (sec)	N/A	0.644	0.443	0.021	0.	0.	0.	0.	78.947

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	143	191	0	0	0	0	82
normalized size	1	1.	1.4	1.87	0.	0.	0.	0.	0.8
time (sec)	N/A	0.141	0.193	0.028	0.	0.	0.	0.	19.443

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	347	390	0	0	0	0	170
normalized size	1	1.	1.66	1.87	0.	0.	0.	0.	0.81
time (sec)	N/A	0.396	1.218	0.039	0.	0.	0.	0.	47.088

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	401	401	427	2068	0	0	0	0	333
normalized size	1	1.	1.06	5.16	0.	0.	0.	0.	0.83
time (sec)	N/A	1.083	6.416	0.055	0.	0.	0.	0.	127.482

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	630	630	584	6245	0	0	0	0	0
normalized size	1	1.	0.93	9.91	0.	0.	0.	0.	0.
time (sec)	N/A	2.048	5.488	0.087	0.	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	659	784	445	1939	0	0	0	0	0
normalized size	1	1.19	0.68	2.94	0.	0.	0.	0.	0.
time (sec)	N/A	2.188	4.51	0.032	0.	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	739	1028	0	0	0	0	360
normalized size	1	1.	1.83	2.55	0.	0.	0.	0.	0.89
time (sec)	N/A	0.945	2.926	0.027	0.	0.	0.	0.	131.911

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	184	300	0	0	0	0	269
normalized size	1	1.	0.56	0.91	0.	0.	0.	0.	0.82
time (sec)	N/A	0.655	0.344	0.031	0.	0.	0.	0.	79.32

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	492	630	0	0	0	0	182
normalized size	1	1.	2.2	2.81	0.	0.	0.	0.	0.81
time (sec)	N/A	0.408	2.207	0.043	0.	0.	0.	0.	50.467

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	999	1879	0	0	0	0	338
normalized size	1	1.	2.55	4.81	0.	0.	0.	0.	0.86
time (sec)	N/A	1.147	3.249	0.055	0.	0.	0.	0.	148.299

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	639	639	570	6211	0	0	0	0	0
normalized size	1	1.	0.89	9.72	0.	0.	0.	0.	0.
time (sec)	N/A	2.156	5.033	0.11	0.	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	621	621	350	988	0	0	0	0	552
normalized size	1	1.	0.56	1.59	0.	0.	0.	0.	0.89
time (sec)	N/A	1.438	2.231	0.036	0.	0.	0.	0.	153.277

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	197	341	0	0	0	0	272
normalized size	1	1.	0.62	1.07	0.	0.	0.	0.	0.85
time (sec)	N/A	0.616	0.364	0.031	0.	0.	0.	0.	80.453

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	143	191	0	0	0	0	82
normalized size	1	1.	1.4	1.87	0.	0.	0.	0.	0.8
time (sec)	N/A	0.137	0.182	0.029	0.	0.	0.	0.	19.48

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	101	118	0	0	0	0	173
normalized size	1	1.	1.01	1.18	0.	0.	0.	0.	1.73
time (sec)	N/A	0.479	0.15	0.03	0.	0.	0.	0.	50.401

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	365	413	0	0	0	0	289
normalized size	1	1.	1.06	1.2	0.	0.	0.	0.	0.84
time (sec)	N/A	0.708	1.215	0.046	0.	0.	0.	0.	94.851

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	435	435	433	2062	0	0	0	0	0
normalized size	1	1.	1.	4.74	0.	0.	0.	0.	0.
time (sec)	N/A	1.698	6.345	0.068	0.	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	980	980	352	1063	0	0	0	0	0
normalized size	1	1.	0.36	1.08	0.	0.	0.	0.	0.
time (sec)	N/A	3.204	2.515	0.047	0.	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	304	594	0	0	0	0	182
normalized size	1	1.	1.36	2.66	0.	0.	0.	0.	0.82
time (sec)	N/A	0.416	1.589	0.043	0.	0.	0.	0.	49.031

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	207	285	0	0	0	0	170
normalized size	1	1.	0.99	1.36	0.	0.	0.	0.	0.81
time (sec)	N/A	0.362	0.621	0.038	0.	0.	0.	0.	47.403

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	221	303	0	0	0	0	289
normalized size	1	1.	0.64	0.88	0.	0.	0.	0.	0.84
time (sec)	N/A	0.711	0.919	0.044	0.	0.	0.	0.	94.239

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	539	539	1284	956	0	0	0	0	0
normalized size	1	1.	2.38	1.77	0.	0.	0.	0.	0.
time (sec)	N/A	1.41	7.186	0.056	0.	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	814	814	2744	4115	0	0	0	0	0
normalized size	1	1.	3.37	5.06	0.	0.	0.	0.	0.
time (sec)	N/A	2.698	9.028	0.088	0.	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	239	204	370	0	0	0	0	221
normalized size	1	0.99	0.84	1.53	0.	0.	0.	0.	0.91
time (sec)	N/A	0.45	0.383	0.052	0.	0.	0.	0.	63.095

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	71	121	0	0	0	0	165
normalized size	1	1.	0.37	0.63	0.	0.	0.	0.	0.86
time (sec)	N/A	0.293	0.184	0.015	0.	0.	0.	0.	42.308

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	50	64	0	0	0	0	58
normalized size	1	1.	0.86	1.1	0.	0.	0.	0.	1.
time (sec)	N/A	0.076	0.101	0.021	0.	0.	0.	0.	13.364

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	122	147	0	0	0	0	109
normalized size	1	1.	1.01	1.21	0.	0.	0.	0.	0.9
time (sec)	N/A	0.195	0.434	0.05	0.	0.	0.	0.	28.465

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	357	477	0	0	0	0	190
normalized size	1	1.	1.66	2.22	0.	0.	0.	0.	0.88
time (sec)	N/A	0.425	0.478	0.051	0.	0.	0.	0.	55.113

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	134	293	0	0	0	0	277
normalized size	1	1.	0.45	0.98	0.	0.	0.	0.	0.93
time (sec)	N/A	0.587	0.284	0.043	0.	0.	0.	0.	66.337

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	94	133	0	0	0	0	85
normalized size	1	1.	1.01	1.43	0.	0.	0.	0.	0.91
time (sec)	N/A	0.135	0.11	0.032	0.	0.	0.	0.	17.251

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	52	53	0	0	0	0	177
normalized size	1	1.	1.06	1.08	0.	0.	0.	0.	3.61
time (sec)	N/A	0.147	0.099	0.032	0.	0.	0.	0.	41.629

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	37	35	0	0	0	0	31
normalized size	1	1.	1.03	0.97	0.	0.	0.	0.	0.86
time (sec)	N/A	0.089	0.07	0.042	0.	0.	0.	0.	19.654

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	114	1622	0	1	0	4	97
normalized size	1	1.	1.01	14.35	0.	0.01	0.	0.04	0.86
time (sec)	N/A	0.337	0.237	0.049	0.	1.613	0.	1.637	34.095

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	422	793	0	0	0	0	308
normalized size	1	1.	1.18	2.21	0.	0.	0.	0.	0.86
time (sec)	N/A	1.171	4.557	0.069	0.	0.	0.	0.	170.75

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	401	765	0	0	0	0	432
normalized size	1	1.	1.05	2.01	0.	0.	0.	0.	1.13
time (sec)	N/A	1.076	5.213	0.059	0.	0.	0.	0.	119.931

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	426	426	773	1105	0	0	0	0	0
normalized size	1	1.	1.81	2.59	0.	0.	0.	0.	0.
time (sec)	N/A	1.332	6.649	0.073	0.	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	485	485	587	1078	0	0	0	0	527
normalized size	1	1.	1.21	2.22	0.	0.	0.	0.	1.09
time (sec)	N/A	1.226	6.161	0.043	0.	0.	0.	0.	160.607

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.165	1.234	0.104	0.	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	545	545	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.641	0.121	0.082	0.	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-1)	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	0	0	0	0	0	0	131
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.8
time (sec)	N/A	0.55	0.134	0.087	0.	0.	0.	0.	90.671

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	0	0	0	0	0	0	580
normalized size	1	1.	0.	0.	0.	0.	0.	0.	3.92
time (sec)	N/A	0.314	1.193	0.099	0.	0.	0.	0.	165.39

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.158	1.334	0.09	0.	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	484	484	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.224	0.856	0.082	0.	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	0	0	0	0	0	0	122
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.38
time (sec)	N/A	1.357	1.206	0.104	0.	0.	0.	0.	51.607

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.163	1.687	0.104	0.	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	541	541	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.583	0.126	0.075	0.	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	F(-1)	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.168	0.203	0.09	0.	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.571	0.122	0.098	0.	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	152	0	0	0	0	0	214
normalized size	1	1.	1.03	0.	0.	0.	0.	0.	1.45
time (sec)	N/A	0.315	0.506	0.093	0.	0.	0.	0.	67.087

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.172	1.111	0.101	0.	0.	0.	0.	0.

2.2 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [64] had the largest ratio of [0.2812]

Table 1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.	24	0.042
2	A	2	1	1.	24	0.042
3	A	2	1	1.	24	0.042
4	A	2	1	1.	22	0.045
5	A	3	3	1.	24	0.125
6	A	3	3	1.	24	0.125

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
7	A	3	3	1.	24	0.125
8	A	4	4	1.	24	0.167
9	A	2	1	1.	26	0.038
10	A	2	1	1.	26	0.038
11	A	2	1	1.	24	0.042
12	A	4	3	1.	26	0.115
13	A	4	4	1.	26	0.154
14	A	4	3	1.	26	0.115
15	A	4	3	1.	26	0.115
16	A	2	1	1.	26	0.038
17	A	2	1	1.	26	0.038
18	A	2	1	1.	24	0.042
19	A	5	3	1.	26	0.115
20	A	5	4	1.	26	0.154
21	A	5	4	1.	26	0.154
22	A	5	3	1.	26	0.115
23	A	7	5	1.	30	0.167
24	A	6	5	1.	30	0.167
25	A	5	5	1.	30	0.167
26	A	5	5	1.	30	0.167
27	A	4	4	1.	30	0.133
28	A	5	5	1.	30	0.167
29	A	7	5	1.	30	0.167
30	A	6	5	1.	30	0.167
31	A	6	6	1.	30	0.2
32	A	6	5	1.	30	0.167
33	A	5	4	1.	30	0.133
34	A	6	5	1.	30	0.167
35	A	7	5	1.	30	0.167
36	A	6	5	1.	30	0.167
37	A	5	5	1.	30	0.167
38	A	4	4	1.	30	0.133
39	A	3	3	1.	30	0.1
40	A	4	4	1.	30	0.133
41	A	5	4	1.	30	0.133
42	A	7	6	1.	30	0.2

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
43	A	6	6	1.	30	0.2
44	A	5	5	1.	30	0.167
45	A	3	3	1.	30	0.1
46	A	4	4	1.	30	0.133
47	A	5	4	1.	30	0.133
48	A	3	3	1.	30	0.1
49	A	8	7	1.	31	0.226
50	A	8	7	1.	31	0.226
51	A	8	7	1.	32	0.219
52	A	4	4	1.	30	0.133
53	A	5	5	1.	30	0.167
54	A	6	5	1.	30	0.167
55	A	2	2	1.	87	0.023
56	A	5	5	1.	81	0.062
57	A	6	6	1.	28	0.214
58	A	14	8	1.	30	0.267
59	A	9	7	1.	30	0.233
60	A	5	5	1.	28	0.179
61	A	2	2	1.	21	0.095
62	A	5	3	1.	30	0.1
63	A	7	5	1.	30	0.167
64	A	14	9	1.28	32	0.281
65	A	7	7	1.	32	0.219
66	A	6	6	1.	32	0.188
67	A	1	1	1.	32	0.031
68	A	3	3	1.	32	0.094
69	A	6	6	1.	32	0.188
70	A	9	8	1.	32	0.25
71	A	14	9	1.19	32	0.281
72	A	7	7	1.	32	0.219
73	A	6	6	1.	32	0.188
74	A	3	3	1.	32	0.094
75	A	6	6	1.	32	0.188
76	A	9	8	1.	32	0.25
77	A	12	8	1.	32	0.25
78	A	6	6	1.	32	0.188

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
79	A	1	1	1.	32	0.031
80	A	3	2	1.	32	0.062
81	A	5	5	1.	32	0.156
82	A	8	7	1.	32	0.219
83	A	14	9	1.	32	0.281
84	A	3	3	1.	32	0.094
85	A	3	3	1.	32	0.094
86	A	5	5	1.	32	0.156
87	A	8	7	1.	32	0.219
88	A	11	6	1.	32	0.188
89	A	7	7	0.99	28	0.25
90	A	6	6	1.	28	0.214
91	A	1	1	1.	28	0.036
92	A	3	3	1.	28	0.107
93	A	6	6	1.	28	0.214
94	A	6	6	1.	32	0.188
95	A	1	1	1.	32	0.031
96	A	1	1	1.	32	0.031
97	A	3	3	1.	30	0.1
98	A	4	4	1.	28	0.143
99	A	11	9	1.	33	0.273
100	A	8	7	1.	32	0.219
101	A	11	9	1.	33	0.273
102	A	8	7	1.	32	0.219
103	A	0	0	0.	0	0.
104	A	7	7	1.	34	0.206
105	A	2	2	1.	34	0.059
106	A	2	2	1.	34	0.059
107	A	0	0	0.	0	0.
108	A	8	8	1.	34	0.235
109	A	5	5	1.	34	0.147
110	A	0	0	0.	0	0.
111	A	7	7	1.	34	0.206
112	A	0	0	0.	0	0.
113	A	2	2	1.	34	0.059
114	A	2	2	1.	34	0.059

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
115	A	0	0	0.	0	0.

3 Listing of integrals

3.1 $\int \left(a+bx^2\right) \left(c+dx^2\right) \left(e+fx^2\right)^4 \, dx$

Optimal. Leaf size=172

$$\frac{1}{3}e^3x^3(4acf+ade+bce)+\frac{1}{5}e^2x^5(2af(3cf+2de)+be(4cf+de))+\frac{1}{11}f^3x^{11}(adf+bcf+4bde)$$
$$+\frac{1}{9}f^2x^9(af(cf+4de)+2be(2cf+3de))+\frac{2}{7}efx^7(af(2cf+3de)+be(3cf+2de))+ace^4x+\frac{1}{13}bdf^4x^{13}$$

```
[Out] a*c*e^4*x + (e^3*(b*c*e + a*d*e + 4*a*c*f)*x^3)/3 + (e^2*(2*a*f*(
2*d*e + 3*c*f) + b*e*(d*e + 4*c*f))*x^5)/5 + (2*e*f*(a*f*(3*d*e +
 2*c*f) + b*e*(2*d*e + 3*c*f))*x^7)/7 + (f^2*(a*f*(4*d*e + c*f) +
 2*b*e*(3*d*e + 2*c*f))*x^9)/9 + (f^3*(4*b*d*e + b*c*f + a*d*f)*x
^11)/11 + (b*d*f^4*x^13)/13
```

Rubi [A] time = 0.540615, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{1}{3}e^3x^3(4acf+ade+bce)+\frac{1}{5}e^2x^5(2af(3cf+2de)+be(4cf+de))+\frac{1}{11}f^3x^{11}(adf+bcf+4bde)$$
$$+\frac{1}{9}f^2x^9(af(cf+4de)+2be(2cf+3de))+\frac{2}{7}efx^7(af(2cf+3de)+be(3cf+2de))+ace^4x+\frac{1}{13}bdf^4x^{13}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^4,x]
```

```
[Out] a*c*e^4*x + (e^3*(b*c*e + a*d*e + 4*a*c*f)*x^3)/3 + (e^2*(2*a*f*(
2*d*e + 3*c*f) + b*e*(d*e + 4*c*f))*x^5)/5 + (2*e*f*(a*f*(3*d*e +
 2*c*f) + b*e*(2*d*e + 3*c*f))*x^7)/7 + (f^2*(a*f*(4*d*e + c*f) +
 2*b*e*(3*d*e + 2*c*f))*x^9)/9 + (f^3*(4*b*d*e + b*c*f + a*d*f)*x
^11)/11 + (b*d*f^4*x^13)/13
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{bdf^4x^{13}}{13}+ce^4\int a\,dx+\frac{e^3x^3\left(4acf+ade+bce\right)}{3}$$
$$+\frac{e^2x^5\left(6acf^2+4adef+4bcef+bde^2\right)}{5}+\frac{2efx^7\left(2acf^2+3adef+3bcef+2bde^2\right)}{7}$$
$$+\frac{f^3x^{11}\left(adf+bcf+4bde\right)}{11}+\frac{f^2x^9\left(acf^2+4adef+4bcef+6bde^2\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)*(f*x**2+e)**4,x)
```

```
[Out] b*d*f**4*x**13/13 + c*e**4*Integral(a, x) + e**3*x**3*(4*a*c*f +
a*d*e + b*c*e)/3 + e**2*x**5*(6*a*c*f**2 + 4*a*d*e*f + 4*b*c*e*f
+ b*d*e**2)/5 + 2*e*f*x**7*(2*a*c*f**2 + 3*a*d*e*f + 3*b*c*e*f +
2*b*d*e**2)/7 + f**3*x**11*(a*d*f + b*c*f + 4*b*d*e)/11 + f**2*x*
*9*(a*c*f**2 + 4*a*d*e*f + 4*b*c*e*f + 6*b*d*e**2)/9
```

Mathematica [A] time = 0.185213, size = 172, normalized size = 1.

$$\frac{1}{3}e^3x^3(4acf+ade+bce)+\frac{1}{5}e^2x^5(2af(3cf+2de)+be(4cf+de))+\frac{1}{11}f^3x^{11}(adf+bcf+4bde)+\frac{1}{9}f^2x^9(af(cf+4de)+2be(2cf+3de))+\frac{2}{7}efx^7(af(2cf+3de)+be(3cf+2de))+ace^4x+\frac{1}{13}bdf^4x^{13}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^4,x]
```

```
[Out] a*c*e^4*x + (e^3*(b*c*e + a*d*e + 4*a*c*f)*x^3)/3 + (e^2*(2*a*f*(
2*d*e + 3*c*f) + b*e*(d*e + 4*c*f))*x^5)/5 + (2*e*f*(a*f*(3*d*e +
 2*c*f) + b*e*(2*d*e + 3*c*f))*x^7)/7 + (f^2*(a*f*(4*d*e + c*f) +
 2*b*e*(3*d*e + 2*c*f))*x^9)/9 + (f^3*(4*b*d*e + b*c*f + a*d*f)*x
^11)/11 + (b*d*f^4*x^13)/13
```

Maple [A] time = 0.002, size = 176, normalized size = 1.

$$\frac{bdf^4x^{13}}{13}+\frac{\left((ad+bc)\,f^4+4\,bdef^3\right)x^{11}}{11}+\frac{\left(acf^4+4\,(ad+bc)\,ef^3+6\,bde^2f^2\right)x^9}{9}+\frac{\left(4\,acef^3+6\,(ad+bc)\,e^2f^2+4\,bde^3f\right)x^7}{7}+\frac{\left(6\,ace^2f^2+4\,(ad+bc)\,e^3f+bde^4\right)x^5}{5}+\frac{\left(4\,ace^3f+(ad+bc)\,e^4\right)x^3}{3}+ace^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^4,x)
```

```
[Out] 1/13*b*d*f^4*x^13+1/11*((a*d+b*c)*f^4+4*b*d*e*f^3)*x^11+1/9*(a*c*
f^4+4*(a*d+b*c)*e*f^3+6*b*d*e^2*f^2)*x^9+1/7*(4*a*c*e*f^3+6*(a*d+
b*c)*e^2*f^2+4*b*d*e^3*f)*x^7+1/5*(6*a*c*e^2*f^2+4*(a*d+b*c)*e^3*
f+b*d*e^4)*x^5+1/3*(4*a*c*e^3*f+(a*d+b*c)*e^4)*x^3+a*c*e^4*x
```

Maxima [A] time = 1.35518, size = 236, normalized size = 1.37

$$\frac{1}{13}\,bdf^4x^{13} + \frac{1}{11}\left(4\,bdef^3 + (bc+ad)f^4\right)x^{11} + \frac{1}{9}\left(6\,bde^2f^2 + acf^4 + 4\,(bc+ad)ef^3\right)x^9 + \frac{2}{7}\left(2\,bde^3f + 2\,acef^3 + 3\,(bc+ad)e^2f^2\right)x^7 + ace^4x + \frac{1}{5}\left(bde^4 + 6\,ace^2f^2 + 4\,(bc+ad)e^3f\right)x^5 + \frac{1}{3}\left(4\,ace^3f + (bc+ad)e^4\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)*(f*x^2 + e)^4,x, algorithm="maxima")
```

```
[Out] 1/13*b*d*f^4*x^13 + 1/11*(4*b*d*e*f^3 + (b*c + a*d)*f^4)*x^11 + 1
/9*(6*b*d*e^2*f^2 + a*c*f^4 + 4*(b*c + a*d)*e*f^3)*x^9 + 2/7*(2*b
*d*e^3*f + 2*a*c*e*f^3 + 3*(b*c + a*d)*e^2*f^2)*x^7 + a*c*e^4*x +
 1/5*(b*d*e^4 + 6*a*c*e^2*f^2 + 4*(b*c + a*d)*e^3*f)*x^5 + 1/3*(4
*a*c*e^3*f + (b*c + a*d)*e^4)*x^3
```

Fricas [A] time = 0.18254, size = 1, normalized size = 0.01

$$\frac{1}{13}x^{13}f^4db + \frac{4}{11}x^{11}f^3edb + \frac{1}{11}x^{11}f^4cb + \frac{1}{11}x^{11}f^4da + \frac{2}{3}x^9f^2e^2db + \frac{4}{9}x^9f^3ecb + \frac{4}{9}x^9f^3eda + \frac{1}{9}x^9f^4ca + \frac{4}{7}x^7fe^3db + \frac{6}{7}x^7f^2e^2cb + \frac{6}{7}x^7f^2e^2da + \frac{4}{7}x^7f^3eca + \frac{1}{5}x^5e^4db + \frac{4}{5}x^5fe^3cb + \frac{4}{5}x^5fe^3da + \frac{6}{5}x^5f^2e^2ca + \frac{1}{3}x^3e^4cb + \frac{1}{3}x^3e^4da + \frac{4}{3}x^3fe^3ca + xe^4ca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)*(f*x^2 + e)^4,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*f^4*d*b + 4/11*x^11*f^3*e*d*b + 1/11*x^11*f^4*c*b + 1/1
1*x^11*f^4*d*a + 2/3*x^9*f^2*e^2*d*b + 4/9*x^9*f^3*e*c*b + 4/9*x^
9*f^3*e*d*a + 1/9*x^9*f^4*c*a + 4/7*x^7*f*e^3*d*b + 6/7*x^7*f^2*e
^2*c*b + 6/7*x^7*f^2*e^2*d*a + 4/7*x^7*f^3*e*c*a + 1/5*x^5*e^4*d*
b + 4/5*x^5*f*e^3*c*b + 4/5*x^5*f*e^3*d*a + 6/5*x^5*f^2*e^2*c*a +
 1/3*x^3*e^4*c*b + 1/3*x^3*e^4*d*a + 4/3*x^3*f*e^3*c*a + x*e^4*c*
a
```

Sympy [A] time = 0.112258, size = 236, normalized size = 1.37

$$ace^4x + \frac{bdf^4x^{13}}{13} + x^{11}\left(\frac{adf^4}{11} + \frac{bcf^4}{11} + \frac{4bdef^3}{11}\right) + x^9\left(\frac{acf^4}{9} + \frac{4adef^3}{9} + \frac{4bcef^3}{9} + \frac{2bde^2f^2}{3}\right) + x^7\left(\frac{4acef^3}{7} + \frac{6ade^2f^2}{7} + \frac{6bce^2f^2}{7} + \frac{4bde^3f}{7}\right) + x^5\left(\frac{6ace^2f^2}{5} + \frac{4ade^3f}{5} + \frac{4bce^3f}{5} + \frac{bde^4}{5}\right) + x^3\left(\frac{4ace^3f}{3} + \frac{ade^4}{3} + \frac{bce^4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x**2+c)*(f*x**2+e)**4,x)
```

```
[Out] a*c*e**4*x + b*d*f**4*x**13/13 + x**11*(a*d*f**4/11 + b*c*f**4/11
 + 4*b*d*e*f**3/11) + x**9*(a*c*f**4/9 + 4*a*d*e*f**3/9 + 4*b*c*e
*f**3/9 + 2*b*d*e**2*f**2/3) + x**7*(4*a*c*e*f**3/7 + 6*a*d*e**2*
f**2/7 + 6*b*c*e**2*f**2/7 + 4*b*d*e**3*f/7) + x**5*(6*a*c*e**2*f
**2/5 + 4*a*d*e**3*f/5 + 4*b*c*e**3*f/5 + b*d*e**4/5) + x**3*(4*a
*c*e**3*f/3 + a*d*e**4/3 + b*c*e**4/3)
```

GIAC/XCAS [A] time = 0.228793, size = 284, normalized size = 1.65

$$\frac{1}{13}bdf^4x^{13} + \frac{1}{11}bcf^4x^{11} + \frac{1}{11}adf^4x^{11} + \frac{4}{11}bdf^3x^{11}e + \frac{1}{9}acf^4x^9 + \frac{4}{9}bcf^3x^9e + \frac{4}{9}adf^3x^9e + \frac{2}{3}bdf^2x^9e^2 + \frac{4}{7}acf^3x^7e + \frac{6}{7}bcf^2x^7e^2 + \frac{6}{7}adf^2x^7e^2 + \frac{4}{7}bdfx^7e^3 + \frac{6}{5}acf^2x^5e^2 + \frac{4}{5}bcfx^5e^3 + \frac{4}{5}adfx^5e^3 + \frac{1}{5}bdx^5e^4 + \frac{4}{3}acfx^3e^3 + \frac{1}{3}bcx^3e^4 + \frac{1}{3}adx^3e^4 + acxe^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)*(f*x^2 + e)^4,x, algorithm="giac")
```

```
[Out] 1/13*b*d*f^4*x^13 + 1/11*b*c*f^4*x^11 + 1/11*a*d*f^4*x^11 + 4/11*
b*d*f^3*x^11*e + 1/9*a*c*f^4*x^9 + 4/9*b*c*f^3*x^9*e + 4/9*a*d*f^
3*x^9*e + 2/3*b*d*f^2*x^9*e^2 + 4/7*a*c*f^3*x^7*e + 6/7*b*c*f^2*x
^7*e^2 + 6/7*a*d*f^2*x^7*e^2 + 4/7*b*d*f*x^7*e^3 + 6/5*a*c*f^2*x^
5*e^2 + 4/5*b*c*f*x^5*e^3 + 4/5*a*d*f*x^5*e^3 + 1/5*b*d*x^5*e^4 +
 4/3*a*c*f*x^3*e^3 + 1/3*b*c*x^3*e^4 + 1/3*a*d*x^3*e^4 + a*c*x*e^
4
```

3.2 $\int \left(a+bx^2\right)\left(c+dx^2\right)\left(e+fx^2\right)^3\,dx$

Optimal. Leaf size=130

$$\frac{1}{3}e^2x^3(3acf+ade+bce)+\frac{1}{9}f^2x^9(adf+bcf+3bde)+\frac{1}{7}fx^7(af(cf+3de)+3be(cf+de))+\frac{1}{5}ex^5(3af(cf+de)+be(3cf+de))+ace^3x+\frac{1}{11}bdf^3x^{11}$$

```
[Out] a*c*e^3*x + (e^2*(b*c*e + a*d*e + 3*a*c*f)*x^3)/3 + (e*(3*a*f*(d*
e + c*f) + b*e*(d*e + 3*c*f))*x^5)/5 + (f*(3*b*e*(d*e + c*f) + a*
f*(3*d*e + c*f))*x^7)/7 + (f^2*(3*b*d*e + b*c*f + a*d*f)*x^9)/9 +
 (b*d*f^3*x^11)/11
```

Rubi [A] time = 0.381312, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{1}{3}e^2x^3(3acf+ade+bce)+\frac{1}{9}f^2x^9(adf+bcf+3bde)+\frac{1}{7}fx^7(af(cf+3de)+3be(cf+de))+\frac{1}{5}ex^5(3af(cf+de)+be(3cf+de))+ace^3x+\frac{1}{11}bdf^3x^{11}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^3,x]
```

```
[Out] a*c*e^3*x + (e^2*(b*c*e + a*d*e + 3*a*c*f)*x^3)/3 + (e*(3*a*f*(d*
e + c*f) + b*e*(d*e + 3*c*f))*x^5)/5 + (f*(3*b*e*(d*e + c*f) + a*
f*(3*d*e + c*f))*x^7)/7 + (f^2*(3*b*d*e + b*c*f + a*d*f)*x^9)/9 +
 (b*d*f^3*x^11)/11
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{bdf^3x^{11}}{11}+ce^3\int a\,dx+\frac{e^2x^3\left(3acf+ade+bce\right)}{3}+\frac{ex^5\left(3acf^2+3adef+3bcef+bde^2\right)}{5}+\frac{f^2x^9\left(adf+bcf+3bde\right)}{9}+\frac{fx^7\left(acf^2+3adef+3bcef+3bde^2\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)*(f*x**2+e)**3,x)
```

```
[Out] b*d*f**3*x**11/11 + c*e**3*Integral(a, x) + e**2*x**3*(3*a*c*f +
a*d*e + b*c*e)/3 + e*x**5*(3*a*c*f**2 + 3*a*d*e*f + 3*b*c*e*f + b
```

```
*d*e**2)/5 + f**2*x**9*(a*d*f + b*c*f + 3*b*d*e)/9 + f*x**7*(a*c*
f**2 + 3*a*d*e*f + 3*b*c*e*f + 3*b*d*e**2)/7
```

Mathematica [A] time = 0.120324, size = 130, normalized size = 1.

$$\frac{1}{3}e^2x^3(3acf+ade+bce)+\frac{1}{9}f^2x^9(adf+bcf+3bde)+\frac{1}{7}fx^7(af(cf+3de)+3be(cf+de))+\frac{1}{5}ex^5(3af(cf+de)+be(3cf+de))+ace^3x+\frac{1}{11}bdf^3x^{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^3,x]
```

```
[Out] a*c*e^3*x + (e^2*(b*c*e + a*d*e + 3*a*c*f)*x^3)/3 + (e*(3*a*f*(d*
e + c*f) + b*e*(d*e + 3*c*f))*x^5)/5 + (f*(3*b*e*(d*e + c*f) + a*
f*(3*d*e + c*f))*x^7)/7 + (f^2*(3*b*d*e + b*c*f + a*d*f)*x^9)/9 +
 (b*d*f^3*x^11)/11
```

Maple [A] time = 0.002, size = 135, normalized size = 1.

$$\frac{bdf^3x^{11}}{11}+\frac{\left((ad+bc)f^3+3\,bdef^2\right)x^9}{9}+\frac{\left(acf^3+3\,(ad+bc)\,ef^2+3\,bde^2f\right)x^7}{7}+\frac{\left(3\,acef^2+3\,(ad+bc)\,e^2f+bde^3\right)x^5}{5}+\frac{\left(3\,ace^2f+(ad+bc)\,e^3\right)x^3}{3}+ace^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^3,x)
```

```
[Out] 1/11*b*d*f^3*x^11+1/9*((a*d+b*c)*f^3+3*b*d*e*f^2)*x^9+1/7*(a*c*f^
3+3*(a*d+b*c)*e*f^2+3*b*d*e^2*f)*x^7+1/5*(3*a*c*e*f^2+3*(a*d+b*c)
*e^2*f+b*d*e^3)*x^5+1/3*(3*a*c*e^2*f+(a*d+b*c)*e^3)*x^3+a*c*e^3*x
```

Maxima [A] time = 1.34708, size = 181, normalized size = 1.39

$$\frac{1}{11}\,bdf^3x^{11}+\frac{1}{9}\left(3\,bdef^2+(bc+ad)f^3\right)x^9+\frac{1}{7}\left(3\,bde^2f+acf^3+3\,(bc+ad)ef^2\right)x^7+ace^3x+\frac{1}{5}\left(bde^3+3\,acef^2+3\,(bc+ad)e^2f\right)x^5+\frac{1}{3}\left(3\,ace^2f+(bc+ad)e^3\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)*(f*x^2 + e)^3,x, algorithm="maxima")
```

```
[Out] 1/11*b*d*f^3*x^11 + 1/9*(3*b*d*e*f^2 + (b*c + a*d)*f^3)*x^9 + 1/7
*(3*b*d*e^2*f + a*c*f^3 + 3*(b*c + a*d)*e*f^2)*x^7 + a*c*e^3*x +
1/5*(b*d*e^3 + 3*a*c*e*f^2 + 3*(b*c + a*d)*e^2*f)*x^5 + 1/3*(3*a*
c*e^2*f + (b*c + a*d)*e^3)*x^3
```

Fricas [A] time = 0.183825, size = 1, normalized size = 0.01

$$\frac{1}{11}x^{11}f^3db + \frac{1}{3}x^9f^2edb + \frac{1}{9}x^9f^3cb + \frac{1}{9}x^9f^3da + \frac{3}{7}x^7fe^2db + \frac{3}{7}x^7f^2ecb + \frac{3}{7}x^7f^2eda + \frac{1}{7}x^7f^3ca + \frac{1}{5}x^5e^3db + \frac{3}{5}x^5fe^2cb + \frac{3}{5}x^5fe^2da + \frac{3}{5}x^5f^2eca + \frac{1}{3}x^3e^3cb + \frac{1}{3}x^3e^3da + x^3fe^2ca + xe^3ca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)*(f*x^2 + e)^3,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*f^3*d*b + 1/3*x^9*f^2*e*d*b + 1/9*x^9*f^3*c*b + 1/9*x^9
*f^3*d*a + 3/7*x^7*f*e^2*d*b + 3/7*x^7*f^2*e*c*b + 3/7*x^7*f^2*e*
d*a + 1/7*x^7*f^3*c*a + 1/5*x^5*e^3*d*b + 3/5*x^5*f*e^2*c*b + 3/5
*x^5*f*e^2*d*a + 3/5*x^5*f^2*e*c*a + 1/3*x^3*e^3*c*b + 1/3*x^3*e^
3*d*a + x^3*f*e^2*c*a + x*e^3*c*a
```

Sympy [A] time = 0.087094, size = 173, normalized size = 1.33

$$ace^3x + \frac{bdf^3x^{11}}{11} + x^9\left(\frac{adf^3}{9} + \frac{bcf^3}{9} + \frac{bdef^2}{3}\right) + x^7\left(\frac{acf^3}{7} + \frac{3adef^2}{7} + \frac{3bcef^2}{7} + \frac{3bde^2f}{7}\right) + x^5\left(\frac{3acef^2}{5} + \frac{3ade^2f}{5} + \frac{3bce^2f}{5} + \frac{bde^3}{5}\right) + x^3\left(ace^2f + \frac{ade^3}{3} + \frac{bce^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x**2+c)*(f*x**2+e)**3,x)
```

```
[Out] a*c*e**3*x + b*d*f**3*x**11/11 + x**9*(a*d*f**3/9 + b*c*f**3/9 +
b*d*e*f**2/3) + x**7*(a*c*f**3/7 + 3*a*d*e*f**2/7 + 3*b*c*e*f**2/
7 + 3*b*d*e**2*f/7) + x**5*(3*a*c*e*f**2/5 + 3*a*d*e**2*f/5 + 3*b
*c*e**2*f/5 + b*d*e**3/5) + x**3*(a*c*e**2*f + a*d*e**3/3 + b*c*e
**3/3)
```

GIAC/XCAS [A] time = 0.229459, size = 217, normalized size = 1.67

$$\frac{1}{11}bdf^3x^{11}+\frac{1}{9}bcf^3x^9+\frac{1}{9}adf^3x^9+\frac{1}{3}bdf^2x^9e+\frac{1}{7}acf^3x^7+\frac{3}{7}bcf^2x^7e+\frac{3}{7}adf^2x^7e+\frac{3}{7}bdfx^7e^2+\frac{3}{5}acf^2x^5e+\frac{3}{5}bcfx^5e^2+\frac{3}{5}adfx^5e^2+\frac{1}{5}bdx^5e^3+acfx^3e^2+\frac{1}{3}bcx^3e^3+\frac{1}{3}adx^3e^3+acxe^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)*(f*x^2 + e)^3,x, algorithm="giac")
```

```
[Out] 1/11*b*d*f^3*x^11 + 1/9*b*c*f^3*x^9 + 1/9*a*d*f^3*x^9 + 1/3*b*d*f
^2*x^9*e + 1/7*a*c*f^3*x^7 + 3/7*b*c*f^2*x^7*e + 3/7*a*d*f^2*x^7*
e + 3/7*b*d*f*x^7*e^2 + 3/5*a*c*f^2*x^5*e + 3/5*b*c*f*x^5*e^2 + 3
/5*a*d*f*x^5*e^2 + 1/5*b*d*x^5*e^3 + a*c*f*x^3*e^2 + 1/3*b*c*x^3*
e^3 + 1/3*a*d*x^3*e^3 + a*c*x*e^3
```

3.3 $\int (a+bx^2)(c+dx^2)(e+fx^2)^2\,dx$

Optimal. Leaf size=94

$$\frac{1}{7}fx^7(adf+bcf+2bde)+\frac{1}{5}x^5(af(cf+2de)+be(2cf+de))+\frac{1}{3}ex^3(2acf+ade+bce)+ace^2x+\frac{1}{9}bdf^2x^9$$

```
[Out] a*c*e^2*x + (e*(b*c*e + a*d*e + 2*a*c*f)*x^3)/3 + ((a*f*(2*d*e +
c*f) + b*e*(d*e + 2*c*f))*x^5)/5 + (f*(2*b*d*e + b*c*f + a*d*f)*x
^7)/7 + (b*d*f^2*x^9)/9
```

Rubi [A] time = 0.249276, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{1}{7}fx^7(adf+bcf+2bde)+\frac{1}{5}x^5(af(cf+2de)+be(2cf+de))+\frac{1}{3}ex^3(2acf+ade+bce)+ace^2x+\frac{1}{9}bdf^2x^9$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^2,x]
```

```
[Out] a*c*e^2*x + (e*(b*c*e + a*d*e + 2*a*c*f)*x^3)/3 + ((a*f*(2*d*e +
c*f) + b*e*(d*e + 2*c*f))*x^5)/5 + (f*(2*b*d*e + b*c*f + a*d*f)*x
^7)/7 + (b*d*f^2*x^9)/9
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{bdf^2x^9}{9}+ce^2\int a\,dx+\frac{ex^3\left(2acf+ade+bce\right)}{3}+\frac{fx^7\left(adf+bcf+2bde\right)}{7}+x^5\left(\frac{acf^2}{5}+\frac{2adef}{5}+\frac{2bcef}{5}+\frac{bde^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)*(f*x**2+e)**2,x)
```

```
[Out] b*d*f**2*x**9/9 + c*e**2*Integral(a, x) + e*x**3*(2*a*c*f + a*d*e
 + b*c*e)/3 + f*x**7*(a*d*f + b*c*f + 2*b*d*e)/7 + x**5*(a*c*f**2
/5 + 2*a*d*e*f/5 + 2*b*c*e*f/5 + b*d*e**2/5)
```

Mathematica [A] time = 0.0703636, size = 96, normalized size = 1.02

$$\frac{1}{5}x^5\left(acf^2+2adef+2bcef+bde^2\right)+\frac{1}{7}fx^7(adf+bcf+2bde)+\frac{1}{3}ex^3(2acf+ade+bce)+ace^2x+\frac{1}{9}bdf^2x^9$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^2,x]
```

```
[Out] a*c*e^2*x + (e*(b*c*e + a*d*e + 2*a*c*f)*x^3)/3 + ((b*d*e^2 + 2*b
*c*e*f + 2*a*d*e*f + a*c*f^2)*x^5)/5 + (f*(2*b*d*e + b*c*f + a*d*
f)*x^7)/7 + (b*d*f^2*x^9)/9
```

Maple [A] time = 0.002, size = 94, normalized size = 1.

$$\frac{bdf^2x^9}{9}+\frac{\left((ad+bc)\,f^2+2\,bdef\right)\,x^7}{7}+\frac{\left(acf^2+2\,(ad+bc)\,ef+bde^2\right)\,x^5}{5}+\frac{\left(2\,acef+(ad+bc)\,e^2\right)\,x^3}{3}+ace^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^2,x)
```

```
[Out] 1/9*b*d*f^2*x^9+1/7*((a*d+b*c)*f^2+2*b*d*e*f)*x^7+1/5*(a*c*f^2+2*
(a*d+b*c)*e*f+b*d*e^2)*x^5+1/3*(2*a*c*e*f+(a*d+b*c)*e^2)*x^3+a*c*
e^2*x
```

Maxima [A] time = 1.35125, size = 126, normalized size = 1.34

$$\frac{1}{9}\,bdf^2x^9+\frac{1}{7}\left(2\,bdef+(bc+ad)f^2\right)x^7+\frac{1}{5}\left(bde^2+acf^2+2\,(bc+ad)ef\right)x^5+ace^2x+\frac{1}{3}\left(2\,acef+(bc+ad)e^2\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)*(f*x^2 + e)^2,x, algorithm="maxima")
```

```
[Out] 1/9*b*d*f^2*x^9 + 1/7*(2*b*d*e*f + (b*c + a*d)*f^2)*x^7 + 1/5*(b*
d*e^2 + a*c*f^2 + 2*(b*c + a*d)*e*f)*x^5 + a*c*e^2*x + 1/3*(2*a*c
*e*f + (b*c + a*d)*e^2)*x^3
```

Fricas [A] time = 0.182044, size = 1, normalized size = 0.01

$$\frac{1}{9}x^9f^2db+\frac{2}{7}x^7fedb+\frac{1}{7}x^7f^2cb+\frac{1}{7}x^7f^2da+\frac{1}{5}x^5e^2db+\frac{2}{5}x^5fecb+\frac{2}{5}x^5feda+\frac{1}{5}x^5f^2ca+\frac{1}{3}x^3e^2cb+\frac{1}{3}x^3e^2da+\frac{2}{3}x^3feca+xe^2ca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)*(f*x^2 + e)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*f^2*d*b + 2/7*x^7*f*e*d*b + 1/7*x^7*f^2*c*b + 1/7*x^7*f^2
*d*a + 1/5*x^5*e^2*d*b + 2/5*x^5*f*e*c*b + 2/5*x^5*f*e*d*a + 1/5*
x^5*f^2*c*a + 1/3*x^3*e^2*c*b + 1/3*x^3*e^2*d*a + 2/3*x^3*f*e*c*a
 + x*e^2*c*a
```

Sympy [A] time = 0.074369, size = 121, normalized size = 1.29

$$ace^2x + \frac{bdf^2x^9}{9} + x^7\left(\frac{adf^2}{7} + \frac{bcf^2}{7} + \frac{2bdef}{7}\right) + x^5\left(\frac{acf^2}{5} + \frac{2adef}{5} + \frac{2bcef}{5} + \frac{bde^2}{5}\right) + x^3\left(\frac{2acef}{3} + \frac{ade^2}{3} + \frac{bce^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x**2+c)*(f*x**2+e)**2,x)
```

```
[Out] a*c*e**2*x + b*d*f**2*x**9/9 + x**7*(a*d*f**2/7 + b*c*f**2/7 + 2*
b*d*e*f/7) + x**5*(a*c*f**2/5 + 2*a*d*e*f/5 + 2*b*c*e*f/5 + b*d*e
**2/5) + x**3*(2*a*c*e*f/3 + a*d*e**2/3 + b*c*e**2/3)
```

GIAC/XCAS [A] time = 0.225645, size = 154, normalized size = 1.64

$$\frac{1}{9}\,bdf^2x^9 + \frac{1}{7}\,bcf^2x^7 + \frac{1}{7}\,adf^2x^7 + \frac{2}{7}\,bdfx^7e + \frac{1}{5}\,acf^2x^5 + \frac{2}{5}\,bcfx^5e + \frac{2}{5}\,adfx^5e + \frac{1}{5}\,bdx^5e^2 + \frac{2}{3}\,acfx^3e + \frac{1}{3}\,bcx^3e^2 + \frac{1}{3}\,adx^3e^2 + acxe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)*(f*x^2 + e)^2,x, algorithm="giac")
```

```
[Out] 1/9*b*d*f^2*x^9 + 1/7*b*c*f^2*x^7 + 1/7*a*d*f^2*x^7 + 2/7*b*d*f*x
^7*e + 1/5*a*c*f^2*x^5 + 2/5*b*c*f*x^5*e + 2/5*a*d*f*x^5*e + 1/5*
b*d*x^5*e^2 + 2/3*a*c*f*x^3*e + 1/3*b*c*x^3*e^2 + 1/3*a*d*x^3*e^2
 + a*c*x*e^2
```

3.4 $\int \left(a+bx^2\right) \left(c+dx^2\right) \left(e+fx^2\right) \, dx$

Optimal. Leaf size=56

$$\frac{1}{5}x^5(adf+bcf+bde)+\frac{1}{3}x^3(acf+ade+bce)+acex+\frac{1}{7}bdfx^7$$

```
[Out] a*c*e*x + ((b*c*e + a*d*e + a*c*f)*x^3)/3 + ((b*d*e + b*c*f + a*d
*f)*x^5)/5 + (b*d*f*x^7)/7
```

Rubi [A] time = 0.115759, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$\frac{1}{5}x^5(adf+bcf+bde)+\frac{1}{3}x^3(acf+ade+bce)+acex+\frac{1}{7}bdfx^7$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)*(c + d*x^2)*(e + f*x^2),x]
```

```
[Out] a*c*e*x + ((b*c*e + a*d*e + a*c*f)*x^3)/3 + ((b*d*e + b*c*f + a*d
*f)*x^5)/5 + (b*d*f*x^7)/7
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{bdfx^7}{7}+ce\int a\,dx+x^5\left(\frac{adf}{5}+\frac{bcf}{5}+\frac{bde}{5}\right)+x^3\left(\frac{acf}{3}+\frac{ade}{3}+\frac{bce}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)*(f*x**2+e),x)
```

```
[Out] b*d*f*x**7/7 + c*e*Integral(a, x) + x**5*(a*d*f/5 + b*c*f/5 + b*d
*e/5) + x**3*(a*c*f/3 + a*d*e/3 + b*c*e/3)
```

Mathematica [A] time = 0.0251049, size = 56, normalized size = 1.

$$\frac{1}{5}x^5(adf+bcf+bde)+\frac{1}{3}x^3(acf+ade+bce)+acex+\frac{1}{7}bdfx^7$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)*(c + d*x^2)*(e + f*x^2),x]
```

```
[Out] a*c*e*x + ((b*c*e + a*d*e + a*c*f)*x^3)/3 + ((b*d*e + b*c*f + a*d
*f)*x^5)/5 + (b*d*f*x^7)/7
```

Maple [A] time = 0.001, size = 53, normalized size = 1.

$$\frac{bdfx^7}{7} + \frac{((ad+bc)f+bde)x^5}{5} + \frac{(acf+(ad+bc)e)x^3}{3} + acex$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)*(f*x^2+e),x)
```

```
[Out] 1/7*b*d*f*x^7+1/5*((a*d+b*c)*f+b*d*e)*x^5+1/3*(a*c*f+(a*d+b*c)*e)
*x^3+a*c*e*x
```

Maxima [A] time = 1.34925, size = 70, normalized size = 1.25

$$\frac{1}{7}bdfx^7 + \frac{1}{5}(bde+(bc+ad)f)x^5 + acex + \frac{1}{3}(acf+(bc+ad)e)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)*(f*x^2 + e),x, algorithm="maxima")
```

```
[Out] 1/7*b*d*f*x^7 + 1/5*(b*d*e + (b*c + a*d)*f)*x^5 + a*c*e*x + 1/3*(
a*c*f + (b*c + a*d)*e)*x^3
```

Fricas [A] time = 0.18361, size = 1, normalized size = 0.02

$$\frac{1}{7}x^7fdb + \frac{1}{5}x^5edb + \frac{1}{5}x^5fcb + \frac{1}{5}x^5fda + \frac{1}{3}x^3ecb + \frac{1}{3}x^3eda + \frac{1}{3}x^3fca + xeca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)*(f*x^2 + e),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*f*d*b + 1/5*x^5*e*d*b + 1/5*x^5*f*c*b + 1/5*x^5*f*d*a + 1
/3*x^3*e*c*b + 1/3*x^3*e*d*a + 1/3*x^3*f*c*a + x*e*c*a
```

Sympy [A] time = 0.055704, size = 63, normalized size = 1.12

$$acex + \frac{bdfx^7}{7} + x^5\left(\frac{adf}{5} + \frac{bcf}{5} + \frac{bde}{5}\right) + x^3\left(\frac{acf}{3} + \frac{ade}{3} + \frac{bce}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x**2+c)*(f*x**2+e),x)
```

```
[Out] a*c*e*x + b*d*f*x**7/7 + x**5*(a*d*f/5 + b*c*f/5 + b*d*e/5) + x**
3*(a*c*f/3 + a*d*e/3 + b*c*e/3)
```

GIAC/XCAS [A] time = 0.227485, size = 89, normalized size = 1.59

$$\frac{1}{7}\,bdfx^7 + \frac{1}{5}\,bcfx^5 + \frac{1}{5}\,adfx^5 + \frac{1}{5}\,bdx^5e + \frac{1}{3}\,acfx^3 + \frac{1}{3}\,bcx^3e + \frac{1}{3}\,adx^3e + acxe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)*(f*x^2 + e),x, algorithm="giac")
```

```
[Out] 1/7*b*d*f*x^7 + 1/5*b*c*f*x^5 + 1/5*a*d*f*x^5 + 1/5*b*d*x^5*e + 1
/3*a*c*f*x^3 + 1/3*b*c*x^3*e + 1/3*a*d*x^3*e + a*c*x*e
```

3.5 $\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2}\,dx$

Optimal. Leaf size=81

$$\frac{(be-af)(de-cf)\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{5/2}}-\frac{x(-2adf-3bcf+3bde)}{3f^2}+\frac{dx\left(a+bx^2\right)}{3f}$$

```
[Out] -((3*b*d*e - 3*b*c*f - 2*a*d*f)*x)/(3*f^2) + (d*x*(a + b*x^2))/(3
*f) + ((b*e - a*f)*(d*e - c*f)*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt
[e]*f^(5/2))
```

Rubi [A] time = 0.206959, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{(be-af)(de-cf)\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{5/2}}-\frac{x(-2adf-3bcf+3bde)}{3f^2}+\frac{dx\left(a+bx^2\right)}{3f}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*(c + d*x^2))/(e + f*x^2),x]
```

```
[Out] -((3*b*d*e - 3*b*c*f - 2*a*d*f)*x)/(3*f^2) + (d*x*(a + b*x^2))/(3
*f) + ((b*e - a*f)*(d*e - c*f)*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt
[e]*f^(5/2))
```

Rubi in Sympy [A] time = 25.9735, size = 76, normalized size = 0.94

$$\frac{dx\left(a+bx^2\right)}{3f}+\frac{x\left(2adf+3bcf-3bde\right)}{3f^2}+\frac{(af-be)\left(cf-de\right)\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)/(f*x**2+e),x)
```

```
[Out] d*x*(a + b*x**2)/(3*f) + x*(2*a*d*f + 3*b*c*f - 3*b*d*e)/(3*f**2)
 + (a*f - b*e)*(c*f - d*e)*atan(sqrt(f)*x/sqrt(e))/(sqrt(e)*f**(5
/2))
```

Mathematica [A] time = 0.107295, size = 72, normalized size = 0.89

$$\frac{(be-af)(de-cf)\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{5/2}}+\frac{x(adf+bcf-bde)}{f^2}+\frac{bdx^3}{3f}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*(c + d*x^2))/(e + f*x^2),x]
```

```
[Out] ((-(b*d*e) + b*c*f + a*d*f)*x)/f^2 + (b*d*x^3)/(3*f) + ((b*e - a*
f)*(d*e - c*f)*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(5/2))
```

Maple [A] time = 0.009, size = 119, normalized size = 1.5

$$\frac{x^3bd}{3f}+\frac{axd}{f}+\frac{bxc}{f}-\frac{bxde}{f^2}+ac\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}-\frac{aed}{f}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}-\frac{bce}{f}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}+\frac{bde^2}{f^2}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)/(f*x^2+e),x)
```

```
[Out] 1/3/f*x^3*b*d+1/f*x*a*d+1/f*x*b*c-1/f^2*x*b*d*e+1/(e*f)^(1/2)*arc
tan(x*f/(e*f)^(1/2))*a*c-1/f/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*
a*d*e-1/f/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*c*e+1/f^2/(e*f)^(
1/2)*arctan(x*f/(e*f)^(1/2))*b*d*e^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)/(f*x^2 + e),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.217235, size = 1, normalized size = 0.01

$$\left[\frac{3\,\left(bde^{2}+acf^{2}-(bc+ad)ef\right)\log\left(\frac{2\,efx+(fx^{2}-e)\sqrt{-ef}}{fx^{2}+e}\right)+2\,\left(bdfx^{3}-3\,(bde-(bc+ad)f)x\right)\sqrt{-ef}}{6\,\sqrt{-ef}f^{2}},\ \frac{3\,\left(bde^{2}+acf^{2}-(bc+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)/(f*x^2 + e),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b*d*e^2 + a*c*f^2 - (b*c + a*d)*e*f)*log((2*e*f*x + (f*x
^2 - e)*sqrt(-e*f))/(f*x^2 + e)) + 2*(b*d*f*x^3 - 3*(b*d*e - (b*c
 + a*d)*f)*x)*sqrt(-e*f))/(sqrt(-e*f)*f^2), 1/3*(3*(b*d*e^2 + a*c
*f^2 - (b*c + a*d)*e*f)*arctan(sqrt(e*f)*x/e) + (b*d*f*x^3 - 3*(b
*d*e - (b*c + a*d)*f)*x)*sqrt(e*f))/(sqrt(e*f)*f^2)]
```

Sympy [A] time = 1.62053, size = 206, normalized size = 2.54

$$\frac{bdx^{3}}{3f}-\frac{\sqrt{-\frac{1}{ef^{5}}}\,(af-be)\,(cf-de)\log\left(-\frac{ef^{2}\sqrt{-\frac{1}{ef^{5}}}(af-be)(cf-de)}{acf^{2}-adef-bcef+bde^{2}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ef^{5}}}\,(af-be)\,(cf-de)\log\left(\frac{ef^{2}\sqrt{-\frac{1}{ef^{5}}}(af-be)(cf-de)}{acf^{2}-adef-bcef+bde^{2}}+x\right)}{2}+\frac{x\,(adf+bcf-bde)}{f^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x**2+c)/(f*x**2+e),x)
```

```
[Out] b*d*x**3/(3*f) - sqrt(-1/(e*f**5))*(a*f - b*e)*(c*f - d*e)*log(-e
*f**2*sqrt(-1/(e*f**5))*(a*f - b*e)*(c*f - d*e)/(a*c*f**2 - a*d*e
*f - b*c*e*f + b*d*e**2) + x)/2 + sqrt(-1/(e*f**5))*(a*f - b*e)*(
c*f - d*e)*log(e*f**2*sqrt(-1/(e*f**5))*(a*f - b*e)*(c*f - d*e)/(
a*c*f**2 - a*d*e*f - b*c*e*f + b*d*e**2) + x)/2 + x*(a*d*f + b*c*
f - b*d*e)/f**2
```

GIAC/XCAS [A] time = 0.227239, size = 108, normalized size = 1.33

$$\frac{\left(acf^{2}-bcfe-adfe+bde^{2}\right)\arctan\left(\sqrt{f}xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{1}{2}\right)}}{f^{\frac{5}{2}}}+\frac{bdf^{2}x^{3}+3\,bcf^{2}x+3\,adf^{2}x-3\,bdfxe}{3\,f^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)/(f*x^2 + e),x, algorithm="giac")
```

```
[Out] (a*c*f^2 - b*c*f*e - a*d*f*e + b*d*e^2)*arctan(sqrt(f)*x*e^(-1/2)
)*e^(-1/2)/f^(5/2) + 1/3*(b*d*f^2*x^3 + 3*b*c*f^2*x + 3*a*d*f^2*x
 - 3*b*d*f*x*e)/f^3
```

3.6 $\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^2}\,dx$

Optimal. Leaf size=108

$$-\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(be(3de-cf)-af(cf+de))}{2e^{3/2}f^{5/2}}-\frac{x(a+bx^2)(de-cf)}{2ef(e+fx^2)}+\frac{bx(3de-cf)}{2ef^2}$$

```
[Out] (b*(3*d*e - c*f)*x)/(2*e*f^2) - ((d*e - c*f)*x*(a + b*x^2))/(2*e*
f*(e + f*x^2)) - ((b*e*(3*d*e - c*f) - a*f*(d*e + c*f))*ArcTan[(S
qrt[f]*x)/Sqrt[e]])/(2*e^(3/2)*f^(5/2))
```

Rubi [A] time = 0.244144, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(be(3de-cf)-af(cf+de))}{2e^{3/2}f^{5/2}}-\frac{x(a+bx^2)(de-cf)}{2ef(e+fx^2)}+\frac{bx(3de-cf)}{2ef^2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^2,x]
```

```
[Out] (b*(3*d*e - c*f)*x)/(2*e*f^2) - ((d*e - c*f)*x*(a + b*x^2))/(2*e*
f*(e + f*x^2)) - ((b*e*(3*d*e - c*f) - a*f*(d*e + c*f))*ArcTan[(S
qrt[f]*x)/Sqrt[e]])/(2*e^(3/2)*f^(5/2))
```

Rubi in Sympy [A] time = 27.5111, size = 92, normalized size = 0.85

$$-\frac{bx(cf-3de)}{2ef^2}+\frac{x(a+bx^2)(cf-de)}{2ef(e+fx^2)}+\frac{(af(cf+de)+be(cf-3de))\,\mathrm{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{\frac{3}{2}}f^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)/(f*x**2+e)**2,x)
```

```
[Out] -b*x*(c*f - 3*d*e)/(2*e*f**2) + x*(a + b*x**2)*(c*f - d*e)/(2*e*f
*(e + f*x**2)) + (a*f*(c*f + d*e) + b*e*(c*f - 3*d*e))*atan(sqrt(
f)*x/sqrt(e))/(2*e**(3/2)*f**(5/2))
```

Mathematica [A] time = 0.127185, size = 95, normalized size = 0.88

$$-\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(be(3de-cf)-af(cf+de))}{2e^{3/2}f^{5/2}}+\frac{x(be-af)(de-cf)}{2ef^2(e+fx^2)}+\frac{bdx}{f^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^2,x]
```

```
[Out] (b*d*x)/f^2 + ((b*e - a*f)*(d*e - c*f)*x)/(2*e*f^2*(e + f*x^2)) -
 ((b*e*(3*d*e - c*f) - a*f*(d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e
]])/(2*e^(3/2)*f^(5/2))
```

Maple [A] time = 0.013, size = 163, normalized size = 1.5

$$\frac{bdx}{f^2}+\frac{axc}{2\,e\,(fx^2+e)}-\frac{axd}{2\,f\,(fx^2+e)}-\frac{bxc}{2\,f\,(fx^2+e)}+\frac{exbd}{2\,f^2\,(fx^2+e)}+\frac{ac}{2\,e}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}+\frac{ad}{2\,f}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}+\frac{bc}{2\,f}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}-\frac{3\,bde}{2\,f^2}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^2,x)
```

```
[Out] b*d/f^2*x+1/2/e*x/(f*x^2+e)*a*c-1/2/f*x/(f*x^2+e)*a*d-1/2/f*x/(f*
x^2+e)*b*c+1/2/f^2*e*x/(f*x^2+e)*b*d+1/2/e/(e*f)^(1/2)*arctan(x*f
/(e*f)^(1/2))*a*c+1/2/f/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a*d+1
/2/f/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*c-3/2/f^2*e/(e*f)^(1/2
)*arctan(x*f/(e*f)^(1/2))*b*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)/(f*x^2 + e)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.218307, size = 1, normalized size = 0.01

$$\left[-\frac{\left(3\,bde^3 - acef^2 - (bc+ad)e^2f + \left(3\,bde^2f - acf^3 - (bc+ad)ef^2\right)x^2\right)\log\left(\frac{2\,efx + (fx^2-e)\sqrt{-ef}}{fx^2+e}\right) - 2\left(2\,bdefx^3 + \left(3\,bde^2\right.\right.}{4\left(ef^3x^2 + e^2f^2\right)\sqrt{-ef}}\right.$$

$$-\frac{\left(3\,bde^3 - acef^2 - (bc+ad)e^2f + \left(3\,bde^2f - acf^3 - (bc+ad)ef^2\right)x^2\right)\arctan\left(\frac{\sqrt{ef}x}{e}\right) - \left(2\,bdefx^3 + \left(3\,bde^2 + acf^2 - (b\right.\right.}{2\left(ef^3x^2 + e^2f^2\right)\sqrt{ef}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)/(f*x^2 + e)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((3*b*d*e^3 - a*c*e*f^2 - (b*c + a*d)*e^2*f + (3*b*d*e^2*f
- a*c*f^3 - (b*c + a*d)*e*f^2)*x^2)*log((2*e*f*x + (f*x^2 - e)*sq
rt(-e*f))/(f*x^2 + e)) - 2*(2*b*d*e*f*x^3 + (3*b*d*e^2 + a*c*f^2
- (b*c + a*d)*e*f)*x)*sqrt(-e*f))/((e*f^3*x^2 + e^2*f^2)*sqrt(-e*
f)), -1/2*((3*b*d*e^3 - a*c*e*f^2 - (b*c + a*d)*e^2*f + (3*b*d*e^
2*f - a*c*f^3 - (b*c + a*d)*e*f^2)*x^2)*arctan(sqrt(e*f)*x/e) - (
2*b*d*e*f*x^3 + (3*b*d*e^2 + a*c*f^2 - (b*c + a*d)*e*f)*x)*sqrt(e
*f))/((e*f^3*x^2 + e^2*f^2)*sqrt(e*f))]
```

Sympy [A] time = 3.17267, size = 190, normalized size = 1.76

$$\frac{bdx}{f^2} + \frac{x\left(acf^2 - adef - bcef + bde^2\right)}{2e^2f^2 + 2ef^3x^2} - \frac{\sqrt{-\frac{1}{e^3f^5}}\left(acf^2 + adef + bcef - 3bde^2\right)\log\left(-e^2f^2\sqrt{-\frac{1}{e^3f^5}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{e^3f^5}}\left(acf^2 + adef + bcef - 3bde^2\right)\log\left(e^2f^2\sqrt{-\frac{1}{e^3f^5}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)/(f*x**2+e)**2,x)
```

```
[Out] b*d*x/f**2 + x*(a*c*f**2 - a*d*e*f - b*c*e*f + b*d*e**2)/(2*e**2*
f**2 + 2*e*f**3*x**2) - sqrt(-1/(e**3*f**5))*(a*c*f**2 + a*d*e*f
+ b*c*e*f - 3*b*d*e**2)*log(-e**2*f**2*sqrt(-1/(e**3*f**5)) + x)/
4 + sqrt(-1/(e**3*f**5))*(a*c*f**2 + a*d*e*f + b*c*e*f - 3*b*d*e*
*2)*log(e**2*f**2*sqrt(-1/(e**3*f**5)) + x)/4
```

GIAC/XCAS [A] time = 0.229069, size = 128, normalized size = 1.19

$$\frac{bdx}{f^2} + \frac{\left(acf^2 + bcfe + adfe - 3\,bde^2\right)\arctan\left(\sqrt{f}xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{3}{2}\right)}}{2\,f^{\frac{5}{2}}} + \frac{\left(acf^2x - bcfxe - adfxe + bdxe^2\right)e^{(-1)}}{2\,(fx^2 + e)f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)/(f*x^2 + e)^2,x, algorithm="giac")
```

```
[Out] b*d*x/f^2 + 1/2*(a*c*f^2 + b*c*f*e + a*d*f*e - 3*b*d*e^2)*arctan(
sqrt(f)*x*e^(-1/2))*e^(-3/2)/f^(5/2) + 1/2*(a*c*f^2*x - b*c*f*x*e
 - a*d*f*x*e + b*d*x*e^2)*e^(-1)/((f*x^2 + e)*f^2)
```

3.7 $\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^3}\,dx$

Optimal. Leaf size=130

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(3cf+de)+be(cf+3de))}{8e^{5/2}f^{5/2}}-\frac{x(be(cf+3de)-af(3cf+de))}{8e^2f^2(e+fx^2)}-\frac{x(a+bx^2)(de-cf)}{4ef(e+fx^2)^2}$$

```
[Out] -((d*e - c*f)*x*(a + b*x^2))/(4*e*f*(e + f*x^2)^2) - ((b*e*(3*d*e
 + c*f) - a*f*(d*e + 3*c*f))*x)/(8*e^2*f^2*(e + f*x^2)) + ((b*e*(
3*d*e + c*f) + a*f*(d*e + 3*c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(8
*e^(5/2)*f^(5/2))
```

Rubi [A] time = 0.313099, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(3cf+de)+be(cf+3de))}{8e^{5/2}f^{5/2}}-\frac{x(be(cf+3de)-af(3cf+de))}{8e^2f^2(e+fx^2)}-\frac{x(a+bx^2)(de-cf)}{4ef(e+fx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^3,x]
```

```
[Out] -((d*e - c*f)*x*(a + b*x^2))/(4*e*f*(e + f*x^2)^2) - ((b*e*(3*d*e
 + c*f) - a*f*(d*e + 3*c*f))*x)/(8*e^2*f^2*(e + f*x^2)) + ((b*e*(
3*d*e + c*f) + a*f*(d*e + 3*c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(8
*e^(5/2)*f^(5/2))
```

Rubi in Sympy [A] time = 29.8022, size = 117, normalized size = 0.9

$$\frac{x\left(a+bx^2\right)(cf-de)}{4ef\left(e+fx^2\right)^2}+\frac{x\left(af\left(3cf+de\right)-be\left(cf+3de\right)\right)}{8e^2f^2\left(e+fx^2\right)}+\frac{\left(af\left(3cf+de\right)+be\left(cf+3de\right)\right)\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{8e^{\frac{5}{2}}f^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)/(f*x**2+e)**3,x)
```

```
[Out] x*(a + b*x**2)*(c*f - d*e)/(4*e*f*(e + f*x**2)**2) + x*(a*f*(3*c*
f + d*e) - b*e*(c*f + 3*d*e))/(8*e**2*f**2*(e + f*x**2)) + (a*f*(
3*c*f + d*e) + b*e*(c*f + 3*d*e))*atan(sqrt(f)*x/sqrt(e))/(8*e**(
```

```
5/2)*f**(5/2))
```

Mathematica [A] time = 0.153391, size = 130, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(3cf+de)+be(cf+3de))}{8e^{5/2}f^{5/2}}+\frac{x(af(3cf+de)+be(cf-5de))}{8e^2f^2\left(e+fx^2\right)}+\frac{x(be-af)(de-cf)}{4ef^2\left(e+fx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^3,x]
```

```
[Out] ((b*e - a*f)*(d*e - c*f)*x)/(4*e*f^2*(e + f*x^2)^2) + ((b*e*(-5*d
*e + c*f) + a*f*(d*e + 3*c*f))*x)/(8*e^2*f^2*(e + f*x^2)) + ((b*e
*(3*d*e + c*f) + a*f*(d*e + 3*c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/
(8*e^(5/2)*f^(5/2))
```

Maple [A] time = 0.011, size = 175, normalized size = 1.4

$$\frac{1}{\left(fx^2+e\right)^2}\left(\frac{\left(3\,acf^2+aedf+bcef-5\,bde^2\right)x^3}{8\,e^2f}+\frac{\left(5\,acf^2-aedf-bcef-3\,bde^2\right)x}{8\,f^2e}\right)+\frac{3\,ac}{8\,e^2}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}+\frac{ad}{8\,ef}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}+\frac{bc}{8\,ef}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}+\frac{3\,bd}{8\,f^2}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^3,x)
```

```
[Out] (1/8*(3*a*c*f^2+a*d*e*f+b*c*e*f-5*b*d*e^2)/e^2/f*x^3+1/8*(5*a*c*f
^2-a*d*e*f-b*c*e*f-3*b*d*e^2)/f^2/e*x)/(f*x^2+e)^2+3/8/e^2/(e*f)^
(1/2)*arctan(x*f/(e*f)^(1/2))*a*c+1/8/e/f/(e*f)^(1/2)*arctan(x*f/
(e*f)^(1/2))*a*d+1/8/e/f/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*c+
3/8/f^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)/(f*x^2 + e)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.217924, size = 1, normalized size = 0.01

$$\left[\frac{\left(3\,bde^4+3\,ace^2f^2+(bc+ad)e^3f+\left(3\,bde^2f^2+3\,acf^4+(bc+ad)ef^3\right)x^4+2\left(3\,bde^3f+3\,acef^3+(bc+ad)e^2f^2\right)x^2\right)\log}{16\left(e^2f^4x^4+2\,e^3f^3x^2+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)/(f*x^2 + e)^3,x, algorithm="fricas")
```

```
[Out] [1/16*((3*b*d*e^4 + 3*a*c*e^2*f^2 + (b*c + a*d)*e^3*f + (3*b*d*e^
2*f^2 + 3*a*c*f^4 + (b*c + a*d)*e*f^3)*x^4 + 2*(3*b*d*e^3*f + 3*a
*c*e*f^3 + (b*c + a*d)*e^2*f^2)*x^2)*log((2*e*f*x + (f*x^2 - e)*s
qrt(-e*f))/(f*x^2 + e)) - 2*((5*b*d*e^2*f - 3*a*c*f^3 - (b*c + a*
d)*e*f^2)*x^3 + (3*b*d*e^3 - 5*a*c*e*f^2 + (b*c + a*d)*e^2*f)*x)*
sqrt(-e*f))/((e^2*f^4*x^4 + 2*e^3*f^3*x^2 + e^4*f^2)*sqrt(-e*f)),
 1/8*((3*b*d*e^4 + 3*a*c*e^2*f^2 + (b*c + a*d)*e^3*f + (3*b*d*e^2
*f^2 + 3*a*c*f^4 + (b*c + a*d)*e*f^3)*x^4 + 2*(3*b*d*e^3*f + 3*a*
c*e*f^3 + (b*c + a*d)*e^2*f^2)*x^2)*arctan(sqrt(e*f)*x/e) - ((5*b
*d*e^2*f - 3*a*c*f^3 - (b*c + a*d)*e*f^2)*x^3 + (3*b*d*e^3 - 5*a*
c*e*f^2 + (b*c + a*d)*e^2*f)*x)*sqrt(e*f))/((e^2*f^4*x^4 + 2*e^3*
f^3*x^2 + e^4*f^2)*sqrt(e*f))]
```

Sympy [A] time = 7.20104, size = 246, normalized size = 1.89

$$-\frac{\sqrt{-\frac{1}{e^5f^5}}\left(3acf^2+adef+bcef+3bde^2\right)\log\left(-e^3f^2\sqrt{-\frac{1}{e^5f^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{e^5f^5}}\left(3acf^2+adef+bcef+3bde^2\right)\log\left(e^3f^2\sqrt{-\frac{1}{e^5f^5}}+x\right)}{16}+\frac{x^3\left(3acf^3+adef^2+bcef^2-5bde^2f\right)+x\left(5acef^2-ade^2f-bce^2f-3bde^3\right)}{8e^4f^2+16e^3f^3x^2+8e^2f^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)/(f*x**2+e)**3,x)
```

```
[Out] -sqrt(-1/(e**5*f**5))*(3*a*c*f**2 + a*d*e*f + b*c*e*f + 3*b*d*e**
2)*log(-e**3*f**2*sqrt(-1/(e**5*f**5)) + x)/16 + sqrt(-1/(e**5*f*
```

```
*5))*(3*a*c*f**2 + a*d*e*f + b*c*e*f + 3*b*d*e**2)*log(e**3*f**2*
sqrt(-1/(e**5*f**5)) + x)/16 + (x**3*(3*a*c*f**3 + a*d*e*f**2 + b
*c*e*f**2 - 5*b*d*e**2*f) + x*(5*a*c*e*f**2 - a*d*e**2*f - b*c*e*
*2*f - 3*b*d*e**3))/(8*e**4*f**2 + 16*e**3*f**3*x**2 + 8*e**2*f**
4*x**4)
```

GIAC/XCAS [A] time = 0.227891, size = 182, normalized size = 1.4

$$\frac{\left(3\,acf^2 + bcfe + adfe + 3\,bde^2\right)\arctan\left(\sqrt{f}xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{5}{2}\right)}}{8\,f^{\frac{5}{2}}} + \frac{\left(3\,acf^3x^3 + bcf^2x^3e + adf^2x^3e - 5\,bdfx^3e^2 + 5\,acf^2xe - bcfxe^2 - adfxe^2 - 3\,bdxe^3\right)e^{(-2)}}{8\,(fx^2+e)^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)/(f*x^2 + e)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*a*c*f^2 + b*c*f*e + a*d*f*e + 3*b*d*e^2)*arctan(sqrt(f)*x*
e^(-1/2))*e^(-5/2)/f^(5/2) + 1/8*(3*a*c*f^3*x^3 + b*c*f^2*x^3*e +
 a*d*f^2*x^3*e - 5*b*d*f*x^3*e^2 + 5*a*c*f^2*x*e - b*c*f*x*e^2 -
a*d*f*x*e^2 - 3*b*d*x*e^3)*e^(-2)/((f*x^2 + e)^2*f^2)
```

3.8 $\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^4}\,dx$

Optimal. Leaf size=171

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(5cf+de)+be(cf+de))}{16e^{7/2}f^{5/2}}+\frac{x(af(5cf+de)+be(cf+de))}{16e^3f^2(e+fx^2)}-\frac{x(3be(cf+de)-af(5cf+de))}{24e^2f^2(e+fx^2)^2}-\frac{x(a+bx^2)(de-cf)}{6ef(e+fx^2)^3}$$

```
[Out] -((d*e - c*f)*x*(a + b*x^2))/(6*e*f*(e + f*x^2)^3) - ((3*b*e*(d*e
 + c*f) - a*f*(d*e + 5*c*f))*x)/(24*e^2*f^2*(e + f*x^2)^2) + ((b*
e*(d*e + c*f) + a*f*(d*e + 5*c*f))*x)/(16*e^3*f^2*(e + f*x^2)) +
((b*e*(d*e + c*f) + a*f*(d*e + 5*c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]
])/(16*e^(7/2)*f^(5/2))
```

Rubi [A] time = 0.479235, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(5cf+de)+be(cf+de))}{16e^{7/2}f^{5/2}}+\frac{x(af(5cf+de)+be(cf+de))}{16e^3f^2(e+fx^2)}-\frac{x(3be(cf+de)-af(5cf+de))}{24e^2f^2(e+fx^2)^2}-\frac{x(a+bx^2)(de-cf)}{6ef(e+fx^2)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^4,x]
```

```
[Out] -((d*e - c*f)*x*(a + b*x^2))/(6*e*f*(e + f*x^2)^3) - ((3*b*e*(d*e
 + c*f) - a*f*(d*e + 5*c*f))*x)/(24*e^2*f^2*(e + f*x^2)^2) + ((b*
e*(d*e + c*f) + a*f*(d*e + 5*c*f))*x)/(16*e^3*f^2*(e + f*x^2)) +
((b*e*(d*e + c*f) + a*f*(d*e + 5*c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]
])/(16*e^(7/2)*f^(5/2))
```

Rubi in Sympy [A] time = 34.6746, size = 156, normalized size = 0.91

$$\frac{x(a+bx^2)(cf-de)}{6ef(e+fx^2)^3}+\frac{x(af(5cf+de)-3be(cf+de))}{24e^2f^2(e+fx^2)^2}+\frac{x(af(5cf+de)+be(cf+de))}{16e^3f^2(e+fx^2)}+\frac{(af(5cf+de)+be(cf+de))\,\mathrm{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{16e^{\frac{7}{2}}f^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)/(f*x**2+e)**4,x)
```

```
[Out] x*(a + b*x**2)*(c*f - d*e)/(6*e*f*(e + f*x**2)**3) + x*(a*f*(5*c*
f + d*e) - 3*b*e*(c*f + d*e))/(24*e**2*f**2*(e + f*x**2)**2) + x*
(a*f*(5*c*f + d*e) + b*e*(c*f + d*e))/(16*e**3*f**2*(e + f*x**2))
 + (a*f*(5*c*f + d*e) + b*e*(c*f + d*e))*atan(sqrt(f)*x/sqrt(e))/
(16*e**(7/2)*f**(5/2))
```

Mathematica [A] time = 0.194682, size = 171, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(5cf+de)+be(cf+de))}{16e^{7/2}f^{5/2}}+\frac{x(af(5cf+de)+be(cf+de))}{16e^3f^2(e+fx^2)}+\frac{x(af(5cf+de)+be(cf-7de))}{24e^2f^2(e+fx^2)^2}+\frac{x(be-af)(de-cf)}{6ef^2(e+fx^2)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^4,x]
```

```
[Out] ((b*e - a*f)*(d*e - c*f)*x)/(6*e*f^2*(e + f*x^2)^3) + ((b*e*(-7*d
*e + c*f) + a*f*(d*e + 5*c*f))*x)/(24*e^2*f^2*(e + f*x^2)^2) + ((
b*e*(d*e + c*f) + a*f*(d*e + 5*c*f))*x)/(16*e^3*f^2*(e + f*x^2))
+ ((b*e*(d*e + c*f) + a*f*(d*e + 5*c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[
e]])/(16*e^(7/2)*f^(5/2))
```

Maple [A] time = 0.014, size = 210, normalized size = 1.2

$$\frac{1}{(fx^2+e)^3}\left(\frac{(5\,acf^2+aedf+bcef+bde^2)\,x^5}{16\,e^3}+\frac{(5\,acf^2+aedf+bcef-bde^2)\,x^3}{6\,e^2f}+\frac{(11\,acf^2-aedf-bcef-bde^2)\,x}{16\,ef^2}\right)+\frac{5\,ac}{16\,e^3}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}+\frac{ad}{16\,e^2f}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}+\frac{bc}{16\,e^2f}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}+\frac{bd}{16\,ef^2}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^4,x)
```

```
[Out] (1/16*(5*a*c*f^2+a*d*e*f+b*c*e*f+b*d*e^2)/e^3*x^5+1/6*(5*a*c*f^2+
a*d*e*f+b*c*e*f-b*d*e^2)/e^2/f*x^3+1/16*(11*a*c*f^2-a*d*e*f-b*c*e
```

```
*f-b*d*e^2)/e/f^2*x)/(f*x^2+e)^3+5/16/e^3/(e*f)^(1/2)*arctan(x*f/
(e*f)^(1/2))*a*c+1/16/e^2/f/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a
*d+1/16/e^2/f/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*c+1/16/e/f^2/
(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)/(f*x^2 + e)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.221683, size = 1, normalized size = 0.01

$$\left[\frac{3\left(bde^5 + 5\,ace^3f^2 + \left(bde^2f^3 + 5\,acf^5 + (bc + ad)ef^4\right)x^6 + (bc + ad)e^4f + 3\left(bde^3f^2 + 5\,acef^4 + (bc + ad)e^2f^3\right)x^4 + 3\left(bc\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)/(f*x^2 + e)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(3*(b*d*e^5 + 5*a*c*e^3*f^2 + (b*d*e^2*f^3 + 5*a*c*f^5 + (b
*c + a*d)*e*f^4)*x^6 + (b*c + a*d)*e^4*f + 3*(b*d*e^3*f^2 + 5*a*c
*e*f^4 + (b*c + a*d)*e^2*f^3)*x^4 + 3*(b*d*e^4*f + 5*a*c*e^2*f^3
+ (b*c + a*d)*e^3*f^2)*x^2)*log((2*e*f*x + (f*x^2 - e)*sqrt(-e*f)
)/(f*x^2 + e)) + 2*(3*(b*d*e^2*f^2 + 5*a*c*f^4 + (b*c + a*d)*e*f^
3)*x^5 - 8*(b*d*e^3*f - 5*a*c*e*f^3 - (b*c + a*d)*e^2*f^2)*x^3 -
3*(b*d*e^4 - 11*a*c*e^2*f^2 + (b*c + a*d)*e^3*f)*x)*sqrt(-e*f))/(
(e^3*f^5*x^6 + 3*e^4*f^4*x^4 + 3*e^5*f^3*x^2 + e^6*f^2)*sqrt(-e*f
)), 1/48*(3*(b*d*e^5 + 5*a*c*e^3*f^2 + (b*d*e^2*f^3 + 5*a*c*f^5 +
 (b*c + a*d)*e*f^4)*x^6 + (b*c + a*d)*e^4*f + 3*(b*d*e^3*f^2 + 5*
a*c*e*f^4 + (b*c + a*d)*e^2*f^3)*x^4 + 3*(b*d*e^4*f + 5*a*c*e^2*f
^3 + (b*c + a*d)*e^3*f^2)*x^2)*arctan(sqrt(e*f)*x/e) + (3*(b*d*e^
2*f^2 + 5*a*c*f^4 + (b*c + a*d)*e*f^3)*x^5 - 8*(b*d*e^3*f - 5*a*c
*e*f^3 - (b*c + a*d)*e^2*f^2)*x^3 - 3*(b*d*e^4 - 11*a*c*e^2*f^2 +
 (b*c + a*d)*e^3*f)*x)*sqrt(e*f))/((e^3*f^5*x^6 + 3*e^4*f^4*x^4 +
 3*e^5*f^3*x^2 + e^6*f^2)*sqrt(e*f))]
```

Sympy [A] time = 14.9241, size = 313, normalized size = 1.83

$$-\frac{\sqrt{-\frac{1}{e^7f^5}}\left(5acf^2+adef+bcef+bde^2\right)\log\left(-e^4f^2\sqrt{-\frac{1}{e^7f^5}}+x\right)}{32}$$
$$+\frac{\sqrt{-\frac{1}{e^7f^5}}\left(5acf^2+adef+bcef+bde^2\right)\log\left(e^4f^2\sqrt{-\frac{1}{e^7f^5}}+x\right)}{32}$$
$$+\frac{x^5\left(15acf^4+3adef^3+3bcef^3+3bde^2f^2\right)+x^3\left(40acef^3+8ade^2f^2+8bce^2f^2-8bde^3f\right)+x\left(33ace^2f^2-3ade^3f-3bc\right.}{48e^6f^2+144e^5f^3x^2+144e^4f^4x^4+48e^3f^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x**2+c)/(f*x**2+e)**4,x)
```

```
[Out] -sqrt(-1/(e**7*f**5))*(5*a*c*f**2 + a*d*e*f + b*c*e*f + b*d*e**2)
*log(-e**4*f**2*sqrt(-1/(e**7*f**5)) + x)/32 + sqrt(-1/(e**7*f**5
))*(5*a*c*f**2 + a*d*e*f + b*c*e*f + b*d*e**2)*log(e**4*f**2*sqrt
(-1/(e**7*f**5)) + x)/32 + (x**5*(15*a*c*f**4 + 3*a*d*e*f**3 + 3*
b*c*e*f**3 + 3*b*d*e**2*f**2) + x**3*(40*a*c*e*f**3 + 8*a*d*e**2*
f**2 + 8*b*c*e**2*f**2 - 8*b*d*e**3*f) + x*(33*a*c*e**2*f**2 - 3*
a*d*e**3*f - 3*b*c*e**3*f - 3*b*d*e**4))/(48*e**6*f**2 + 144*e**5
*f**3*x**2 + 144*e**4*f**4*x**4 + 48*e**3*f**5*x**6)
```

GIAC/XCAS [A] time = 0.231918, size = 248, normalized size = 1.45

$$\frac{\left(5\,acf^2+bcfe+adfe+bde^2\right)\arctan\left(\sqrt{f}xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{7}{2}\right)}}{16\,f^{\frac{5}{2}}}$$
$$+\frac{\left(15\,acf^4x^5+3\,bcf^3x^5e+3\,adf^3x^5e+3\,bdf^2x^5e^2+40\,acf^3x^3e+8\,bcf^2x^3e^2+8\,adf^2x^3e^2-8\,bdfx^3e^3+33\,acf^2xe^2-3\right.}{48\left(fx^2+e\right)^3f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)/(f*x^2 + e)^4,x, algorithm="giac")
```

```
[Out] 1/16*(5*a*c*f^2 + b*c*f*e + a*d*f*e + b*d*e^2)*arctan(sqrt(f)*x*e
^(-1/2))*e^(-7/2)/f^(5/2) + 1/48*(15*a*c*f^4*x^5 + 3*b*c*f^3*x^5*
e + 3*a*d*f^3*x^5*e + 3*b*d*f^2*x^5*e^2 + 40*a*c*f^3*x^3*e + 8*b*
c*f^2*x^3*e^2 + 8*a*d*f^2*x^3*e^2 - 8*b*d*f*x^3*e^3 + 33*a*c*f^2*
x*e^2 - 3*b*c*f*x*e^3 - 3*a*d*f*x*e^3 - 3*b*d*x*e^4)*e^(-3)/((f*x
^2 + e)^3*f^2)
```

3.9 $\int \left(a+bx^2\right) \left(c+dx^2\right)^2 \left(e+fx^2\right)^3 \, dx$

Optimal. Leaf size=226

$$\frac{1}{9}fx^9\left(adf(2cf+3de)+b\left(c^2f^2+6cdef+3d^2e^2\right)\right)+\frac{1}{7}x^7\left(af\left(c^2f^2+6cdef+3d^2e^2\right)+be\left(3c^2f^2+6cdef+d^2e^2\right)\right)+\frac{1}{5}ex^5\left(a\left(3c^2f^2+6cdef+d^2e^2\right)+bce(3cf+2de)\right)+\frac{1}{3}ce^2x^3(3acf+2ade+bce)+\frac{1}{11}df^2x^{11}(adf+2bcf+3bde)+ac^2e^3x+\frac{1}{13}bd^2f^3x^{13}$$

```
[Out] a*c^2*e^3*x + (c*e^2*(b*c*e + 2*a*d*e + 3*a*c*f)*x^3)/3 + (e*(b*c
*e*(2*d*e + 3*c*f) + a*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^5)/5
+ ((a*f*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2) + b*e*(d^2*e^2 + 6*c*d*
e*f + 3*c^2*f^2))*x^7)/7 + (f*(a*d*f*(3*d*e + 2*c*f) + b*(3*d^2*e
^2 + 6*c*d*e*f + c^2*f^2))*x^9)/9 + (d*f^2*(3*b*d*e + 2*b*c*f + a
*d*f)*x^11)/11 + (b*d^2*f^3*x^13)/13
```

Rubi [A] time = 0.625718, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$

$$\frac{1}{9}fx^9\left(adf(2cf+3de)+b\left(c^2f^2+6cdef+3d^2e^2\right)\right)+\frac{1}{7}x^7\left(af\left(c^2f^2+6cdef+3d^2e^2\right)+be\left(3c^2f^2+6cdef+d^2e^2\right)\right)+\frac{1}{5}ex^5\left(a\left(3c^2f^2+6cdef+d^2e^2\right)+bce(3cf+2de)\right)+\frac{1}{3}ce^2x^3(3acf+2ade+bce)+\frac{1}{11}df^2x^{11}(adf+2bcf+3bde)+ac^2e^3x+\frac{1}{13}bd^2f^3x^{13}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^3,x]
```

```
[Out] a*c^2*e^3*x + (c*e^2*(b*c*e + 2*a*d*e + 3*a*c*f)*x^3)/3 + (e*(b*c
*e*(2*d*e + 3*c*f) + a*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^5)/5
+ ((a*f*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2) + b*e*(d^2*e^2 + 6*c*d*
e*f + 3*c^2*f^2))*x^7)/7 + (f*(a*d*f*(3*d*e + 2*c*f) + b*(3*d^2*e
^2 + 6*c*d*e*f + c^2*f^2))*x^9)/9 + (d*f^2*(3*b*d*e + 2*b*c*f + a
*d*f)*x^11)/11 + (b*d^2*f^3*x^13)/13
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{bd^2f^3x^{13}}{13}+c^2e^3\int a\,dx+\frac{ce^2x^3(3acf+2ade+bce)}{3}+\frac{df^2x^{11}(adf+2bcf+3bde)}{11}+\frac{ex^5\left(3ac^2f^2+6acdef+ad^2e^2+3bc^2ef+2bcde^2\right)}{5}+\frac{fx^9\left(2acdf^2+3ad^2ef+bc^2f^2+6bcdef+3bd^2e^2\right)}{9}+x^7\left(\frac{ac^2f^3}{7}+\frac{6acdef^2}{7}+\frac{3ad^2e^2f}{7}+\frac{3bc^2ef^2}{7}+\frac{6bcde^2f}{7}+\frac{bd^2e^3}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)**2*(f*x**2+e)**3,x)
```

```
[Out] b*d**2*f**3*x**13/13 + c**2*e**3*Integral(a, x) + c*e**2*x**3*(3*
a*c*f + 2*a*d*e + b*c*e)/3 + d*f**2*x**11*(a*d*f + 2*b*c*f + 3*b*
d*e)/11 + e*x**5*(3*a*c**2*f**2 + 6*a*c*d*e*f + a*d**2*e**2 + 3*b
*c**2*e*f + 2*b*c*d*e**2)/5 + f*x**9*(2*a*c*d*f**2 + 3*a*d**2*e*f
 + b*c**2*f**2 + 6*b*c*d*e*f + 3*b*d**2*e**2)/9 + x**7*(a*c**2*f*
*3/7 + 6*a*c*d*e*f**2/7 + 3*a*d**2*e**2*f/7 + 3*b*c**2*e*f**2/7 +
 6*b*c*d*e**2*f/7 + b*d**2*e**3/7)
```

Mathematica [A] time = 0.165082, size = 226, normalized size = 1.

$$\frac{1}{9}fx^9\left(adf(2cf+3de)+b\left(c^2f^2+6cdef+3d^2e^2\right)\right)+\frac{1}{7}x^7\left(af\left(c^2f^2+6cdef+3d^2e^2\right)+be\left(3c^2f^2+6cdef+d^2e^2\right)\right)+\frac{1}{5}ex^5\left(a\left(3c^2f^2+6cdef+d^2e^2\right)+bce(3cf+2de)\right)+\frac{1}{3}ce^2x^3(3acf+2ade+bce)+\frac{1}{11}df^2x^{11}(adf+2bcf+3bde)+ac^2e^3x+\frac{1}{13}bd^2f^3x^{13}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^3,x]
```

```
[Out] a*c^2*e^3*x + (c*e^2*(b*c*e + 2*a*d*e + 3*a*c*f)*x^3)/3 + (e*(b*c
*e*(2*d*e + 3*c*f) + a*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^5)/5
+ ((a*f*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2) + b*e*(d^2*e^2 + 6*c*d*
e*f + 3*c^2*f^2))*x^7)/7 + (f*(a*d*f*(3*d*e + 2*c*f) + b*(3*d^2*e
^2 + 6*c*d*e*f + c^2*f^2))*x^9)/9 + (d*f^2*(3*b*d*e + 2*b*c*f + a
*d*f)*x^11)/11 + (b*d^2*f^3*x^13)/13
```

Maple [A] time = 0.001, size = 237, normalized size = 1.1

$$\frac{bd^2f^3x^{13}}{13}+\frac{\left(\left(ad^2+2\,bcd\right)\,f^3+3\,bd^2ef^2\right)\,x^{11}}{11}+\frac{\left(\left(2\,acd+bc^2\right)\,f^3+3\,\left(ad^2+2\,bcd\right)\,ef^2+3\,bd^2e^2f\right)\,x^9}{9}+\frac{\left(ac^2f^3+3\,\left(2\,acd+bc^2\right)\,ef^2+3\,\left(ad^2+2\,bcd\right)\,e^2f+bd^2e^3\right)\,x^7}{7}+\frac{\left(3\,ac^2ef^2+3\,\left(2\,acd+bc^2\right)\,e^2f+\left(ad^2+2\,bcd\right)\,e^3\right)\,x^5}{5}+\frac{\left(3\,ac^2e^2f+\left(2\,acd+bc^2\right)\,e^3\right)\,x^3}{3}+ac^2e^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^3,x)
```

```
[Out] 1/13*b*d^2*f^3*x^13+1/11*((a*d^2+2*b*c*d)*f^3+3*b*d^2*e*f^2)*x^11
+1/9*((2*a*c*d+b*c^2)*f^3+3*(a*d^2+2*b*c*d)*e*f^2+3*b*d^2*e^2*f)*
x^9+1/7*(a*c^2*f^3+3*(2*a*c*d+b*c^2)*e*f^2+3*(a*d^2+2*b*c*d)*e^2*
f+b*d^2*e^3)*x^7+1/5*(3*a*c^2*e*f^2+3*(2*a*c*d+b*c^2)*e^2*f+(a*d^
2+2*b*c*d)*e^3)*x^5+1/3*(3*a*c^2*e^2*f+(2*a*c*d+b*c^2)*e^3)*x^3+a
*c^2*e^3*x
```

Maxima [A] time = 1.35544, size = 319, normalized size = 1.41

$$\frac{1}{13}\,bd^2f^3x^{13}+\frac{1}{11}\,\left(3\,bd^2ef^2+\left(2\,bcd+ad^2\right)f^3\right)x^{11}+\frac{1}{9}\,\left(3\,bd^2e^2f+3\,\left(2\,bcd+ad^2\right)ef^2+\left(bc^2+2\,acd\right)f^3\right)x^9+\frac{1}{7}\,\left(bd^2e^3+ac^2f^3+3\,\left(2\,bcd+ad^2\right)e^2f+3\,\left(bc^2+2\,acd\right)ef^2\right)x^7+ac^2e^3x+\frac{1}{5}\,\left(3\,ac^2ef^2+\left(2\,bcd+ad^2\right)e^3+3\,\left(bc^2+2\,acd\right)e^2f\right)x^5+\frac{1}{3}\,\left(3\,ac^2e^2f+\left(bc^2+2\,acd\right)e^3\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)^2*(f*x^2 + e)^3,x, algorithm="maxima")
```

```
[Out] 1/13*b*d^2*f^3*x^13 + 1/11*(3*b*d^2*e*f^2 + (2*b*c*d + a*d^2)*f^3
)*x^11 + 1/9*(3*b*d^2*e^2*f + 3*(2*b*c*d + a*d^2)*e*f^2 + (b*c^2
+ 2*a*c*d)*f^3)*x^9 + 1/7*(b*d^2*e^3 + a*c^2*f^3 + 3*(2*b*c*d + a
*d^2)*e^2*f + 3*(b*c^2 + 2*a*c*d)*e*f^2)*x^7 + a*c^2*e^3*x + 1/5*
(3*a*c^2*e*f^2 + (2*b*c*d + a*d^2)*e^3 + 3*(b*c^2 + 2*a*c*d)*e^2*
f)*x^5 + 1/3*(3*a*c^2*e^2*f + (b*c^2 + 2*a*c*d)*e^3)*x^3
```

Fricas [A] time = 0.183982, size = 1, normalized size = 0.

$$\frac{1}{13}x^{13}f^3d^2b + \frac{3}{11}x^{11}f^2ed^2b + \frac{2}{11}x^{11}f^3dcb + \frac{1}{11}x^{11}f^3d^2a + \frac{1}{3}x^9fe^2d^2b + \frac{2}{3}x^9f^2edcb + \frac{1}{9}x^9f^3c^2b + \frac{1}{3}x^9f^2ed^2a + \frac{2}{9}x^9f^3dca + \frac{1}{7}x^7e^3d^2b + \frac{6}{7}x^7fe^2dcb + \frac{3}{7}x^7f^2ec^2b + \frac{3}{7}x^7fe^2d^2a + \frac{6}{7}x^7f^2edca + \frac{1}{7}x^7f^3c^2a + \frac{2}{5}x^5e^3dcb + \frac{3}{5}x^5fe^2c^2b + \frac{1}{5}x^5e^3d^2a + \frac{6}{5}x^5fe^2dca + \frac{3}{5}x^5f^2ec^2a + \frac{1}{3}x^3e^3c^2b + \frac{2}{3}x^3e^3dca + x^3fe^2c^2a + xe^3c^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^2*(f*x^2 + e)^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*f^3*d^2*b + 3/11*x^11*f^2*e*d^2*b + 2/11*x^11*f^3*d*c*b
 + 1/11*x^11*f^3*d^2*a + 1/3*x^9*f*e^2*d^2*b + 2/3*x^9*f^2*e*d*c*
b + 1/9*x^9*f^3*c^2*b + 1/3*x^9*f^2*e*d^2*a + 2/9*x^9*f^3*d*c*a +
 1/7*x^7*e^3*d^2*b + 6/7*x^7*f*e^2*d*c*b + 3/7*x^7*f^2*e*c^2*b +
3/7*x^7*f*e^2*d^2*a + 6/7*x^7*f^2*e*d*c*a + 1/7*x^7*f^3*c^2*a + 2
/5*x^5*e^3*d*c*b + 3/5*x^5*f*e^2*c^2*b + 1/5*x^5*e^3*d^2*a + 6/5*
x^5*f*e^2*d*c*a + 3/5*x^5*f^2*e*c^2*a + 1/3*x^3*e^3*c^2*b + 2/3*x
^3*e^3*d*c*a + x^3*f*e^2*c^2*a + x*e^3*c^2*a
```

Sympy [A] time = 0.119166, size = 304, normalized size = 1.35

$$ac^2e^3x + \frac{bd^2f^3x^{13}}{13} + x^{11}\left(\frac{ad^2f^3}{11} + \frac{2bcdf^3}{11} + \frac{3bd^2ef^2}{11}\right) + x^9\left(\frac{2acdf^3}{9} + \frac{ad^2ef^2}{3} + \frac{bc^2f^3}{9} + \frac{2bcdef^2}{3} + \frac{bd^2e^2f}{3}\right) + x^7\left(\frac{ac^2f^3}{7} + \frac{6acdef^2}{7} + \frac{3ad^2e^2f}{7} + \frac{3bc^2ef^2}{7} + \frac{6bcde^2f}{7} + \frac{bd^2e^3}{7}\right) + x^5\left(\frac{3ac^2ef^2}{5} + \frac{6acde^2f}{5} + \frac{ad^2e^3}{5} + \frac{3bc^2e^2f}{5} + \frac{2bcde^3}{5}\right) + x^3\left(ac^2e^2f + \frac{2acde^3}{3} + \frac{bc^2e^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x**2+c)**2*(f*x**2+e)**3,x)
```

```
[Out] a*c**2*e**3*x + b*d**2*f**3*x**13/13 + x**11*(a*d**2*f**3/11 + 2*
b*c*d*f**3/11 + 3*b*d**2*e*f**2/11) + x**9*(2*a*c*d*f**3/9 + a*d*
*2*e*f**2/3 + b*c**2*f**3/9 + 2*b*c*d*e*f**2/3 + b*d**2*e**2*f/3)
 + x**7*(a*c**2*f**3/7 + 6*a*c*d*e*f**2/7 + 3*a*d**2*e**2*f/7 + 3
*b*c**2*e*f**2/7 + 6*b*c*d*e**2*f/7 + b*d**2*e**3/7) + x**5*(3*a*
c**2*e*f**2/5 + 6*a*c*d*e**2*f/5 + a*d**2*e**3/5 + 3*b*c**2*e**2*
f/5 + 2*b*c*d*e**3/5) + x**3*(a*c**2*e**2*f + 2*a*c*d*e**3/3 + b*
c**2*e**3/3)
```

GIAC/XCAS [A] time = 0.226921, size = 382, normalized size = 1.69

$$\frac{1}{13}\,bd^2f^3x^{13} + \frac{2}{11}\,bcdf^3x^{11} + \frac{1}{11}\,ad^2f^3x^{11} + \frac{3}{11}\,bd^2f^2x^{11}e + \frac{1}{9}\,bc^2f^3x^9 + \frac{2}{9}\,acdf^3x^9 + \frac{2}{3}\,bcdf^2x^9e + \frac{1}{3}\,ad^2f^2x^9e + \frac{1}{3}\,bd^2fx^9e^2 + \frac{1}{7}\,ac^2f^3x^7 + \frac{3}{7}\,bc^2f^2x^7e + \frac{6}{7}\,acdf^2x^7e + \frac{6}{7}\,bcdfx^7e^2 + \frac{3}{7}\,ad^2fx^7e^2 + \frac{1}{7}\,bd^2x^7e^3 + \frac{3}{5}\,ac^2f^2x^5e + \frac{3}{5}\,bc^2fx^5e^2 + \frac{6}{5}\,acdfx^5e^2 + \frac{2}{5}\,bcdx^5e^3 + \frac{1}{5}\,ad^2x^5e^3 + ac^2fx^3e^2 + \frac{1}{3}\,bc^2x^3e^3 + \frac{2}{3}\,acdx^3e^3 + ac^2xe^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^2*(f*x^2 + e)^3,x, algorithm="giac")
```

```
[Out] 1/13*b*d^2*f^3*x^13 + 2/11*b*c*d*f^3*x^11 + 1/11*a*d^2*f^3*x^11 +
 3/11*b*d^2*f^2*x^11*e + 1/9*b*c^2*f^3*x^9 + 2/9*a*c*d*f^3*x^9 +
2/3*b*c*d*f^2*x^9*e + 1/3*a*d^2*f^2*x^9*e + 1/3*b*d^2*f*x^9*e^2 +
 1/7*a*c^2*f^3*x^7 + 3/7*b*c^2*f^2*x^7*e + 6/7*a*c*d*f^2*x^7*e +
6/7*b*c*d*f*x^7*e^2 + 3/7*a*d^2*f*x^7*e^2 + 1/7*b*d^2*x^7*e^3 + 3
/5*a*c^2*f^2*x^5*e + 3/5*b*c^2*f*x^5*e^2 + 6/5*a*c*d*f*x^5*e^2 +
2/5*b*c*d*x^5*e^3 + 1/5*a*d^2*x^5*e^3 + a*c^2*f*x^3*e^2 + 1/3*b*c
^2*x^3*e^3 + 2/3*a*c*d*x^3*e^3 + a*c^2*x*e^3
```

3.10 $\int (a+bx^2)(c+dx^2)^2(e+fx^2)^2\,dx$

Optimal. Leaf size=158

$$\frac{1}{7}x^7\left(2adf(cf+de)+b\left(c^2f^2+4cdef+d^2e^2\right)\right)+\frac{1}{5}x^5\left(a\left(c^2f^2+4cdef+d^2e^2\right)+2bce(cf+de)\right)+\frac{1}{9}dfx^9(adf+2b(cf+de))+\frac{1}{3}cex^3(2a(cf+de)+bce)+ac^2e^2x+\frac{1}{11}bd^2f^2x^{11}$$

```
[Out] a*c^2*e^2*x + (c*e*(b*c*e + 2*a*(d*e + c*f))*x^3)/3 + ((2*b*c*e*(
d*e + c*f) + a*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^5)/5 + ((2*a*d*
f*(d*e + c*f) + b*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^7)/7 + (d*f*
(a*d*f + 2*b*(d*e + c*f))*x^9)/9 + (b*d^2*f^2*x^11)/11
```

Rubi [A] time = 0.496061, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$

$$\frac{1}{7}x^7\left(2adf(cf+de)+b\left(c^2f^2+4cdef+d^2e^2\right)\right)+\frac{1}{5}x^5\left(a\left(c^2f^2+4cdef+d^2e^2\right)+2bce(cf+de)\right)+\frac{1}{9}dfx^9(adf+2b(cf+de))+\frac{1}{3}cex^3(2a(cf+de)+bce)+ac^2e^2x+\frac{1}{11}bd^2f^2x^{11}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^2,x]
```

```
[Out] a*c^2*e^2*x + (c*e*(b*c*e + 2*a*(d*e + c*f))*x^3)/3 + ((2*b*c*e*(
d*e + c*f) + a*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^5)/5 + ((2*a*d*
f*(d*e + c*f) + b*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^7)/7 + (d*f*
(a*d*f + 2*b*(d*e + c*f))*x^9)/9 + (b*d^2*f^2*x^11)/11
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{bd^2f^2x^{11}}{11}+c^2e^2\int a\,dx+\frac{cex^3\left(2acf+2ade+bce\right)}{3}+\frac{dfx^9\left(adf+2bcf+2bde\right)}{9}+x^7\left(\frac{2acdf^2}{7}+\frac{2ad^2ef}{7}+\frac{bc^2f^2}{7}+\frac{4bcdef}{7}+\frac{bd^2e^2}{7}\right)+x^5\left(\frac{ac^2f^2}{5}+\frac{4acdef}{5}+\frac{ad^2e^2}{5}+\frac{2bc^2ef}{5}+\frac{2bcde^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)**2*(f*x**2+e)**2,x)
```

```
[Out] b*d**2*f**2*x**11/11 + c**2*e**2*Integral(a, x) + c*e*x**3*(2*a*c
*f + 2*a*d*e + b*c*e)/3 + d*f*x**9*(a*d*f + 2*b*c*f + 2*b*d*e)/9
```

```
+ x**7*(2*a*c*d*f**2/7 + 2*a*d**2*e*f/7 + b*c**2*f**2/7 + 4*b*c*d
*e*f/7 + b*d**2*e**2/7) + x**5*(a*c**2*f**2/5 + 4*a*c*d*e*f/5 + a
*d**2*e**2/5 + 2*b*c**2*e*f/5 + 2*b*c*d*e**2/5)
```

Mathematica [A] time = 0.113676, size = 158, normalized size = 1.

$$\frac{1}{7}x^7\left(2adf(cf+de)+b\left(c^2f^2+4cdef+d^2e^2\right)\right)+\frac{1}{5}x^5\left(a\left(c^2f^2+4cdef+d^2e^2\right)+2bce(cf+de)\right)+\frac{1}{9}dfx^9(adf+2b(cf+de))+\frac{1}{3}cex^3(2a(cf+de)+bce)+ac^2e^2x+\frac{1}{11}bd^2f^2x^{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^2,x]
```

```
[Out] a*c^2*e^2*x + (c*e*(b*c*e + 2*a*(d*e + c*f))*x^3)/3 + ((2*b*c*e*(
d*e + c*f) + a*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^5)/5 + ((2*a*d*
f*(d*e + c*f) + b*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^7)/7 + (d*f*
(a*d*f + 2*b*(d*e + c*f))*x^9)/9 + (b*d^2*f^2*x^11)/11
```

Maple [A] time = 0.002, size = 169, normalized size = 1.1

$$\frac{bd^2f^2x^{11}}{11}+\frac{\left(\left(ad^2+2\,bcd\right)f^2+2\,bd^2ef\right)x^9}{9}+\frac{\left(\left(2\,acd+bc^2\right)f^2+2\,\left(ad^2+2\,bcd\right)ef+bd^2e^2\right)x^7}{7}+\frac{\left(ac^2f^2+2\,\left(2\,acd+bc^2\right)ef+\left(ad^2+2\,bcd\right)e^2\right)x^5}{5}+\frac{\left(2\,ac^2ef+\left(2\,acd+bc^2\right)e^2\right)x^3}{3}+ac^2e^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^2,x)
```

```
[Out] 1/11*b*d^2*f^2*x^11+1/9*((a*d^2+2*b*c*d)*f^2+2*b*d^2*e*f)*x^9+1/7
*((2*a*c*d+b*c^2)*f^2+2*(a*d^2+2*b*c*d)*e*f+b*d^2*e^2)*x^7+1/5*(a
*c^2*f^2+2*(2*a*c*d+b*c^2)*e*f+(a*d^2+2*b*c*d)*e^2)*x^5+1/3*(2*a*
c^2*e*f+(2*a*c*d+b*c^2)*e^2)*x^3+a*c^2*e^2*x
```

Maxima [A] time = 1.34805, size = 227, normalized size = 1.44

$$\frac{1}{11}\,bd^2f^2x^{11}+\frac{1}{9}\left(2\,bd^2ef+\left(2\,bcd+ad^2\right)f^2\right)x^9+\frac{1}{7}\left(bd^2e^2+2\left(2\,bcd+ad^2\right)ef+\left(bc^2+2\,acd\right)f^2\right)x^7+ac^2e^2x+\frac{1}{5}\left(ac^2f^2+\left(2\,bcd+ad^2\right)e^2+2\left(bc^2+2\,acd\right)ef\right)x^5+\frac{1}{3}\left(2\,ac^2ef+\left(bc^2+2\,acd\right)e^2\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^2*(f*x^2 + e)^2,x, algorithm="maxima")
```

```
[Out] 1/11*b*d^2*f^2*x^11 + 1/9*(2*b*d^2*e*f + (2*b*c*d + a*d^2)*f^2)*x
^9 + 1/7*(b*d^2*e^2 + 2*(2*b*c*d + a*d^2)*e*f + (b*c^2 + 2*a*c*d)
*f^2)*x^7 + a*c^2*e^2*x + 1/5*(a*c^2*f^2 + (2*b*c*d + a*d^2)*e^2
+ 2*(b*c^2 + 2*a*c*d)*e*f)*x^5 + 1/3*(2*a*c^2*e*f + (b*c^2 + 2*a*
c*d)*e^2)*x^3
```

Fricas [A] time = 0.1828, size = 1, normalized size = 0.01

$$\frac{1}{11}x^{11}f^2d^2b + \frac{2}{9}x^9fed^2b + \frac{2}{9}x^9f^2dcb + \frac{1}{9}x^9f^2d^2a + \frac{1}{7}x^7e^2d^2b + \frac{4}{7}x^7fedcb + \frac{1}{7}x^7f^2c^2b + \frac{2}{7}x^7fed^2a + \frac{2}{7}x^7f^2dca + \frac{2}{5}x^5e^2dcb + \frac{2}{5}x^5fec^2b + \frac{1}{5}x^5e^2d^2a + \frac{4}{5}x^5fedca + \frac{1}{5}x^5f^2c^2a + \frac{1}{3}x^3e^2c^2b + \frac{2}{3}x^3e^2dca + \frac{2}{3}x^3fec^2a + xe^2c^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^2*(f*x^2 + e)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*f^2*d^2*b + 2/9*x^9*f*e*d^2*b + 2/9*x^9*f^2*d*c*b + 1/9
*x^9*f^2*d^2*a + 1/7*x^7*e^2*d^2*b + 4/7*x^7*f*e*d*c*b + 1/7*x^7*
f^2*c^2*b + 2/7*x^7*f*e*d^2*a + 2/7*x^7*f^2*d*c*a + 2/5*x^5*e^2*d
*c*b + 2/5*x^5*f*e*c^2*b + 1/5*x^5*e^2*d^2*a + 4/5*x^5*f*e*d*c*a
+ 1/5*x^5*f^2*c^2*a + 1/3*x^3*e^2*c^2*b + 2/3*x^3*e^2*d*c*a + 2/3
*x^3*f*e*c^2*a + x*e^2*c^2*a
```

Sympy [A] time = 0.09932, size = 216, normalized size = 1.37

$$ac^2e^2x + \frac{bd^2f^2x^{11}}{11} + x^9\left(\frac{ad^2f^2}{9} + \frac{2bcdf^2}{9} + \frac{2bd^2ef}{9}\right) + x^7\left(\frac{2acdf^2}{7} + \frac{2ad^2ef}{7} + \frac{bc^2f^2}{7} + \frac{4bcdef}{7} + \frac{bd^2e^2}{7}\right) + x^5\left(\frac{ac^2f^2}{5} + \frac{4acdef}{5} + \frac{ad^2e^2}{5} + \frac{2bc^2ef}{5} + \frac{2bcde^2}{5}\right) + x^3\left(\frac{2ac^2ef}{3} + \frac{2acde^2}{3} + \frac{bc^2e^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x**2+c)**2*(f*x**2+e)**2,x)
```

```
[Out] a*c**2*e**2*x + b*d**2*f**2*x**11/11 + x**9*(a*d**2*f**2/9 + 2*b*
c*d*f**2/9 + 2*b*d**2*e*f/9) + x**7*(2*a*c*d*f**2/7 + 2*a*d**2*e*
f/7 + b*c**2*f**2/7 + 4*b*c*d*e*f/7 + b*d**2*e**2/7) + x**5*(a*c*
*2*f**2/5 + 4*a*c*d*e*f/5 + a*d**2*e**2/5 + 2*b*c**2*e*f/5 + 2*b*
```

```
c*d*e**2/5) + x**3*(2*a*c**2*e*f/3 + 2*a*c*d*e**2/3 + b*c**2*e**2
/3)
```

GIAC/XCAS [A] time = 0.227538, size = 273, normalized size = 1.73

$$\frac{1}{11}\,bd^2f^2x^{11} + \frac{2}{9}\,bcdf^2x^9 + \frac{1}{9}\,ad^2f^2x^9 + \frac{2}{9}\,bd^2fx^9e + \frac{1}{7}\,bc^2f^2x^7 + \frac{2}{7}\,acdf^2x^7 + \frac{4}{7}\,bcdfx^7e + \frac{2}{7}\,ad^2fx^7e + \frac{1}{7}\,bd^2x^7e^2 + \frac{1}{5}\,ac^2f^2x^5 + \frac{2}{5}\,bc^2fx^5e + \frac{4}{5}\,acdfx^5e + \frac{2}{5}\,bcdx^5e^2 + \frac{1}{5}\,ad^2x^5e^2 + \frac{2}{3}\,ac^2fx^3e + \frac{1}{3}\,bc^2x^3e^2 + \frac{2}{3}\,acdx^3e^2 + ac^2xe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)^2*(f*x^2 + e)^2,x, algorithm="giac")
```

```
[Out] 1/11*b*d^2*f^2*x^11 + 2/9*b*c*d*f^2*x^9 + 1/9*a*d^2*f^2*x^9 + 2/9
*b*d^2*f*x^9*e + 1/7*b*c^2*f^2*x^7 + 2/7*a*c*d*f^2*x^7 + 4/7*b*c*
d*f*x^7*e + 2/7*a*d^2*f*x^7*e + 1/7*b*d^2*x^7*e^2 + 1/5*a*c^2*f^2
*x^5 + 2/5*b*c^2*f*x^5*e + 4/5*a*c*d*f*x^5*e + 2/5*b*c*d*x^5*e^2
+ 1/5*a*d^2*x^5*e^2 + 2/3*a*c^2*f*x^3*e + 1/3*b*c^2*x^3*e^2 + 2/3
*a*c*d*x^3*e^2 + a*c^2*x*e^2
```

3.11 $\int \left(a+bx^2\right)\left(c+dx^2\right)^2\left(e+fx^2\right)\,dx$

Optimal. Leaf size=94

$$\frac{1}{7}dx^7(adf+2bcf+bde)+\frac{1}{5}x^5(ad(2cf+de)+bc(cf+2de))+\frac{1}{3}cx^3(acf+2ade+bce)+ac^2ex+\frac{1}{9}bd^2fx^9$$

```
[Out] a*c^2*e*x + (c*(b*c*e + 2*a*d*e + a*c*f)*x^3)/3 + ((b*c*(2*d*e +
c*f) + a*d*(d*e + 2*c*f))*x^5)/5 + (d*(b*d*e + 2*b*c*f + a*d*f)*x
^7)/7 + (b*d^2*f*x^9)/9
```

Rubi [A] time = 0.250629, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{1}{7}dx^7(adf+2bcf+bde)+\frac{1}{5}x^5(ad(2cf+de)+bc(cf+2de))+\frac{1}{3}cx^3(acf+2ade+bce)+ac^2ex+\frac{1}{9}bd^2fx^9$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2),x]
```

```
[Out] a*c^2*e*x + (c*(b*c*e + 2*a*d*e + a*c*f)*x^3)/3 + ((b*c*(2*d*e +
c*f) + a*d*(d*e + 2*c*f))*x^5)/5 + (d*(b*d*e + 2*b*c*f + a*d*f)*x
^7)/7 + (b*d^2*f*x^9)/9
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{bd^2fx^9}{9}+c^2e\int a\,dx+\frac{cx^3\left(acf+2ade+bce\right)}{3}+\frac{dx^7\left(adf+2bcf+bde\right)}{7}+x^5\left(\frac{2acdf}{5}+\frac{ad^2e}{5}+\frac{bc^2f}{5}+\frac{2bcde}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)**2*(f*x**2+e),x)
```

```
[Out] b*d**2*f*x**9/9 + c**2*e*Integral(a, x) + c*x**3*(a*c*f + 2*a*d*e
 + b*c*e)/3 + d*x**7*(a*d*f + 2*b*c*f + b*d*e)/7 + x**5*(2*a*c*d*
f/5 + a*d**2*e/5 + b*c**2*f/5 + 2*b*c*d*e/5)
```

Mathematica [A] time = 0.0581505, size = 96, normalized size = 1.02

$$\frac{1}{5}x^5\left(2acdf+ad^2e+bc^2f+2bcde\right)+\frac{1}{7}dx^7(adf+2bcf+bde)+\frac{1}{3}cx^3(acf+2ade+bce)+ac^2ex+\frac{1}{9}bd^2fx^9$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2),x]
```

```
[Out] a*c^2*e*x + (c*(b*c*e + 2*a*d*e + a*c*f)*x^3)/3 + ((2*b*c*d*e + a
*d^2*e + b*c^2*f + 2*a*c*d*f)*x^5)/5 + (d*(b*d*e + 2*b*c*f + a*d*
f)*x^7)/7 + (b*d^2*f*x^9)/9
```

Maple [A]   time = 0.001, size = 101, normalized size = 1.1

$$\frac{bd^2fx^9}{9}+\frac{\left(\left(ad^2+2\,bcd\right)f+bd^2e\right)x^7}{7}+\frac{\left(\left(2\,acd+bc^2\right)f+\left(ad^2+2\,bcd\right)e\right)x^5}{5}+\frac{\left(ac^2f+\left(2\,acd+bc^2\right)e\right)x^3}{3}+ac^2ex$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e),x)
```

```
[Out] 1/9*b*d^2*f*x^9+1/7*((a*d^2+2*b*c*d)*f+b*d^2*e)*x^7+1/5*((2*a*c*d
+b*c^2)*f+(a*d^2+2*b*c*d)*e)*x^5+1/3*(a*c^2*f+(2*a*c*d+b*c^2)*e)*
x^3+a*c^2*e*x
```

Maxima [A]   time = 1.35665, size = 135, normalized size = 1.44

$$\frac{1}{9}\,bd^2fx^9+\frac{1}{7}\left(bd^2e+\left(2\,bcd+ad^2\right)f\right)x^7+\frac{1}{5}\left(\left(2\,bcd+ad^2\right)e+\left(bc^2+2\,acd\right)f\right)x^5+ac^2ex+\frac{1}{3}\left(ac^2f+\left(bc^2+2\,acd\right)e\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)^2*(f*x^2 + e),x, algorithm="maxima")
```

```
[Out] 1/9*b*d^2*f*x^9 + 1/7*(b*d^2*e + (2*b*c*d + a*d^2)*f)*x^7 + 1/5*(
(2*b*c*d + a*d^2)*e + (b*c^2 + 2*a*c*d)*f)*x^5 + a*c^2*e*x + 1/3*
(a*c^2*f + (b*c^2 + 2*a*c*d)*e)*x^3
```

Fricas [A]   time = 0.182081, size = 1, normalized size = 0.01

$$\frac{1}{9}x^9fd^2b+\frac{1}{7}x^7ed^2b+\frac{2}{7}x^7fdcb+\frac{1}{7}x^7fd^2a+\frac{2}{5}x^5edcb+\frac{1}{5}x^5fc^2b+\frac{1}{5}x^5ed^2a+\frac{2}{5}x^5fdca+\frac{1}{3}x^3ec^2b+\frac{2}{3}x^3edca+\frac{1}{3}x^3fc^2a+xec^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)^2*(f*x^2 + e),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*f*d^2*b + 1/7*x^7*e*d^2*b + 2/7*x^7*f*d*c*b + 1/7*x^7*f*d
^2*a + 2/5*x^5*e*d*c*b + 1/5*x^5*f*c^2*b + 1/5*x^5*e*d^2*a + 2/5*
x^5*f*d*c*a + 1/3*x^3*e*c^2*b + 2/3*x^3*e*d*c*a + 1/3*x^3*f*c^2*a
 + x*e*c^2*a
```

Sympy [A] time = 0.074659, size = 121, normalized size = 1.29

$$ac^2ex + \frac{bd^2fx^9}{9} + x^7\left(\frac{ad^2f}{7} + \frac{2bcdf}{7} + \frac{bd^2e}{7}\right) + x^5\left(\frac{2acdf}{5} + \frac{ad^2e}{5} + \frac{bc^2f}{5} + \frac{2bcde}{5}\right) + x^3\left(\frac{ac^2f}{3} + \frac{2acde}{3} + \frac{bc^2e}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x**2+c)**2*(f*x**2+e),x)
```

```
[Out] a*c**2*e*x + b*d**2*f*x**9/9 + x**7*(a*d**2*f/7 + 2*b*c*d*f/7 + b
*d**2*e/7) + x**5*(2*a*c*d*f/5 + a*d**2*e/5 + b*c**2*f/5 + 2*b*c*
d*e/5) + x**3*(a*c**2*f/3 + 2*a*c*d*e/3 + b*c**2*e/3)
```

GIAC/XCAS [A] time = 0.227169, size = 162, normalized size = 1.72

$$\frac{1}{9}bd^2fx^9 + \frac{2}{7}bcdfx^7 + \frac{1}{7}ad^2fx^7 + \frac{1}{7}bd^2x^7e + \frac{1}{5}bc^2fx^5 + \frac{2}{5}acdfx^5 + \frac{2}{5}bcdx^5e + \frac{1}{5}ad^2x^5e + \frac{1}{3}ac^2fx^3 + \frac{1}{3}bc^2x^3e + \frac{2}{3}acdx^3e + ac^2xe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)^2*(f*x^2 + e),x, algorithm="giac")
```

```
[Out] 1/9*b*d^2*f*x^9 + 2/7*b*c*d*f*x^7 + 1/7*a*d^2*f*x^7 + 1/7*b*d^2*x
^7*e + 1/5*b*c^2*f*x^5 + 2/5*a*c*d*f*x^5 + 2/5*b*c*d*x^5*e + 1/5*
a*d^2*x^5*e + 1/3*a*c^2*f*x^3 + 1/3*b*c^2*x^3*e + 2/3*a*c*d*x^3*e
 + a*c^2*x*e
```

3.12 $\int \frac{(a+bx^2)(c+dx^2)^2}{e+fx^2}\,dx$

Optimal. Leaf size=142

$$-\frac{x\left(5adf(3de-5cf)-b\left(8c^2f^2-25cdef+15d^2e^2\right)\right)}{15f^3}-\frac{(be-af)(de-cf)^2\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{7/2}}-\frac{x\left(c+dx^2\right)(-5adf-4bcf+5bde)}{15f^2}+\frac{bx\left(c+dx^2\right)^2}{5f}$$

```
[Out] -((5*a*d*f*(3*d*e - 5*c*f) - b*(15*d^2*e^2 - 25*c*d*e*f + 8*c^2*f
^2))*x)/(15*f^3) - ((5*b*d*e - 4*b*c*f - 5*a*d*f)*x*(c + d*x^2))/
(15*f^2) + (b*x*(c + d*x^2)^2)/(5*f) - ((b*e - a*f)*(d*e - c*f)^2
*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(7/2))
```

Rubi [A] time = 0.534499, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{x\left(5adf(3de-5cf)-b\left(8c^2f^2-25cdef+15d^2e^2\right)\right)}{15f^3}-\frac{(be-af)(de-cf)^2\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{7/2}}-\frac{x\left(c+dx^2\right)(-5adf-4bcf+5bde)}{15f^2}+\frac{bx\left(c+dx^2\right)^2}{5f}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2),x]
```

```
[Out] -((5*a*d*f*(3*d*e - 5*c*f) - b*(15*d^2*e^2 - 25*c*d*e*f + 8*c^2*f
^2))*x)/(15*f^3) - ((5*b*d*e - 4*b*c*f - 5*a*d*f)*x*(c + d*x^2))/
(15*f^2) + (b*x*(c + d*x^2)^2)/(5*f) - ((b*e - a*f)*(d*e - c*f)^2
*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(7/2))
```

Rubi in Sympy [A] time = 60.7377, size = 153, normalized size = 1.08

$$\frac{bx\left(c+dx^2\right)^2}{5f}+\frac{dx\left(c(5af-be)+x^2(5adf+4bcf-5bde)\right)}{15f^2}+\frac{x\left(25acdf^2-15ad^2ef+12bc^2f^2-29bcdef+15bd^2e^2\right)}{15f^3}+\frac{(af-be)(cf-de)^2\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)**2/(f*x**2+e),x)
```

```
[Out] b*x*(c + d*x**2)**2/(5*f) + d*x*(c*(5*a*f - b*e) + x**2*(5*a*d*f
+ 4*b*c*f - 5*b*d*e))/(15*f**2) + x*(25*a*c*d*f**2 - 15*a*d**2*e*
f + 12*b*c**2*f**2 - 29*b*c*d*e*f + 15*b*d**2*e**2)/(15*f**3) + (
a*f - b*e)*(c*f - d*e)**2*atan(sqrt(f)*x/sqrt(e))/(sqrt(e)*f**(7/
2))
```

Mathematica [A] time = 0.12025, size = 115, normalized size = 0.81

$$-\frac{(be-af)(de-cf)^2\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{7/2}}+\frac{x\left(adf(2cf-de)+b(de-cf)^2\right)}{f^3}+\frac{dx^3(adf+2bcf-bde)}{3f^2}+\frac{bd^2x^5}{5f}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2),x]
```

```
[Out] ((b*(d*e - c*f)^2 + a*d*f*(-(d*e) + 2*c*f))*x)/f^3 + (d*(-(b*d*e)
 + 2*b*c*f + a*d*f)*x^3)/(3*f^2) + (b*d^2*x^5)/(5*f) - ((b*e - a*
f)*(d*e - c*f)^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(7/2))
```

Maple [A] time = 0.007, size = 243, normalized size = 1.7

$$\frac{bd^2x^5}{5f}+\frac{x^3ad^2}{3f}+\frac{2x^3bcd}{3f}-\frac{x^3bd^2e}{3f^2}+2\frac{acdx}{f}-\frac{ad^2ex}{f^2}+\frac{bc^2x}{f}-2\frac{bcdex}{f^2}+\frac{bd^2e^2x}{f^3}$$
$$+ac^2\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}-2\frac{acde}{f\sqrt{ef}}\arctan\left(\frac{fx}{\sqrt{ef}}\right)+\frac{ad^2e^2}{f^2}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}$$
$$-\frac{bc^2e}{f}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}+2\frac{bcde^2}{f^2\sqrt{ef}}\arctan\left(\frac{fx}{\sqrt{ef}}\right)-\frac{bd^2e^3}{f^3}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e),x)
```

```
[Out] 1/5/f*b*d^2*x^5+1/3/f*x^3*a*d^2+2/3/f*x^3*b*c*d-1/3/f^2*x^3*b*d^2
*e+2/f*a*c*d*x-1/f^2*a*d^2*e*x+1/f*b*c^2*x-2/f^2*b*c*d*e*x+1/f^3*
b*d^2*e^2*x+1/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a*c^2-2/f/(e*f)
^(1/2)*arctan(x*f/(e*f)^(1/2))*a*c*d*e+1/f^2/(e*f)^(1/2)*arctan(x
*f/(e*f)^(1/2))*a*d^2*e^2-1/f/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))
*b*c^2*e+2/f^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*c*d*e^2-1/f^
3/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*d^2*e^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^2/(f*x^2 + e),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.215693, size = 1, normalized size = 0.01

$$\left[-\frac{15\left(bd^2e^3 - ac^2f^3 - \left(2\,bcd + ad^2\right)e^2f + \left(bc^2 + 2\,acd\right)ef^2\right)\log\left(\frac{2\,efx + \left(fx^2 - e\right)\sqrt{-ef}}{fx^2 + e}\right) - 2\left(3\,bd^2f^2x^5 - 5\left(bd^2ef - \left(2\,bcd\right.\right.\right.}{30\sqrt{-ef}f^3}\right.$$

$$-\frac{15\left(bd^2e^3 - ac^2f^3 - \left(2\,bcd + ad^2\right)e^2f + \left(bc^2 + 2\,acd\right)ef^2\right)\arctan\left(\frac{\sqrt{ef}x}{e}\right) - \left(3\,bd^2f^2x^5 - 5\left(bd^2ef - \left(2\,bcd + ad^2\right)f^2\right)x\right.}{15\sqrt{ef}f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^2/(f*x^2 + e),x, algorithm="fricas")
```

```
[Out] [-1/30*(15*(b*d^2*e^3 - a*c^2*f^3 - (2*b*c*d + a*d^2)*e^2*f + (b*
c^2 + 2*a*c*d)*e*f^2)*log((2*e*f*x + (f*x^2 - e)*sqrt(-e*f))/(f*x
^2 + e)) - 2*(3*b*d^2*f^2*x^5 - 5*(b*d^2*e*f - (2*b*c*d + a*d^2)*
f^2)*x^3 + 15*(b*d^2*e^2 - (2*b*c*d + a*d^2)*e*f + (b*c^2 + 2*a*c
*d)*f^2)*x)*sqrt(-e*f))/(sqrt(-e*f)*f^3), -1/15*(15*(b*d^2*e^3 -
a*c^2*f^3 - (2*b*c*d + a*d^2)*e^2*f + (b*c^2 + 2*a*c*d)*e*f^2)*ar
ctan(sqrt(e*f)*x/e) - (3*b*d^2*f^2*x^5 - 5*(b*d^2*e*f - (2*b*c*d
+ a*d^2)*f^2)*x^3 + 15*(b*d^2*e^2 - (2*b*c*d + a*d^2)*e*f + (b*c^
2 + 2*a*c*d)*f^2)*x)*sqrt(e*f))/(sqrt(e*f)*f^3)]
```

Sympy [A] time = 2.51235, size = 343, normalized size = 2.42

$$\frac{bd^2x^5}{5f} - \frac{\sqrt{-\frac{1}{ef^7}}\,(af-be)\,(cf-de)^2\log\left(-\frac{ef^3\sqrt{-\frac{1}{ef^7}}(af-be)(cf-de)^2}{ac^2f^3-2acdef^2+ad^2e^2f-bc^2ef^2+2bcde^2f-bd^2e^3}+x\right)}{2} + \frac{\sqrt{-\frac{1}{ef^7}}\,(af-be)\,(cf-de)^2\log\left(\frac{ef^3\sqrt{-\frac{1}{ef^7}}(af-be)(cf-de)^2}{ac^2f^3-2acdef^2+ad^2e^2f-bc^2ef^2+2bcde^2f-bd^2e^3}+x\right)}{2} + \frac{x^3\left(ad^2f+2bcdf-bd^2e\right)}{3f^2} + \frac{x\left(2acdf^2-ad^2ef+bc^2f^2-2bcdef+bd^2e^2\right)}{f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x**2+c)**2/(f*x**2+e),x)
```

```
[Out] b*d**2*x**5/(5*f) - sqrt(-1/(e*f**7))*(a*f - b*e)*(c*f - d*e)**2*
log(-e*f**3*sqrt(-1/(e*f**7))*(a*f - b*e)*(c*f - d*e)**2/(a*c**2*
f**3 - 2*a*c*d*e*f**2 + a*d**2*e**2*f - b*c**2*e*f**2 + 2*b*c*d*e
**2*f - b*d**2*e**3) + x)/2 + sqrt(-1/(e*f**7))*(a*f - b*e)*(c*f
- d*e)**2*log(e*f**3*sqrt(-1/(e*f**7))*(a*f - b*e)*(c*f - d*e)**2
/(a*c**2*f**3 - 2*a*c*d*e*f**2 + a*d**2*e**2*f - b*c**2*e*f**2 +
2*b*c*d*e**2*f - b*d**2*e**3) + x)/2 + x**3*(a*d**2*f + 2*b*c*d*f
 - b*d**2*e)/(3*f**2) + x*(2*a*c*d*f**2 - a*d**2*e*f + b*c**2*f**
2 - 2*b*c*d*e*f + b*d**2*e**2)/f**3
```

GIAC/XCAS [A] time = 0.228411, size = 240, normalized size = 1.69

$$\frac{\left(ac^2f^3 - bc^2f^2e - 2\,acdf^2e + 2\,bcdfe^2 + ad^2fe^2 - bd^2e^3\right)\arctan\left(\sqrt{f}xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{1}{2}\right)}}{f^{\frac{7}{2}}} + \frac{3\,bd^2f^4x^5 + 10\,bcdf^4x^3 + 5\,ad^2f^4x^3 - 5\,bd^2f^3x^3e + 15\,bc^2f^4x + 30\,acdf^4x - 30\,bcdf^3xe - 15\,ad^2f^3xe + 15\,bd^2f^2xe^2}{15\,f^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)^2/(f*x^2 + e),x, algorithm="giac")
```

```
[Out] (a*c^2*f^3 - b*c^2*f^2*e - 2*a*c*d*f^2*e + 2*b*c*d*f*e^2 + a*d^2*
f*e^2 - b*d^2*e^3)*arctan(sqrt(f)*x*e^(-1/2))*e^(-1/2)/f^(7/2) +
1/15*(3*b*d^2*f^4*x^5 + 10*b*c*d*f^4*x^3 + 5*a*d^2*f^4*x^3 - 5*b*
d^2*f^3*x^3*e + 15*b*c^2*f^4*x + 30*a*c*d*f^4*x - 30*b*c*d*f^3*x*
e - 15*a*d^2*f^3*x*e + 15*b*d^2*f^2*x*e^2)/f^5
```

3.13 $\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^2} dx$

Optimal. Leaf size=164

$$\frac{(de-cf)\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(be(5de-cf)-af(cf+3de))}{2e^{3/2}f^{7/2}}-\frac{dx(be(15de-13cf)-3af(3de-cf))}{6ef^3}+\frac{dx\left(c+dx^2\right)(5be-3af)}{6ef^2}-\frac{x\left(c+dx^2\right)^2(be-af)}{2ef\left(e+fx^2\right)}$$

```
[Out] -(d*(b*e*(15*d*e - 13*c*f) - 3*a*f*(3*d*e - c*f))*x)/(6*e*f^3) +
(d*(5*b*e - 3*a*f)*x*(c + d*x^2))/(6*e*f^2) - ((b*e - a*f)*x*(c +
 d*x^2)^2)/(2*e*f*(e + f*x^2)) + ((d*e - c*f)*(b*e*(5*d*e - c*f)
- a*f*(3*d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(2*e^(3/2)*f^(7
/2))
```

Rubi [A] time = 0.610264, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{(de-cf)\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(be(5de-cf)-af(cf+3de))}{2e^{3/2}f^{7/2}}-\frac{dx(be(15de-13cf)-3af(3de-cf))}{6ef^3}+\frac{dx\left(c+dx^2\right)(5be-3af)}{6ef^2}-\frac{x\left(c+dx^2\right)^2(be-af)}{2ef\left(e+fx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^2,x]
```

```
[Out] -(d*(b*e*(15*d*e - 13*c*f) - 3*a*f*(3*d*e - c*f))*x)/(6*e*f^3) +
(d*(5*b*e - 3*a*f)*x*(c + d*x^2))/(6*e*f^2) - ((b*e - a*f)*x*(c +
 d*x^2)^2)/(2*e*f*(e + f*x^2)) + ((d*e - c*f)*(b*e*(5*d*e - c*f)
- a*f*(3*d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(2*e^(3/2)*f^(7
/2))
```

Rubi in Sympy [A] time = 58.1949, size = 168, normalized size = 1.02

$$\frac{dx\left(c\left(af+be\right)-dx^2\left(3af-5be\right)\right)}{6ef^2}-\frac{dx\left(7acf^2-9adef-17bcef+15bde^2\right)}{6ef^3}+\frac{x\left(c+dx^2\right)^2\left(af-be\right)}{2ef\left(e+fx^2\right)}+\frac{\left(cf-de\right)\left(acf^2+3adef+bcef-5bde^2\right)\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{\frac{3}{2}}f^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)**2/(f*x**2+e)**2,x)
```

```
[Out] d*x*(c*(a*f + b*e) - d*x**2*(3*a*f - 5*b*e))/(6*e*f**2) - d*x*(7*
a*c*f**2 - 9*a*d*e*f - 17*b*c*e*f + 15*b*d*e**2)/(6*e*f**3) + x*(
c + d*x**2)**2*(a*f - b*e)/(2*e*f*(e + f*x**2)) + (c*f - d*e)*(a*
c*f**2 + 3*a*d*e*f + b*c*e*f - 5*b*d*e**2)*atan(sqrt(f)*x/sqrt(e)
)/(2*e**(3/2)*f**(7/2))
```

Mathematica [A]   time = 0.171502, size = 134, normalized size = 0.82

$$\frac{(de-cf)\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(be(5de-cf)-af(cf+3de))}{2e^{3/2}f^{7/2}}-\frac{x(be-af)(de-cf)^2}{2ef^3\left(e+fx^2\right)}+\frac{dx(adf+2bcf-2bde)}{f^3}+\frac{bd^2x^3}{3f^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^2,x]
```

```
[Out] (d*(-2*b*d*e + 2*b*c*f + a*d*f)*x)/f^3 + (b*d^2*x^3)/(3*f^2) - ((
b*e - a*f)*(d*e - c*f)^2*x)/(2*e*f^3*(e + f*x^2)) + ((d*e - c*f)*
(b*e*(5*d*e - c*f) - a*f*(3*d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e
]])/(2*e^(3/2)*f^(7/2))
```

Maple [B]   time = 0.014, size = 299, normalized size = 1.8

$$\frac{d^2x^3b}{3f^2}+\frac{d^2xa}{f^2}+2\frac{dxbc}{f^2}-2\frac{d^2xbe}{f^3}+\frac{axc^2}{2e\left(fx^2+e\right)}-\frac{axcd}{f\left(fx^2+e\right)}+\frac{exad^2}{2f^2\left(fx^2+e\right)}-\frac{bxc^2}{2f\left(fx^2+e\right)}+\frac{exbcd}{f^2\left(fx^2+e\right)}-\frac{e^2xbd^2}{2f^3\left(fx^2+e\right)}+\frac{ac^2}{2e}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}+\frac{acd}{f}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}-\frac{3ad^2e}{2f^2}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}+\frac{bc^2}{2f}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}-3\frac{bcde}{f^2\sqrt{ef}}\arctan\left(\frac{fx}{\sqrt{ef}}\right)+\frac{5bd^2e^2}{2f^3}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^2,x)
```

```
[Out] 1/3*d^2/f^2*x^3*b+d^2/f^2*x*a+2*d/f^2*x*b*c-2*d^2/f^3*x*b*e+1/2/e
*x/(f*x^2+e)*a*c^2-1/f*x/(f*x^2+e)*a*c*d+1/2/f^2*e*x/(f*x^2+e)*a*
d^2-1/2/f*x/(f*x^2+e)*b*c^2+1/f^2*e*x/(f*x^2+e)*b*c*d-1/2/f^3*e^2
*x/(f*x^2+e)*b*d^2+1/2/e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a*c^
2+1/f/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a*c*d-3/2/f^2*e/(e*f)^(
1/2)*arctan(x*f/(e*f)^(1/2))*a*d^2+1/2/f/(e*f)^(1/2)*arctan(x*f/(
e*f)^(1/2))*b*c^2-3/f^2*e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*c
*d+5/2/f^3*e^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^2/(f*x^2 + e)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.221101, size = 1, normalized size = 0.01

$$\left[\frac{3\left(5\,bd^2e^4 + ac^2ef^3 - 3\left(2\,bcd + ad^2\right)e^3f + \left(bc^2 + 2\,acd\right)e^2f^2 + \left(5\,bd^2e^3f + ac^2f^4 - 3\left(2\,bcd + ad^2\right)e^2f^2 + \left(bc^2 + 2\,acd\right)e\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^2/(f*x^2 + e)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(5*b*d^2*e^4 + a*c^2*e*f^3 - 3*(2*b*c*d + a*d^2)*e^3*f +
 (b*c^2 + 2*a*c*d)*e^2*f^2 + (5*b*d^2*e^3*f + a*c^2*f^4 - 3*(2*b*
c*d + a*d^2)*e^2*f^2 + (b*c^2 + 2*a*c*d)*e*f^3)*x^2)*log((2*e*f*x
 + (f*x^2 - e)*sqrt(-e*f))/(f*x^2 + e)) + 2*(2*b*d^2*e*f^2*x^5 -
2*(5*b*d^2*e^2*f - 3*(2*b*c*d + a*d^2)*e*f^2)*x^3 - 3*(5*b*d^2*e^
3 - a*c^2*f^3 - 3*(2*b*c*d + a*d^2)*e^2*f + (b*c^2 + 2*a*c*d)*e*f
^2)*x)*sqrt(-e*f))/((e*f^4*x^2 + e^2*f^3)*sqrt(-e*f)), 1/6*(3*(5*
b*d^2*e^4 + a*c^2*e*f^3 - 3*(2*b*c*d + a*d^2)*e^3*f + (b*c^2 + 2*
a*c*d)*e^2*f^2 + (5*b*d^2*e^3*f + a*c^2*f^4 - 3*(2*b*c*d + a*d^2)
*e^2*f^2 + (b*c^2 + 2*a*c*d)*e*f^3)*x^2)*arctan(sqrt(e*f)*x/e) +
(2*b*d^2*e*f^2*x^5 - 2*(5*b*d^2*e^2*f - 3*(2*b*c*d + a*d^2)*e*f^2
)*x^3 - 3*(5*b*d^2*e^3 - a*c^2*f^3 - 3*(2*b*c*d + a*d^2)*e^2*f +
(b*c^2 + 2*a*c*d)*e*f^2)*x)*sqrt(e*f))/((e*f^4*x^2 + e^2*f^3)*sqr
t(e*f))]
```

Sympy [A] time = 6.53106, size = 479, normalized size = 2.92

$$\frac{bd^2x^3}{3f^2} + \frac{x\left(ac^2f^3 - 2acdef^2 + ad^2e^2f - bc^2ef^2 + 2bcde^2f - bd^2e^3\right)}{2e^2f^3 + 2ef^4x^2}$$
$$- \frac{\sqrt{-\frac{1}{e^3f^7}}\left(cf - de\right)\left(acf^2 + 3adef + bcef - 5bde^2\right)\log\left(-\frac{e^2f^3\sqrt{-\frac{1}{e^3f^7}}(cf-de)\left(acf^2+3adef+bcef-5bde^2\right)}{ac^2f^3+2acdef^2-3ad^2e^2f+bc^2ef^2-6bcde^2f+5bd^2e^3} + x\right)}{4}$$
$$+ \frac{\sqrt{-\frac{1}{e^3f^7}}\left(cf - de\right)\left(acf^2 + 3adef + bcef - 5bde^2\right)\log\left(\frac{e^2f^3\sqrt{-\frac{1}{e^3f^7}}(cf-de)\left(acf^2+3adef+bcef-5bde^2\right)}{ac^2f^3+2acdef^2-3ad^2e^2f+bc^2ef^2-6bcde^2f+5bd^2e^3} + x\right)}{4}$$
$$+ \frac{x\left(ad^2f + 2bcdf - 2bd^2e\right)}{f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x**2+c)**2/(f*x**2+e)**2,x)
```

```
[Out] b*d**2*x**3/(3*f**2) + x*(a*c**2*f**3 - 2*a*c*d*e*f**2 + a*d**2*e
**2*f - b*c**2*e*f**2 + 2*b*c*d*e**2*f - b*d**2*e**3)/(2*e**2*f**
3 + 2*e*f**4*x**2) - sqrt(-1/(e**3*f**7))*(c*f - d*e)*(a*c*f**2 +
 3*a*d*e*f + b*c*e*f - 5*b*d*e**2)*log(-e**2*f**3*sqrt(-1/(e**3*f
**7))*(c*f - d*e)*(a*c*f**2 + 3*a*d*e*f + b*c*e*f - 5*b*d*e**2)/(
a*c**2*f**3 + 2*a*c*d*e*f**2 - 3*a*d**2*e**2*f + b*c**2*e*f**2 -
6*b*c*d*e**2*f + 5*b*d**2*e**3) + x)/4 + sqrt(-1/(e**3*f**7))*(c*
f - d*e)*(a*c*f**2 + 3*a*d*e*f + b*c*e*f - 5*b*d*e**2)*log(e**2*f
**3*sqrt(-1/(e**3*f**7))*(c*f - d*e)*(a*c*f**2 + 3*a*d*e*f + b*c*
e*f - 5*b*d*e**2)/(a*c**2*f**3 + 2*a*c*d*e*f**2 - 3*a*d**2*e**2*f
 + b*c**2*e*f**2 - 6*b*c*d*e**2*f + 5*b*d**2*e**3) + x)/4 + x*(a*
d**2*f + 2*b*c*d*f - 2*b*d**2*e)/f**3
```

GIAC/XCAS [A] time = 0.230253, size = 263, normalized size = 1.6

$$\frac{\left(ac^2f^3 + bc^2f^2e + 2\,acdf^2e - 6\,bcdfe^2 - 3\,ad^2fe^2 + 5\,bd^2e^3\right)\arctan\left(\sqrt{f}xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{3}{2}\right)}}{2\,f^{\frac{7}{2}}}$$
$$+ \frac{\left(ac^2f^3x - bc^2f^2xe - 2\,acdf^2xe + 2\,bcdfxe^2 + ad^2fxe^2 - bd^2xe^3\right)e^{(-1)}}{2\left(fx^2 + e\right)f^3}$$
$$+ \frac{bd^2f^4x^3 + 6\,bcdf^4x + 3\,ad^2f^4x - 6\,bd^2f^3xe}{3\,f^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^2/(f*x^2 + e)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a*c^2*f^3 + b*c^2*f^2*e + 2*a*c*d*f^2*e - 6*b*c*d*f*e^2 - 3*
a*d^2*f*e^2 + 5*b*d^2*e^3)*arctan(sqrt(f)*x*e^(-1/2))*e^(-3/2)/f^
```

```
(7/2) + 1/2*(a*c^2*f^3*x - b*c^2*f^2*x*e - 2*a*c*d*f^2*x*e + 2*b*
c*d*f*x*e^2 + a*d^2*f*x*e^2 - b*d^2*x*e^3)*e^(-1)/((f*x^2 + e)*f^
3) + 1/3*(b*d^2*f^4*x^3 + 6*b*c*d*f^4*x + 3*a*d^2*f^4*x - 6*b*d^2
*f^3*x*e)/f^6
```

3.14 $\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3}\,dx$

Optimal. Leaf size=207

$$-\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^2f^2-6cdef+15d^2e^2\right)-af\left(3c^2f^2+2cdef+3d^2e^2\right)\right)}{8e^{5/2}f^{7/2}}+\frac{dx(be(15de-cf)-3af(cf+de))}{8e^2f^3}-\frac{x\left(c+dx^2\right)(be(5de-cf)-af(3cf+de))}{8e^2f^2\left(e+fx^2\right)}-\frac{x\left(c+dx^2\right)^2(be-af)}{4ef\left(e+fx^2\right)^2}$$

```
[Out] (d*(b*e*(15*d*e - c*f) - 3*a*f*(d*e + c*f))*x)/(8*e^2*f^3) - ((b*
e - a*f)*x*(c + d*x^2)^2)/(4*e*f*(e + f*x^2)^2) - ((b*e*(5*d*e -
c*f) - a*f*(d*e + 3*c*f))*x*(c + d*x^2))/(8*e^2*f^2*(e + f*x^2))
- ((b*e*(15*d^2*e^2 - 6*c*d*e*f - c^2*f^2) - a*f*(3*d^2*e^2 + 2*c
*d*e*f + 3*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(8*e^(5/2)*f^(7
/2))
```

Rubi [A] time = 0.643559, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$-\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^2f^2-6cdef+15d^2e^2\right)-af\left(3c^2f^2+2cdef+3d^2e^2\right)\right)}{8e^{5/2}f^{7/2}}+\frac{dx(be(15de-cf)-3af(cf+de))}{8e^2f^3}-\frac{x\left(c+dx^2\right)(be(5de-cf)-af(3cf+de))}{8e^2f^2\left(e+fx^2\right)}-\frac{x\left(c+dx^2\right)^2(be-af)}{4ef\left(e+fx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^3,x]
```

```
[Out] (d*(b*e*(15*d*e - c*f) - 3*a*f*(d*e + c*f))*x)/(8*e^2*f^3) - ((b*
e - a*f)*x*(c + d*x^2)^2)/(4*e*f*(e + f*x^2)^2) - ((b*e*(5*d*e -
c*f) - a*f*(d*e + 3*c*f))*x*(c + d*x^2))/(8*e^2*f^2*(e + f*x^2))
- ((b*e*(15*d^2*e^2 - 6*c*d*e*f - c^2*f^2) - a*f*(3*d^2*e^2 + 2*c
*d*e*f + 3*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(8*e^(5/2)*f^(7
/2))
```

Rubi in Sympy [A] time = 63.1617, size = 196, normalized size = 0.95

$$-\frac{dx\left(cf\left(3af+be\right)+3de\left(af-5be\right)\right)}{8e^{2}f^{3}}+\frac{x\left(c+dx^{2}\right)^{2}\left(af-be\right)}{4ef\left(e+fx^{2}\right)^{2}}+\frac{x\left(c+dx^{2}\right)\left(cf\left(3af+be\right)+de\left(af-5be\right)\right)}{8e^{2}f^{2}\left(e+fx^{2}\right)}+\frac{\left(cf\left(cf\left(3af+be\right)-de\left(af-5be\right)\right)+de\left(cf\left(3af+be\right)+3de\left(af-5be\right)\right)\right)\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{8e^{\frac{5}{2}}f^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)**2/(f*x**2+e)**3,x)
```

```
[Out] -d*x*(c*f*(3*a*f + b*e) + 3*d*e*(a*f - 5*b*e))/(8*e**2*f**3) + x*
(c + d*x**2)**2*(a*f - b*e)/(4*e*f*(e + f*x**2)**2) + x*(c + d*x*
*2)*(c*f*(3*a*f + b*e) + d*e*(a*f - 5*b*e))/(8*e**2*f**2*(e + f*x
**2)) + (c*f*(c*f*(3*a*f + b*e) - d*e*(a*f - 5*b*e)) + d*e*(c*f*(
3*a*f + b*e) + 3*d*e*(a*f - 5*b*e)))*atan(sqrt(f)*x/sqrt(e))/(8*e
**(5/2)*f**(7/2))
```

Mathematica [A] time = 0.238367, size = 183, normalized size = 0.88

$$-\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^{2}f^{2}-6cdef+15d^{2}e^{2}\right)-af\left(3c^{2}f^{2}+2cdef+3d^{2}e^{2}\right)\right)}{8e^{5/2}f^{7/2}}+\frac{x(de-cf)(be(9de-cf)-af(3cf+5de))}{8e^{2}f^{3}\left(e+fx^{2}\right)}-\frac{x(be-af)(de-cf)^{2}}{4ef^{3}\left(e+fx^{2}\right)^{2}}+\frac{bd^{2}x}{f^{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^3,x]
```

```
[Out] (b*d^2*x)/f^3 - ((b*e - a*f)*(d*e - c*f)^2*x)/(4*e*f^3*(e + f*x^2
)^2) + ((d*e - c*f)*(b*e*(9*d*e - c*f) - a*f*(5*d*e + 3*c*f))*x)/
(8*e^2*f^3*(e + f*x^2)) - ((b*e*(15*d^2*e^2 - 6*c*d*e*f - c^2*f^2
) - a*f*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2))*ArcTan[(Sqrt[f]*x)/S
qrt[e]])/(8*e^(5/2)*f^(7/2))
```

Maple [B] time = 0.016, size = 397, normalized size = 1.9

$$\frac{bd^{2}x}{f^{3}}+\frac{3\,f\,x^{3}ac^{2}}{8\,\left(f\,x^{2}+e\right)^{2}e^{2}}+\frac{x^{3}acd}{4\,\left(f\,x^{2}+e\right)^{2}e}-\frac{5\,x^{3}ad^{2}}{8\,f\left(f\,x^{2}+e\right)^{2}}+\frac{x^{3}bc^{2}}{8\,\left(f\,x^{2}+e\right)^{2}e}-\frac{5\,x^{3}bcd}{4\,f\left(f\,x^{2}+e\right)^{2}}+\frac{9\,x^{3}bd^{2}e}{8\,f^{2}\left(f\,x^{2}+e\right)^{2}}+\frac{5\,axc^{2}}{8\,\left(f\,x^{2}+e\right)^{2}e}-\frac{acdx}{4\,f\left(f\,x^{2}+e\right)^{2}}-\frac{3\,ad^{2}ex}{8\,f^{2}\left(f\,x^{2}+e\right)^{2}}-\frac{bc^{2}x}{8\,f\left(f\,x^{2}+e\right)^{2}}-\frac{3\,bcdex}{4\,f^{2}\left(f\,x^{2}+e\right)^{2}}+\frac{7\,bd^{2}e^{2}x}{8\,f^{3}\left(f\,x^{2}+e\right)^{2}}+\frac{3\,ac^{2}}{8\,e^{2}}\arctan\left(f\,x\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}+\frac{acd}{4\,ef}\arctan\left(f\,x\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}+\frac{3\,ad^{2}}{8\,f^{2}}\arctan\left(f\,x\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}+\frac{bc^{2}}{8\,ef}\arctan\left(f\,x\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}+\frac{3\,bcd}{4\,f^{2}}\arctan\left(f\,x\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}-\frac{15\,bd^{2}e}{8\,f^{3}}\arctan\left(f\,x\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^3,x)
```

```
[Out] b*d^2/f^3*x+3/8*f/(f*x^2+e)^2/e^2*x^3*a*c^2+1/4/(f*x^2+e)^2/e*x^3
*a*c*d-5/8/f/(f*x^2+e)^2*x^3*a*d^2+1/8/(f*x^2+e)^2/e*x^3*b*c^2-5/
4/f/(f*x^2+e)^2*x^3*b*c*d+9/8/f^2/(f*x^2+e)^2*x^3*b*d^2*e+5/8/(f*
x^2+e)^2/e*x*a*c^2-1/4/f/(f*x^2+e)^2*a*c*d*x-3/8/f^2/(f*x^2+e)^2*
a*d^2*e*x-1/8/f/(f*x^2+e)^2*b*c^2*x-3/4/f^2/(f*x^2+e)^2*b*c*d*e*x
+7/8/f^3/(f*x^2+e)^2*b*d^2*e^2*x+3/8/e^2/(e*f)^(1/2)*arctan(x*f/(
e*f)^(1/2))*a*c^2+1/4/f/e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a*c
*d+3/8/f^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a*d^2+1/8/f/e/(e*f
)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*c^2+3/4/f^2/(e*f)^(1/2)*arctan(
x*f/(e*f)^(1/2))*b*c*d-15/8/f^3*e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1
/2))*b*d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)^2/(f*x^2 + e)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.226148, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^2/(f*x^2 + e)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*((15*b*d^2*e^5 - 3*a*c^2*e^2*f^3 - 3*(2*b*c*d + a*d^2)*e^4
*f - (b*c^2 + 2*a*c*d)*e^3*f^2 + (15*b*d^2*e^3*f^2 - 3*a*c^2*f^5
- 3*(2*b*c*d + a*d^2)*e^2*f^3 - (b*c^2 + 2*a*c*d)*e*f^4)*x^4 + 2*
(15*b*d^2*e^4*f - 3*a*c^2*e*f^4 - 3*(2*b*c*d + a*d^2)*e^3*f^2 - (
b*c^2 + 2*a*c*d)*e^2*f^3)*x^2)*log((2*e*f*x + (f*x^2 - e)*sqrt(-e
*f))/(f*x^2 + e)) - 2*(8*b*d^2*e^2*f^2*x^5 + (25*b*d^2*e^3*f + 3*
a*c^2*f^4 - 5*(2*b*c*d + a*d^2)*e^2*f^2 + (b*c^2 + 2*a*c*d)*e*f^3
)*x^3 + (15*b*d^2*e^4 + 5*a*c^2*e*f^3 - 3*(2*b*c*d + a*d^2)*e^3*f
 - (b*c^2 + 2*a*c*d)*e^2*f^2)*x)*sqrt(-e*f))/((e^2*f^5*x^4 + 2*e^
3*f^4*x^2 + e^4*f^3)*sqrt(-e*f)), -1/8*((15*b*d^2*e^5 - 3*a*c^2*e
^2*f^3 - 3*(2*b*c*d + a*d^2)*e^4*f - (b*c^2 + 2*a*c*d)*e^3*f^2 +
(15*b*d^2*e^3*f^2 - 3*a*c^2*f^5 - 3*(2*b*c*d + a*d^2)*e^2*f^3 - (
b*c^2 + 2*a*c*d)*e*f^4)*x^4 + 2*(15*b*d^2*e^4*f - 3*a*c^2*e*f^4 -
 3*(2*b*c*d + a*d^2)*e^3*f^2 - (b*c^2 + 2*a*c*d)*e^2*f^3)*x^2)*ar
ctan(sqrt(e*f)*x/e) - (8*b*d^2*e^2*f^2*x^5 + (25*b*d^2*e^3*f + 3*
a*c^2*f^4 - 5*(2*b*c*d + a*d^2)*e^2*f^2 + (b*c^2 + 2*a*c*d)*e*f^3
)*x^3 + (15*b*d^2*e^4 + 5*a*c^2*e*f^3 - 3*(2*b*c*d + a*d^2)*e^3*f
 - (b*c^2 + 2*a*c*d)*e^2*f^2)*x)*sqrt(e*f))/((e^2*f^5*x^4 + 2*e^3
*f^4*x^2 + e^4*f^3)*sqrt(e*f))]
```

Sympy [A] time = 29.0162, size = 400, normalized size = 1.93

$$\frac{bd^2x}{f^3} - \frac{\sqrt{-\frac{1}{e^5f^7}}\left(3ac^2f^3 + 2acdef^2 + 3ad^2e^2f + bc^2ef^2 + 6bcde^2f - 15bd^2e^3\right)\log\left(-e^3f^3\sqrt{-\frac{1}{e^5f^7}} + x\right)}{16}$$
$$+ \frac{\sqrt{-\frac{1}{e^5f^7}}\left(3ac^2f^3 + 2acdef^2 + 3ad^2e^2f + bc^2ef^2 + 6bcde^2f - 15bd^2e^3\right)\log\left(e^3f^3\sqrt{-\frac{1}{e^5f^7}} + x\right)}{16}$$
$$+ \frac{x^3\left(3ac^2f^4 + 2acdef^3 - 5ad^2e^2f^2 + bc^2ef^3 - 10bcde^2f^2 + 9bd^2e^3f\right) + x\left(5ac^2ef^3 - 2acde^2f^2 - 3ad^2e^3f - bc^2e^2f^2 - 6b\right.}{8e^4f^3 + 16e^3f^4x^2 + 8e^2f^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x**2+c)**2/(f*x**2+e)**3,x)
```

```
[Out] b*d**2*x/f**3 - sqrt(-1/(e**5*f**7))*(3*a*c**2*f**3 + 2*a*c*d*e*f
**2 + 3*a*d**2*e**2*f + b*c**2*e*f**2 + 6*b*c*d*e**2*f - 15*b*d**
2*e**3)*log(-e**3*f**3*sqrt(-1/(e**5*f**7)) + x)/16 + sqrt(-1/(e*
*5*f**7))*(3*a*c**2*f**3 + 2*a*c*d*e*f**2 + 3*a*d**2*e**2*f + b*c
**2*e*f**2 + 6*b*c*d*e**2*f - 15*b*d**2*e**3)*log(e**3*f**3*sqrt(
-1/(e**5*f**7)) + x)/16 + (x**3*(3*a*c**2*f**4 + 2*a*c*d*e*f**3 -
 5*a*d**2*e**2*f**2 + b*c**2*e*f**3 - 10*b*c*d*e**2*f**2 + 9*b*d*
*2*e**3*f) + x*(5*a*c**2*e*f**3 - 2*a*c*d*e**2*f**2 - 3*a*d**2*e*
*3*f - b*c**2*e**2*f**2 - 6*b*c*d*e**3*f + 7*b*d**2*e**4))/(8*e**
4*f**3 + 16*e**3*f**4*x**2 + 8*e**2*f**5*x**4)
```

GIAC/XCAS [A] time = 0.229108, size = 321, normalized size = 1.55

$$\frac{bd^2x}{f^3} + \frac{\left(3\,ac^2f^3 + bc^2f^2e + 2\,acdf^2e + 6\,bcdfe^2 + 3\,ad^2fe^2 - 15\,bd^2e^3\right)\arctan\left(\sqrt{f}xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{5}{2}\right)}}{8\,f^{\frac{7}{2}}} + \frac{\left(3\,ac^2f^4x^3 + bc^2f^3x^3e + 2\,acdf^3x^3e - 10\,bcdf^2x^3e^2 - 5\,ad^2f^2x^3e^2 + 9\,bd^2fx^3e^3 + 5\,ac^2f^3xe - bc^2f^2xe^2 - 2\,acdf^2xe^2 - \cdots}{8\,(fx^2 + e)^2f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^2/(f*x^2 + e)^3,x, algorithm="giac")
```

```
[Out] b*d^2*x/f^3 + 1/8*(3*a*c^2*f^3 + b*c^2*f^2*e + 2*a*c*d*f^2*e + 6*
b*c*d*f*e^2 + 3*a*d^2*f*e^2 - 15*b*d^2*e^3)*arctan(sqrt(f)*x*e^(-
1/2))*e^(-5/2)/f^(7/2) + 1/8*(3*a*c^2*f^4*x^3 + b*c^2*f^3*x^3*e +
 2*a*c*d*f^3*x^3*e - 10*b*c*d*f^2*x^3*e^2 - 5*a*d^2*f^2*x^3*e^2 +
 9*b*d^2*f*x^3*e^3 + 5*a*c^2*f^3*x*e - b*c^2*f^2*x*e^2 - 2*a*c*d*
f^2*x*e^2 - 6*b*c*d*f*x*e^3 - 3*a*d^2*f*x*e^3 + 7*b*d^2*x*e^4)*e^
(-2)/((f*x^2 + e)^2*f^3)
```

3.15 $\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^4}\,dx$

Optimal. Leaf size=240

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(af\left(5c^2f^2+2cdef+d^2e^2\right)+be\left(c^2f^2+2cdef+5d^2e^2\right)\right)}{16e^{7/2}f^{7/2}} - \frac{x\left(af\left(-15c^2f^2+4cdef+3d^2e^2\right)+be\left(-3c^2f^2-4cdef+15d^2e^2\right)\right)}{48e^3f^3(e+fx^2)} - \frac{x(c+dx^2)(de(af+5be)-cf(5af+be))}{24e^2f^2(e+fx^2)^2} - \frac{x(c+dx^2)^2(be-af)}{6ef(e+fx^2)^3}$$

```
[Out] -((b*e - a*f)*x*(c + d*x^2)^2)/(6*e*f*(e + f*x^2)^3) - ((d*e*(5*b
*e + a*f) - c*f*(b*e + 5*a*f))*x*(c + d*x^2))/(24*e^2*f^2*(e + f*
x^2)^2) - ((a*f*(3*d^2*e^2 + 4*c*d*e*f - 15*c^2*f^2) + b*e*(15*d^
2*e^2 - 4*c*d*e*f - 3*c^2*f^2))*x)/(48*e^3*f^3*(e + f*x^2)) + ((b
*e*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) + a*f*(d^2*e^2 + 2*c*d*e*f +
 5*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(16*e^(7/2)*f^(7/2))
```

Rubi [A] time = 0.754023, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(af\left(5c^2f^2+2cdef+d^2e^2\right)+be\left(c^2f^2+2cdef+5d^2e^2\right)\right)}{16e^{7/2}f^{7/2}} - \frac{x\left(af\left(-15c^2f^2+4cdef+3d^2e^2\right)+be\left(-3c^2f^2-4cdef+15d^2e^2\right)\right)}{48e^3f^3(e+fx^2)} - \frac{x(c+dx^2)(de(af+5be)-cf(5af+be))}{24e^2f^2(e+fx^2)^2} - \frac{x(c+dx^2)^2(be-af)}{6ef(e+fx^2)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^4,x]
```


```
[Out] -((b*e - a*f)*x*(c + d*x^2)^2)/(6*e*f*(e + f*x^2)^3) - ((d*e*(5*b
*e + a*f) - c*f*(b*e + 5*a*f))*x*(c + d*x^2))/(24*e^2*f^2*(e + f*
x^2)^2) - ((a*f*(3*d^2*e^2 + 4*c*d*e*f - 15*c^2*f^2) + b*e*(15*d^
2*e^2 - 4*c*d*e*f - 3*c^2*f^2))*x)/(48*e^3*f^3*(e + f*x^2)) + ((b
*e*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) + a*f*(d^2*e^2 + 2*c*d*e*f +
 5*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(16*e^(7/2)*f^(7/2))
```

Rubi in Sympy [A] time = 66.1663, size = 236, normalized size = 0.98

$$\frac{x\left(c+dx^2\right)^2(af-be)}{6ef\left(e+fx^2\right)^3}+\frac{x\left(c+dx^2\right)\left(cf\left(5af+be\right)-de\left(af+5be\right)\right)}{24e^2f^2\left(e+fx^2\right)^2}+\frac{x\left(cf\left(3cf\left(5af+be\right)+de\left(af+5be\right)\right)-de\left(cf\left(5af+be\right)+3de\left(af+5be\right)\right)\right)}{48e^3f^3\left(e+fx^2\right)}+\frac{\left(cf\left(3cf\left(5af+be\right)+de\left(af+5be\right)\right)+de\left(cf\left(5af+be\right)+3de\left(af+5be\right)\right)\right)\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{48e^{\frac{7}{2}}f^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)**2/(f*x**2+e)**4,x)
```

```
[Out] x*(c + d*x**2)**2*(a*f - b*e)/(6*e*f*(e + f*x**2)**3) + x*(c + d*
x**2)*(c*f*(5*a*f + b*e) - d*e*(a*f + 5*b*e))/(24*e**2*f**2*(e +
f*x**2)**2) + x*(c*f*(3*c*f*(5*a*f + b*e) + d*e*(a*f + 5*b*e)) -
d*e*(c*f*(5*a*f + b*e) + 3*d*e*(a*f + 5*b*e)))/(48*e**3*f**3*(e +
 f*x**2)) + (c*f*(3*c*f*(5*a*f + b*e) + d*e*(a*f + 5*b*e)) + d*e*
(c*f*(5*a*f + b*e) + 3*d*e*(a*f + 5*b*e)))*atan(sqrt(f)*x/sqrt(e)
)/(48*e**(7/2)*f**(7/2))
```

Mathematica [A] time = 0.312869, size = 242, normalized size = 1.01

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(af\left(5c^2f^2+2cdef+d^2e^2\right)+be\left(c^2f^2+2cdef+5d^2e^2\right)\right)}{16e^{7/2}f^{7/2}}+\frac{x\left(af\left(5c^2f^2+2cdef+d^2e^2\right)+be\left(c^2f^2+2cdef-11d^2e^2\right)\right)}{16e^3f^3\left(e+fx^2\right)}+\frac{x(de-cf)(be(13de-cf)-af(5cf+7de))}{24e^2f^3\left(e+fx^2\right)^2}-\frac{x(be-af)(de-cf)^2}{6ef^3\left(e+fx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^4,x]
```

```
[Out] -((b*e - a*f)*(d*e - c*f)^2*x)/(6*e*f^3*(e + f*x^2)^3) + ((d*e -
c*f)*(b*e*(13*d*e - c*f) - a*f*(7*d*e + 5*c*f))*x)/(24*e^2*f^3*(e
 + f*x^2)^2) + ((b*e*(-11*d^2*e^2 + 2*c*d*e*f + c^2*f^2) + a*f*(d
^2*e^2 + 2*c*d*e*f + 5*c^2*f^2))*x)/(16*e^3*f^3*(e + f*x^2)) + ((
b*e*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) + a*f*(d^2*e^2 + 2*c*d*e*f
+ 5*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(16*e^(7/2)*f^(7/2))
```

Maple [A] time = 0.015, size = 360, normalized size = 1.5

$$\frac{1}{(fx^2+e)^3}\left(\frac{(5\,ac^2f^3+2\,acdef^2+ad^2e^2f+bc^2ef^2+2\,bcde^2f-11\,bd^2e^3)\,x^5}{16\,e^3f}+\frac{(5\,ac^2f^3+2\,acdef^2-ad^2e^2f+bc^2ef^2-}{6\,e^2f^2}\right.$$
$$+\frac{5\,ac^2}{16\,e^3}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}+\frac{acd}{8\,e^2f}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}+\frac{ad^2}{16\,ef^2}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}$$
$$+\frac{bc^2}{16\,e^2f}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}+\frac{bcd}{8\,ef^2}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}+\frac{5\,bd^2}{16\,f^3}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^4,x)
```

```
[Out] (1/16*(5*a*c^2*f^3+2*a*c*d*e*f^2+a*d^2*e^2*f+b*c^2*e*f^2+2*b*c*d*
e^2*f-11*b*d^2*e^3)/e^3/f*x^5+1/6*(5*a*c^2*f^3+2*a*c*d*e*f^2-a*d^
2*e^2*f+b*c^2*e*f^2-2*b*c*d*e^2*f-5*b*d^2*e^3)/e^2/f^2*x^3+1/16*(
11*a*c^2*f^3-2*a*c*d*e*f^2-a*d^2*e^2*f-b*c^2*e*f^2-2*b*c*d*e^2*f-
5*b*d^2*e^3)/f^3/e*x)/(f*x^2+e)^3+5/16/e^3/(e*f)^(1/2)*arctan(x*f
/(e*f)^(1/2))*a*c^2+1/8/e^2/f/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))
*a*c*d+1/16/e/f^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a*d^2+1/16/
e^2/f/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*c^2+1/8/e/f^2/(e*f)^(
1/2)*arctan(x*f/(e*f)^(1/2))*b*c*d+5/16/f^3/(e*f)^(1/2)*arctan(x*
f/(e*f)^(1/2))*b*d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)^2/(f*x^2 + e)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.223613, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)^2/(f*x^2 + e)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(3*(5*b*d^2*e^6 + 5*a*c^2*e^3*f^3 + (2*b*c*d + a*d^2)*e^5*f
 + (b*c^2 + 2*a*c*d)*e^4*f^2 + (5*b*d^2*e^3*f^3 + 5*a*c^2*f^6 + (
2*b*c*d + a*d^2)*e^2*f^4 + (b*c^2 + 2*a*c*d)*e*f^5)*x^6 + 3*(5*b*
d^2*e^4*f^2 + 5*a*c^2*e*f^5 + (2*b*c*d + a*d^2)*e^3*f^3 + (b*c^2
+ 2*a*c*d)*e^2*f^4)*x^4 + 3*(5*b*d^2*e^5*f + 5*a*c^2*e^2*f^4 + (2
*b*c*d + a*d^2)*e^4*f^2 + (b*c^2 + 2*a*c*d)*e^3*f^3)*x^2)*log((2*
e*f*x + (f*x^2 - e)*sqrt(-e*f))/(f*x^2 + e)) - 2*(3*(11*b*d^2*e^3
*f^2 - 5*a*c^2*f^5 - (2*b*c*d + a*d^2)*e^2*f^3 - (b*c^2 + 2*a*c*d
)*e*f^4)*x^5 + 8*(5*b*d^2*e^4*f - 5*a*c^2*e*f^4 + (2*b*c*d + a*d^
2)*e^3*f^2 - (b*c^2 + 2*a*c*d)*e^2*f^3)*x^3 + 3*(5*b*d^2*e^5 - 11
*a*c^2*e^2*f^3 + (2*b*c*d + a*d^2)*e^4*f + (b*c^2 + 2*a*c*d)*e^3*
f^2)*x)*sqrt(-e*f))/((e^3*f^6*x^6 + 3*e^4*f^5*x^4 + 3*e^5*f^4*x^2
 + e^6*f^3)*sqrt(-e*f)), 1/48*(3*(5*b*d^2*e^6 + 5*a*c^2*e^3*f^3 +
 (2*b*c*d + a*d^2)*e^5*f + (b*c^2 + 2*a*c*d)*e^4*f^2 + (5*b*d^2*e
^3*f^3 + 5*a*c^2*f^6 + (2*b*c*d + a*d^2)*e^2*f^4 + (b*c^2 + 2*a*c
*d)*e*f^5)*x^6 + 3*(5*b*d^2*e^4*f^2 + 5*a*c^2*e*f^5 + (2*b*c*d +
a*d^2)*e^3*f^3 + (b*c^2 + 2*a*c*d)*e^2*f^4)*x^4 + 3*(5*b*d^2*e^5*
f + 5*a*c^2*e^2*f^4 + (2*b*c*d + a*d^2)*e^4*f^2 + (b*c^2 + 2*a*c*
d)*e^3*f^3)*x^2)*arctan(sqrt(e*f)*x/e) - (3*(11*b*d^2*e^3*f^2 - 5
*a*c^2*f^5 - (2*b*c*d + a*d^2)*e^2*f^3 - (b*c^2 + 2*a*c*d)*e*f^4)
*x^5 + 8*(5*b*d^2*e^4*f - 5*a*c^2*e*f^4 + (2*b*c*d + a*d^2)*e^3*f
^2 - (b*c^2 + 2*a*c*d)*e^2*f^3)*x^3 + 3*(5*b*d^2*e^5 - 11*a*c^2*e
^2*f^3 + (2*b*c*d + a*d^2)*e^4*f + (b*c^2 + 2*a*c*d)*e^3*f^2)*x)*
sqrt(e*f))/((e^3*f^6*x^6 + 3*e^4*f^5*x^4 + 3*e^5*f^4*x^2 + e^6*f^
3)*sqrt(e*f))]
```

Sympy [A] time = 120.765, size = 486, normalized size = 2.02

$$-\frac{\sqrt{-\frac{1}{e^7f^7}}\left(5ac^2f^3+2acdef^2+ad^2e^2f+bc^2ef^2+2bcde^2f+5bd^2e^3\right)\log\left(-e^4f^3\sqrt{-\frac{1}{e^7f^7}}+x\right)}{32}+\frac{\sqrt{-\frac{1}{e^7f^7}}\left(5ac^2f^3+2acdef^2+ad^2e^2f+bc^2ef^2+2bcde^2f+5bd^2e^3\right)\log\left(e^4f^3\sqrt{-\frac{1}{e^7f^7}}+x\right)}{32}+\frac{x^5\left(15ac^2f^5+6acdef^4+3ad^2e^2f^3+3bc^2ef^4+6bcde^2f^3-33bd^2e^3f^2\right)+x^3\left(40ac^2ef^4+16acde^2f^3-8ad^2e^3f^2+8bc^2e\right.}{48e^6f^3+144e^5f^4x^2+144e^4f^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x**2+c)**2/(f*x**2+e)**4,x)
```

```
[Out] -sqrt(-1/(e**7*f**7))*(5*a*c**2*f**3 + 2*a*c*d*e*f**2 + a*d**2*e*
*2*f + b*c**2*e*f**2 + 2*b*c*d*e**2*f + 5*b*d**2*e**3)*log(-e**4*
f**3*sqrt(-1/(e**7*f**7)) + x)/32 + sqrt(-1/(e**7*f**7))*(5*a*c**
2*f**3 + 2*a*c*d*e*f**2 + a*d**2*e**2*f + b*c**2*e*f**2 + 2*b*c*d
*e**2*f + 5*b*d**2*e**3)*log(e**4*f**3*sqrt(-1/(e**7*f**7)) + x)/
32 + (x**5*(15*a*c**2*f**5 + 6*a*c*d*e*f**4 + 3*a*d**2*e**2*f**3
+ 3*b*c**2*e*f**4 + 6*b*c*d*e**2*f**3 - 33*b*d**2*e**3*f**2) + x*
*3*(40*a*c**2*e*f**4 + 16*a*c*d*e**2*f**3 - 8*a*d**2*e**3*f**2 +
8*b*c**2*e**2*f**3 - 16*b*c*d*e**3*f**2 - 40*b*d**2*e**4*f) + x*(
33*a*c**2*e**2*f**3 - 6*a*c*d*e**3*f**2 - 3*a*d**2*e**4*f - 3*b*c
```

```
**2*e**3*f**2 - 6*b*c*d*e**4*f - 15*b*d**2*e**5))/(48*e**6*f**3 +
 144*e**5*f**4*x**2 + 144*e**4*f**5*x**4 + 48*e**3*f**6*x**6)
```

GIAC/XCAS [A] time = 0.231742, size = 420, normalized size = 1.75

$$\frac{\left(5\,ac^2f^3 + bc^2f^2e + 2\,acdf^2e + 2\,bcdfe^2 + ad^2fe^2 + 5\,bd^2e^3\right)\arctan\left(\sqrt{f}xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{7}{2}\right)}}{16\,f^{\frac{7}{2}}} + \frac{\left(15\,ac^2f^5x^5 + 3\,bc^2f^4x^5e + 6\,acdf^4x^5e + 6\,bcdf^3x^5e^2 + 3\,ad^2f^3x^5e^2 - 33\,bd^2f^2x^5e^3 + 40\,ac^2f^4x^3e + 8\,bc^2f^3x^3e^2 + 16\,ac\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^2/(f*x^2 + e)^4,x, algorithm="giac")
```

```
[Out] 1/16*(5*a*c^2*f^3 + b*c^2*f^2*e + 2*a*c*d*f^2*e + 2*b*c*d*f*e^2 +
 a*d^2*f*e^2 + 5*b*d^2*e^3)*arctan(sqrt(f)*x*e^(-1/2))*e^(-7/2)/f
^(7/2) + 1/48*(15*a*c^2*f^5*x^5 + 3*b*c^2*f^4*x^5*e + 6*a*c*d*f^4
*x^5*e + 6*b*c*d*f^3*x^5*e^2 + 3*a*d^2*f^3*x^5*e^2 - 33*b*d^2*f^2
*x^5*e^3 + 40*a*c^2*f^4*x^3*e + 8*b*c^2*f^3*x^3*e^2 + 16*a*c*d*f^
3*x^3*e^2 - 16*b*c*d*f^2*x^3*e^3 - 8*a*d^2*f^2*x^3*e^3 - 40*b*d^2
*f*x^3*e^4 + 33*a*c^2*f^3*x*e^2 - 3*b*c^2*f^2*x*e^3 - 6*a*c*d*f^2
*x*e^3 - 6*b*c*d*f*x*e^4 - 3*a*d^2*f*x*e^4 - 15*b*d^2*x*e^5)*e^(-
3)/((f*x^2 + e)^3*f^3)
```

3.16 $\int (a+bx^2)(c+dx^2)^3(e+fx^2)^3\,dx$

Optimal. Leaf size=310

$$\frac{3}{11}dfx^{11}\left(adf(cf+de)+b\left(c^2f^2+3cdef+d^2e^2\right)\right)+\frac{3}{5}cex^5\left(a\left(c^2f^2+3cdef+d^2e^2\right)+bce(cf+de)\right)+\frac{1}{3}c^2e^2x^3(3a(cf+de)+bce)+\frac{1}{9}x^9\left(3adf\left(c^2f^2+3cdef+d^2e^2\right)+b\left(c^3f^3+9c^2def^2+9cd^2e^2f+d^3e^3\right)\right)+\frac{1}{7}x^7\left(a\left(c^3f^3+9c^2def^2+9cd^2e^2f+d^3e^3\right)+3bce\left(c^2f^2+3cdef+d^2e^2\right)\right)+\frac{1}{13}d^2f^2x^{13}(adf+3b(cf+de))+ac^3e^3x+\frac{1}{15}bd^3f^3x^{15}$$

```
[Out] a*c^3*e^3*x + (c^2*e^2*(b*c*e + 3*a*(d*e + c*f))*x^3)/3 + (3*c*e*
(b*c*e*(d*e + c*f) + a*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^5)/5 +
((3*b*c*e*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + a*(d^3*e^3 + 9*c*d^2*
e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^7)/7 + ((3*a*d*f*(d^2*e^2 + 3
*c*d*e*f + c^2*f^2) + b*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2
+ c^3*f^3))*x^9)/9 + (3*d*f*(a*d*f*(d*e + c*f) + b*(d^2*e^2 + 3*c
*d*e*f + c^2*f^2))*x^11)/11 + (d^2*f^2*(a*d*f + 3*b*(d*e + c*f))*
x^13)/13 + (b*d^3*f^3*x^15)/15
```

Rubi [A] time = 0.900497, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$

$$\frac{3}{11}dfx^{11}\left(adf(cf+de)+b\left(c^2f^2+3cdef+d^2e^2\right)\right)+\frac{3}{5}cex^5\left(a\left(c^2f^2+3cdef+d^2e^2\right)+bce(cf+de)\right)+\frac{1}{3}c^2e^2x^3(3a(cf+de)+bce)+\frac{1}{9}x^9\left(3adf\left(c^2f^2+3cdef+d^2e^2\right)+b\left(c^3f^3+9c^2def^2+9cd^2e^2f+d^3e^3\right)\right)+\frac{1}{7}x^7\left(a\left(c^3f^3+9c^2def^2+9cd^2e^2f+d^3e^3\right)+3bce\left(c^2f^2+3cdef+d^2e^2\right)\right)+\frac{1}{13}d^2f^2x^{13}(adf+3b(cf+de))+ac^3e^3x+\frac{1}{15}bd^3f^3x^{15}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)*(c + d*x^2)^3*(e + f*x^2)^3,x]
```


```
[Out] a*c^3*e^3*x + (c^2*e^2*(b*c*e + 3*a*(d*e + c*f))*x^3)/3 + (3*c*e*
(b*c*e*(d*e + c*f) + a*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^5)/5 +
((3*b*c*e*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + a*(d^3*e^3 + 9*c*d^2*
e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^7)/7 + ((3*a*d*f*(d^2*e^2 + 3
*c*d*e*f + c^2*f^2) + b*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2
+ c^3*f^3))*x^9)/9 + (3*d*f*(a*d*f*(d*e + c*f) + b*(d^2*e^2 + 3*c
*d*e*f + c^2*f^2))*x^11)/11 + (d^2*f^2*(a*d*f + 3*b*(d*e + c*f))*
```

```
x^13)/13 + (b*d^3*f^3*x^15)/15
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{bd^3f^3x^{15}}{15}+c^3e^3\int a\,dx+\frac{c^2e^2x^3\,(3acf+3ade+bce)}{3}+\frac{3cex^5\,\left(ac^2f^2+3acdef+ad^2e^2+bc^2ef+bcde^2\right)}{5}+\frac{d^2f^2x^{13}\,(adf+3bcf+3bde)}{13}+\frac{3dfx^{11}\,\left(acdf^2+ad^2ef+bc^2f^2+3bcdef+bd^2e^2\right)}{11}+x^9\left(\frac{ac^2df^3}{3}+acd^2ef^2+\frac{ad^3e^2f}{3}+\frac{bc^3f^3}{9}+bc^2def^2+bcd^2e^2f+\frac{bd^3e^3}{9}\right)+x^7\left(\frac{ac^3f^3}{7}+\frac{9ac^2def^2}{7}+\frac{9acd^2e^2f}{7}+\frac{ad^3e^3}{7}+\frac{3bc^3ef^2}{7}+\frac{9bc^2de^2f}{7}+\frac{3bcd^2e^3}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)**3*(f*x**2+e)**3,x)
```

```
[Out] b*d**3*f**3*x**15/15 + c**3*e**3*Integral(a, x) + c**2*e**2*x**3*
(3*a*c*f + 3*a*d*e + b*c*e)/3 + 3*c*e*x**5*(a*c**2*f**2 + 3*a*c*d
*e*f + a*d**2*e**2 + b*c**2*e*f + b*c*d*e**2)/5 + d**2*f**2*x**13
*(a*d*f + 3*b*c*f + 3*b*d*e)/13 + 3*d*f*x**11*(a*c*d*f**2 + a*d**
2*e*f + b*c**2*f**2 + 3*b*c*d*e*f + b*d**2*e**2)/11 + x**9*(a*c**
2*d*f**3/3 + a*c*d**2*e*f**2 + a*d**3*e**2*f/3 + b*c**3*f**3/9 +
b*c**2*d*e*f**2 + b*c*d**2*e**2*f + b*d**3*e**3/9) + x**7*(a*c**3
*f**3/7 + 9*a*c**2*d*e*f**2/7 + 9*a*c*d**2*e**2*f/7 + a*d**3*e**3
/7 + 3*b*c**3*e*f**2/7 + 9*b*c**2*d*e**2*f/7 + 3*b*c*d**2*e**3/7)
```

Mathematica [A] time = 0.241811, size = 310, normalized size = 1.

$$\frac{3}{11}dfx^{11}\left(adf(cf+de)+b\left(c^2f^2+3cdef+d^2e^2\right)\right)+\frac{3}{5}cex^5\left(a\left(c^2f^2+3cdef+d^2e^2\right)+bce(cf+de)\right)+\frac{1}{3}c^2e^2x^3(3a(cf+de)+bce)+\frac{1}{9}x^9\left(3adf\left(c^2f^2+3cdef+d^2e^2\right)+b\left(c^3f^3+9c^2def^2+9cd^2e^2f+d^3e^3\right)\right)+\frac{1}{7}x^7\left(a\left(c^3f^3+9c^2def^2+9cd^2e^2f+d^3e^3\right)+3bce\left(c^2f^2+3cdef+d^2e^2\right)\right)+\frac{1}{13}d^2f^2x^{13}(adf+3b(cf+de))+ac^3e^3x+\frac{1}{15}bd^3f^3x^{15}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)*(c + d*x^2)^3*(e + f*x^2)^3,x]
```

```
[Out] a*c^3*e^3*x + (c^2*e^2*(b*c*e + 3*a*(d*e + c*f))*x^3)/3 + (3*c*e*
(b*c*e*(d*e + c*f) + a*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^5)/5 +
((3*b*c*e*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + a*(d^3*e^3 + 9*c*d^2*
e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^7)/7 + ((3*a*d*f*(d^2*e^2 + 3
*c*d*e*f + c^2*f^2) + b*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2
+ c^3*f^3))*x^9)/9 + (3*d*f*(a*d*f*(d*e + c*f) + b*(d^2*e^2 + 3*c
*d*e*f + c^2*f^2))*x^11)/11 + (d^2*f^2*(a*d*f + 3*b*(d*e + c*f))*
x^13)/13 + (b*d^3*f^3*x^15)/15
```

Maple [A] time = 0.002, size = 339, normalized size = 1.1

$$\frac{bd^3f^3x^{15}}{15}+\frac{\left(\left(ad^3+3\,bcd^2\right)f^3+3\,bd^3ef^2\right)x^{13}}{13}+\frac{\left(\left(3\,acd^2+3\,bc^2d\right)f^3+3\,\left(ad^3+3\,bcd^2\right)ef^2+3\,bd^3e^2f\right)x^{11}}{11}+\frac{\left(\left(3\,ac^2d+bc^3\right)f^3+3\,\left(3\,acd^2+3\,bc^2d\right)ef^2+3\,\left(ad^3+3\,bcd^2\right)e^2f+bd^3e^3\right)x^9}{9}+\frac{\left(ac^3f^3+3\,\left(3\,ac^2d+bc^3\right)ef^2+3\,\left(3\,acd^2+3\,bc^2d\right)e^2f+\left(ad^3+3\,bcd^2\right)e^3\right)x^7}{7}+\frac{\left(3\,ac^3ef^2+3\,\left(3\,ac^2d+bc^3\right)e^2f+\left(3\,acd^2+3\,bc^2d\right)e^3\right)x^5}{5}+\frac{\left(3\,ac^3e^2f+\left(3\,ac^2d+bc^3\right)e^3\right)x^3}{3}+ac^3e^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^3,x)
```

```
[Out] 1/15*b*d^3*f^3*x^15+1/13*((a*d^3+3*b*c*d^2)*f^3+3*b*d^3*e*f^2)*x^
13+1/11*((3*a*c*d^2+3*b*c^2*d)*f^3+3*(a*d^3+3*b*c*d^2)*e*f^2+3*b*
d^3*e^2*f)*x^11+1/9*((3*a*c^2*d+b*c^3)*f^3+3*(3*a*c*d^2+3*b*c^2*d
)*e*f^2+3*(a*d^3+3*b*c*d^2)*e^2*f+b*d^3*e^3)*x^9+1/7*(a*c^3*f^3+3
*(3*a*c^2*d+b*c^3)*e*f^2+3*(3*a*c*d^2+3*b*c^2*d)*e^2*f+(a*d^3+3*b
*c*d^2)*e^3)*x^7+1/5*(3*a*c^3*e*f^2+3*(3*a*c^2*d+b*c^3)*e^2*f+(3*
a*c*d^2+3*b*c^2*d)*e^3)*x^5+1/3*(3*a*c^3*e^2*f+(3*a*c^2*d+b*c^3)*
e^3)*x^3+a*c^3*e^3*x
```

Maxima [A] time = 1.36075, size = 440, normalized size = 1.42

$$\frac{1}{15}\,bd^3f^3x^{15} + \frac{1}{13}\,\left(3\,bd^3ef^2 + \left(3\,bcd^2 + ad^3\right)f^3\right)x^{13} + \frac{3}{11}\,\left(bd^3e^2f + \left(3\,bcd^2 + ad^3\right)ef^2 + \left(bc^2d + acd^2\right)f^3\right)x^{11} + \frac{1}{9}\,\left(bd^3e^3 + 3\,\left(3\,bcd^2 + ad^3\right)e^2f + 9\,\left(bc^2d + acd^2\right)ef^2 + \left(bc^3 + 3\,ac^2d\right)f^3\right)x^9 + ac^3e^3x + \frac{1}{7}\,\left(ac^3f^3 + \left(3\,bcd^2 + ad^3\right)e^3 + 9\,\left(bc^2d + acd^2\right)e^2f + 3\,\left(bc^3 + 3\,ac^2d\right)ef^2\right)x^7 + \frac{3}{5}\,\left(ac^3ef^2 + \left(bc^2d + acd^2\right)e^3 + \left(bc^3 + 3\,ac^2d\right)e^2f\right)x^5 + \frac{1}{3}\,\left(3\,ac^3e^2f + \left(bc^3 + 3\,ac^2d\right)e^3\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^3*(f*x^2 + e)^3,x, algorithm="maxima")
```

```
[Out] 1/15*b*d^3*f^3*x^15 + 1/13*(3*b*d^3*e*f^2 + (3*b*c*d^2 + a*d^3)*f
^3)*x^13 + 3/11*(b*d^3*e^2*f + (3*b*c*d^2 + a*d^3)*e*f^2 + (b*c^2
*d + a*c*d^2)*f^3)*x^11 + 1/9*(b*d^3*e^3 + 3*(3*b*c*d^2 + a*d^3)*
e^2*f + 9*(b*c^2*d + a*c*d^2)*e*f^2 + (b*c^3 + 3*a*c^2*d)*f^3)*x^
9 + a*c^3*e^3*x + 1/7*(a*c^3*f^3 + (3*b*c*d^2 + a*d^3)*e^3 + 9*(b
*c^2*d + a*c*d^2)*e^2*f + 3*(b*c^3 + 3*a*c^2*d)*e*f^2)*x^7 + 3/5*
(a*c^3*e*f^2 + (b*c^2*d + a*c*d^2)*e^3 + (b*c^3 + 3*a*c^2*d)*e^2*
f)*x^5 + 1/3*(3*a*c^3*e^2*f + (b*c^3 + 3*a*c^2*d)*e^3)*x^3
```

Fricas [A] time = 0.183341, size = 1, normalized size = 0.

$$\frac{1}{15}x^{15}f^3d^3b + \frac{3}{13}x^{13}f^2ed^3b + \frac{3}{13}x^{13}f^3d^2cb + \frac{1}{13}x^{13}f^3d^3a + \frac{3}{11}x^{11}fe^2d^3b + \frac{9}{11}x^{11}f^2ed^2cb + \frac{3}{11}x^{11}f^3dc^2b + \frac{3}{11}x^{11}f^2ed^3a + \frac{3}{11}x^{11}f^3d^2ca + \frac{1}{9}x^9e^3d^3b + x^9fe^2d^2cb + x^9f^2edc^2b + \frac{1}{9}x^9f^3c^3b + \frac{1}{3}x^9fe^2d^3a + x^9f^2ed^2ca + \frac{1}{3}x^9f^3dc^2a + \frac{3}{7}x^7e^3d^2cb + \frac{9}{7}x^7fe^2dc^2b + \frac{3}{7}x^7f^2ec^3b + \frac{1}{7}x^7e^3d^3a + \frac{9}{7}x^7fe^2d^2ca + \frac{9}{7}x^7f^2edc^2a + \frac{1}{7}x^7f^3c^3a + \frac{3}{5}x^5e^3dc^2b + \frac{3}{5}x^5fe^2c^3b + \frac{3}{5}x^5e^3d^2ca + \frac{9}{5}x^5fe^2dc^2a + \frac{3}{5}x^5f^2ec^3a + \frac{1}{3}x^3e^3c^3b + x^3e^3dc^2a + x^3fe^2c^3a + xe^3c^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^3*(f*x^2 + e)^3,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*f^3*d^3*b + 3/13*x^13*f^2*e*d^3*b + 3/13*x^13*f^3*d^2*c
*b + 1/13*x^13*f^3*d^3*a + 3/11*x^11*f*e^2*d^3*b + 9/11*x^11*f^2*
e*d^2*c*b + 3/11*x^11*f^3*d*c^2*b + 3/11*x^11*f^2*e*d^3*a + 3/11*
x^11*f^3*d^2*c*a + 1/9*x^9*e^3*d^3*b + x^9*f*e^2*d^2*c*b + x^9*f^
2*e*d*c^2*b + 1/9*x^9*f^3*c^3*b + 1/3*x^9*f*e^2*d^3*a + x^9*f^2*e
*d^2*c*a + 1/3*x^9*f^3*d*c^2*a + 3/7*x^7*e^3*d^2*c*b + 9/7*x^7*f*
e^2*d*c^2*b + 3/7*x^7*f^2*e*c^3*b + 1/7*x^7*e^3*d^3*a + 9/7*x^7*f
```

```
*e^2*d^2*c*a + 9/7*x^7*f^2*e*d*c^2*a + 1/7*x^7*f^3*c^3*a + 3/5*x^
5*e^3*d*c^2*b + 3/5*x^5*f*e^2*c^3*b + 3/5*x^5*e^3*d^2*c*a + 9/5*x
^5*f*e^2*d*c^2*a + 3/5*x^5*f^2*e*c^3*a + 1/3*x^3*e^3*c^3*b + x^3*
e^3*d*c^2*a + x^3*f*e^2*c^3*a + x*e^3*c^3*a
```

Sympy [A] time = 0.147185, size = 423, normalized size = 1.36

$$ac^3e^3x + \frac{bd^3f^3x^{15}}{15} + x^{13}\left(\frac{ad^3f^3}{13} + \frac{3bcd^2f^3}{13} + \frac{3bd^3ef^2}{13}\right) + x^{11}\left(\frac{3acd^2f^3}{11} + \frac{3ad^3ef^2}{11} + \frac{3bc^2df^3}{11} + \frac{9bcd^2ef^2}{11} + \frac{3bd^3e^2f}{11}\right) + x^9\left(\frac{ac^2df^3}{3} + acd^2ef^2 + \frac{ad^3e^2f}{3} + \frac{bc^3f^3}{9} + bc^2def^2 + bcd^2e^2f + \frac{bd^3e^3}{9}\right) + x^7\left(\frac{ac^3f^3}{7} + \frac{9ac^2def^2}{7} + \frac{9acd^2e^2f}{7} + \frac{ad^3e^3}{7} + \frac{3bc^3ef^2}{7} + \frac{9bc^2de^2f}{7} + \frac{3bcd^2e^3}{7}\right) + x^5\left(\frac{3ac^3ef^2}{5} + \frac{9ac^2de^2f}{5} + \frac{3acd^2e^3}{5} + \frac{3bc^3e^2f}{5} + \frac{3bc^2de^3}{5}\right) + x^3\left(ac^3e^2f + ac^2de^3 + \frac{bc^3e^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x**2+c)**3*(f*x**2+e)**3,x)
```

```
[Out] a*c**3*e**3*x + b*d**3*f**3*x**15/15 + x**13*(a*d**3*f**3/13 + 3*
b*c*d**2*f**3/13 + 3*b*d**3*e*f**2/13) + x**11*(3*a*c*d**2*f**3/1
1 + 3*a*d**3*e*f**2/11 + 3*b*c**2*d*f**3/11 + 9*b*c*d**2*e*f**2/1
1 + 3*b*d**3*e**2*f/11) + x**9*(a*c**2*d*f**3/3 + a*c*d**2*e*f**2
 + a*d**3*e**2*f/3 + b*c**3*f**3/9 + b*c**2*d*e*f**2 + b*c*d**2*e
**2*f + b*d**3*e**3/9) + x**7*(a*c**3*f**3/7 + 9*a*c**2*d*e*f**2/
7 + 9*a*c*d**2*e**2*f/7 + a*d**3*e**3/7 + 3*b*c**3*e*f**2/7 + 9*b
*c**2*d*e**2*f/7 + 3*b*c*d**2*e**3/7) + x**5*(3*a*c**3*e*f**2/5 +
 9*a*c**2*d*e**2*f/5 + 3*a*c*d**2*e**3/5 + 3*b*c**3*e**2*f/5 + 3*
b*c**2*d*e**3/5) + x**3*(a*c**3*e**2*f + a*c**2*d*e**3 + b*c**3*e
**3/3)
```

GIAC/XCAS [A] time = 0.226791, size = 541, normalized size = 1.75

$$\frac{1}{15}bd^3f^3x^{15} + \frac{3}{13}bcd^2f^3x^{13} + \frac{1}{13}ad^3f^3x^{13} + \frac{3}{13}bd^3f^2x^{13}e + \frac{3}{11}bc^2df^3x^{11} + \frac{3}{11}acd^2f^3x^{11} + \frac{9}{11}bcd^2f^2x^{11}e + \frac{3}{11}ad^3f^2x^{11}e + \frac{3}{11}bd^3fx^{11}e^2 + \frac{1}{9}bc^3f^3x^9 + \frac{1}{3}ac^2df^3x^9 + bc^2df^2x^9e + acd^2f^2x^9e + bcd^2fx^9e^2 + \frac{1}{3}ad^3fx^9e^2 + \frac{1}{7}ac^3f^3x^7 + \frac{1}{9}bd^3x^9e^3 + \frac{3}{7}bc^3f^2x^7e + \frac{9}{7}ac^2df^2x^7e + \frac{9}{7}bc^2dfx^7e^2 + \frac{9}{7}acd^2fx^7e^2 + \frac{3}{7}bcd^2x^7e^3 + \frac{1}{7}ad^3x^7e^3 + \frac{3}{5}ac^3f^2x^5e + \frac{3}{5}bc^3fx^5e^2 + \frac{9}{5}ac^2dfx^5e^2 + \frac{3}{5}bc^2dx^5e^3 + \frac{3}{5}acd^2x^5e^3 + ac^3fx^3e^2 + \frac{1}{3}bc^3x^3e^3 + ac^2dx^3e^3 + ac^3xe^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)^3*(f*x^2 + e)^3,x, algorithm="giac")
```

```
[Out] 1/15*b*d^3*f^3*x^15 + 3/13*b*c*d^2*f^3*x^13 + 1/13*a*d^3*f^3*x^13
 + 3/13*b*d^3*f^2*x^13*e + 3/11*b*c^2*d*f^3*x^11 + 3/11*a*c*d^2*f
^3*x^11 + 9/11*b*c*d^2*f^2*x^11*e + 3/11*a*d^3*f^2*x^11*e + 3/11*
b*d^3*f*x^11*e^2 + 1/9*b*c^3*f^3*x^9 + 1/3*a*c^2*d*f^3*x^9 + b*c^
2*d*f^2*x^9*e + a*c*d^2*f^2*x^9*e + b*c*d^2*f*x^9*e^2 + 1/3*a*d^3
*f*x^9*e^2 + 1/7*a*c^3*f^3*x^7 + 1/9*b*d^3*x^9*e^3 + 3/7*b*c^3*f^
2*x^7*e + 9/7*a*c^2*d*f^2*x^7*e + 9/7*b*c^2*d*f*x^7*e^2 + 9/7*a*c
*d^2*f*x^7*e^2 + 3/7*b*c*d^2*x^7*e^3 + 1/7*a*d^3*x^7*e^3 + 3/5*a*
c^3*f^2*x^5*e + 3/5*b*c^3*f*x^5*e^2 + 9/5*a*c^2*d*f*x^5*e^2 + 3/5
*b*c^2*d*x^5*e^3 + 3/5*a*c*d^2*x^5*e^3 + a*c^3*f*x^3*e^2 + 1/3*b*
c^3*x^3*e^3 + a*c^2*d*x^3*e^3 + a*c^3*x*e^3
```

3.17 $\int \left(a+b x^2\right) \left(c+d x^2\right)^3 \left(e+f x^2\right)^2 \, dx$

Optimal. Leaf size=226

$$\frac{1}{9}dx^9\left(adf(3cf+2de)+b\left(3c^2f^2+6cdef+d^2e^2\right)\right)+\frac{1}{7}x^7\left(ad\left(3c^2f^2+6cdef+d^2e^2\right)+bc\left(c^2f^2+6cdef+3d^2e^2\right)\right)+\frac{1}{5}cx^5\left(a\left(c^2f^2+6cdef+3d^2e^2\right)+bce(2cf+3de)\right)+\frac{1}{3}c^2ex^3(2acf+3ade+bce)+\frac{1}{11}d^2fx^{11}(adf+3bcf+2bde)+ac^3e^2x+\frac{1}{13}bd^3f^2x^{13}$$

```
[Out] a*c^3*e^2*x + (c^2*e*(b*c*e + 3*a*d*e + 2*a*c*f)*x^3)/3 + (c*(b*c
*e*(3*d*e + 2*c*f) + a*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2))*x^5)/5
+ ((b*c*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2) + a*d*(d^2*e^2 + 6*c*d*
e*f + 3*c^2*f^2))*x^7)/7 + (d*(a*d*f*(2*d*e + 3*c*f) + b*(d^2*e^2
 + 6*c*d*e*f + 3*c^2*f^2))*x^9)/9 + (d^2*f*(2*b*d*e + 3*b*c*f + a
*d*f)*x^11)/11 + (b*d^3*f^2*x^13)/13
```

Rubi [A] time = 0.616779, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$\frac{1}{9}dx^9\left(adf(3cf+2de)+b\left(3c^2f^2+6cdef+d^2e^2\right)\right)+\frac{1}{7}x^7\left(ad\left(3c^2f^2+6cdef+d^2e^2\right)+bc\left(c^2f^2+6cdef+3d^2e^2\right)\right)+\frac{1}{5}cx^5\left(a\left(c^2f^2+6cdef+3d^2e^2\right)+bce(2cf+3de)\right)+\frac{1}{3}c^2ex^3(2acf+3ade+bce)+\frac{1}{11}d^2fx^{11}(adf+3bcf+2bde)+ac^3e^2x+\frac{1}{13}bd^3f^2x^{13}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)*(c + d*x^2)^3*(e + f*x^2)^2,x]
```

```
[Out] a*c^3*e^2*x + (c^2*e*(b*c*e + 3*a*d*e + 2*a*c*f)*x^3)/3 + (c*(b*c
*e*(3*d*e + 2*c*f) + a*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2))*x^5)/5
+ ((b*c*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2) + a*d*(d^2*e^2 + 6*c*d*
e*f + 3*c^2*f^2))*x^7)/7 + (d*(a*d*f*(2*d*e + 3*c*f) + b*(d^2*e^2
 + 6*c*d*e*f + 3*c^2*f^2))*x^9)/9 + (d^2*f*(2*b*d*e + 3*b*c*f + a
*d*f)*x^11)/11 + (b*d^3*f^2*x^13)/13
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{bd^3f^2x^{13}}{13}+c^3e^2\int a\,dx+\frac{c^2ex^3\,(2acf+3ade+bce)}{3}+\frac{cx^5\left(ac^2f^2+6acdef+3ad^2e^2+2bc^2ef+3bcde^2\right)}{5}+\frac{d^2fx^{11}\,(adf+3bcf+2bde)}{11}+\frac{dx^9\left(3acdf^2+2ad^2ef+3bc^2f^2+6bcdef+bd^2e^2\right)}{9}+x^7\left(\frac{3ac^2df^2}{7}+\frac{6acd^2ef}{7}+\frac{ad^3e^2}{7}+\frac{bc^3f^2}{7}+\frac{6bc^2def}{7}+\frac{3bcd^2e^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)**3*(f*x**2+e)**2,x)
```

```
[Out] b*d**3*f**2*x**13/13 + c**3*e**2*Integral(a, x) + c**2*e*x**3*(2*
a*c*f + 3*a*d*e + b*c*e)/3 + c*x**5*(a*c**2*f**2 + 6*a*c*d*e*f +
3*a*d**2*e**2 + 2*b*c**2*e*f + 3*b*c*d*e**2)/5 + d**2*f*x**11*(a*
d*f + 3*b*c*f + 2*b*d*e)/11 + d*x**9*(3*a*c*d*f**2 + 2*a*d**2*e*f
 + 3*b*c**2*f**2 + 6*b*c*d*e*f + b*d**2*e**2)/9 + x**7*(3*a*c**2*
d*f**2/7 + 6*a*c*d**2*e*f/7 + a*d**3*e**2/7 + b*c**3*f**2/7 + 6*b
*c**2*d*e*f/7 + 3*b*c*d**2*e**2/7)
```

Mathematica [A] time = 0.169033, size = 226, normalized size = 1.

$$\frac{1}{9}dx^9\left(adf(3cf+2de)+b\left(3c^2f^2+6cdef+d^2e^2\right)\right)+\frac{1}{7}x^7\left(ad\left(3c^2f^2+6cdef+d^2e^2\right)+bc\left(c^2f^2+6cdef+3d^2e^2\right)\right)+\frac{1}{5}cx^5\left(a\left(c^2f^2+6cdef+3d^2e^2\right)+bce(2cf+3de)\right)+\frac{1}{3}c^2ex^3(2acf+3ade+bce)+\frac{1}{11}d^2fx^{11}(adf+3bcf+2bde)+ac^3e^2x+\frac{1}{13}bd^3f^2x^{13}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)*(c + d*x^2)^3*(e + f*x^2)^2,x]
```

```
[Out] a*c^3*e^2*x + (c^2*e*(b*c*e + 3*a*d*e + 2*a*c*f)*x^3)/3 + (c*(b*c
*e*(3*d*e + 2*c*f) + a*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2))*x^5)/5
+ ((b*c*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2) + a*d*(d^2*e^2 + 6*c*d*
e*f + 3*c^2*f^2))*x^7)/7 + (d*(a*d*f*(2*d*e + 3*c*f) + b*(d^2*e^2
 + 6*c*d*e*f + 3*c^2*f^2))*x^9)/9 + (d^2*f*(2*b*d*e + 3*b*c*f + a
*d*f)*x^11)/11 + (b*d^3*f^2*x^13)/13
```

Maple [A] time = 0.002, size = 244, normalized size = 1.1

$$\frac{bd^3f^2x^{13}}{13}+\frac{\left(\left(ad^3+3\,bcd^2\right)f^2+2\,bd^3ef\right)x^{11}}{11}+\frac{\left(\left(3\,acd^2+3\,bc^2d\right)f^2+2\,\left(ad^3+3\,bcd^2\right)ef+bd^3e^2\right)x^9}{9}+\frac{\left(\left(3\,ac^2d+bc^3\right)f^2+2\,\left(3\,acd^2+3\,bc^2d\right)ef+\left(ad^3+3\,bcd^2\right)e^2\right)x^7}{7}+\frac{\left(ac^3f^2+2\,\left(3\,ac^2d+bc^3\right)ef+\left(3\,acd^2+3\,bc^2d\right)e^2\right)x^5}{5}+\frac{\left(2\,ac^3ef+\left(3\,ac^2d+bc^3\right)e^2\right)x^3}{3}+ac^3e^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^2,x)
```

```
[Out] 1/13*b*d^3*f^2*x^13+1/11*((a*d^3+3*b*c*d^2)*f^2+2*b*d^3*e*f)*x^11
+1/9*((3*a*c*d^2+3*b*c^2*d)*f^2+2*(a*d^3+3*b*c*d^2)*e*f+b*d^3*e^2
)*x^9+1/7*((3*a*c^2*d+b*c^3)*f^2+2*(3*a*c*d^2+3*b*c^2*d)*e*f+(a*d
^3+3*b*c*d^2)*e^2)*x^7+1/5*(a*c^3*f^2+2*(3*a*c^2*d+b*c^3)*e*f+(3*
a*c*d^2+3*b*c^2*d)*e^2)*x^5+1/3*(2*a*c^3*e*f+(3*a*c^2*d+b*c^3)*e^
2)*x^3+a*c^3*e^2*x
```

Maxima [A] time = 1.34575, size = 323, normalized size = 1.43

$$\frac{1}{13}\,bd^3f^2x^{13}+\frac{1}{11}\left(2\,bd^3ef+\left(3\,bcd^2+ad^3\right)f^2\right)x^{11}+\frac{1}{9}\left(bd^3e^2+2\left(3\,bcd^2+ad^3\right)ef+3\left(bc^2d+acd^2\right)f^2\right)x^9+\frac{1}{7}\left(\left(3\,bcd^2+ad^3\right)e^2+6\left(bc^2d+acd^2\right)ef+\left(bc^3+3\,ac^2d\right)f^2\right)x^7+ac^3e^2x+\frac{1}{5}\left(ac^3f^2+3\left(bc^2d+acd^2\right)e^2+2\left(bc^3+3\,ac^2d\right)ef\right)x^5+\frac{1}{3}\left(2\,ac^3ef+\left(bc^3+3\,ac^2d\right)e^2\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)^3*(f*x^2 + e)^2,x, algorithm="maxima")
```

```
[Out] 1/13*b*d^3*f^2*x^13 + 1/11*(2*b*d^3*e*f + (3*b*c*d^2 + a*d^3)*f^2
)*x^11 + 1/9*(b*d^3*e^2 + 2*(3*b*c*d^2 + a*d^3)*e*f + 3*(b*c^2*d
+ a*c*d^2)*f^2)*x^9 + 1/7*((3*b*c*d^2 + a*d^3)*e^2 + 6*(b*c^2*d +
 a*c*d^2)*e*f + (b*c^3 + 3*a*c^2*d)*f^2)*x^7 + a*c^3*e^2*x + 1/5*
(a*c^3*f^2 + 3*(b*c^2*d + a*c*d^2)*e^2 + 2*(b*c^3 + 3*a*c^2*d)*e*
f)*x^5 + 1/3*(2*a*c^3*e*f + (b*c^3 + 3*a*c^2*d)*e^2)*x^3
```

Fricas [A] time = 0.184445, size = 1, normalized size = 0.

$$\frac{1}{13}x^{13}f^2d^3b+\frac{2}{11}x^{11}fed^3b+\frac{3}{11}x^{11}f^2d^2cb+\frac{1}{11}x^{11}f^2d^3a+\frac{1}{9}x^9e^2d^3b+\frac{2}{3}x^9fed^2cb+\frac{1}{3}x^9f^2dc^2b+\frac{2}{9}x^9fed^3a+\frac{1}{3}x^9f^2d^2ca+\frac{3}{7}x^7e^2d^2cb+\frac{6}{7}x^7fedc^2b+\frac{1}{7}x^7f^2c^3b+\frac{1}{7}x^7e^2d^3a+\frac{6}{7}x^7fed^2ca+\frac{3}{7}x^7f^2dc^2a+\frac{3}{5}x^5e^2dc^2b+\frac{2}{5}x^5fec^3b+\frac{3}{5}x^5e^2d^2ca+\frac{6}{5}x^5fedc^2a+\frac{1}{5}x^5f^2c^3a+\frac{1}{3}x^3e^2c^3b+x^3e^2dc^2a+\frac{2}{3}x^3fec^3a+xe^2c^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^3*(f*x^2 + e)^2,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*f^2*d^3*b + 2/11*x^11*f*e*d^3*b + 3/11*x^11*f^2*d^2*c*b
 + 1/11*x^11*f^2*d^3*a + 1/9*x^9*e^2*d^3*b + 2/3*x^9*f*e*d^2*c*b
+ 1/3*x^9*f^2*d*c^2*b + 2/9*x^9*f*e*d^3*a + 1/3*x^9*f^2*d^2*c*a +
 3/7*x^7*e^2*d^2*c*b + 6/7*x^7*f*e*d*c^2*b + 1/7*x^7*f^2*c^3*b +
1/7*x^7*e^2*d^3*a + 6/7*x^7*f*e*d^2*c*a + 3/7*x^7*f^2*d*c^2*a + 3
/5*x^5*e^2*d*c^2*b + 2/5*x^5*f*e*c^3*b + 3/5*x^5*e^2*d^2*c*a + 6/
5*x^5*f*e*d*c^2*a + 1/5*x^5*f^2*c^3*a + 1/3*x^3*e^2*c^3*b + x^3*e
^2*d*c^2*a + 2/3*x^3*f*e*c^3*a + x*e^2*c^3*a
```

Sympy [A] time = 0.11686, size = 304, normalized size = 1.35

$$ac^3e^2x+\frac{bd^3f^2x^{13}}{13}+x^{11}\left(\frac{ad^3f^2}{11}+\frac{3bcd^2f^2}{11}+\frac{2bd^3ef}{11}\right)+x^9\left(\frac{acd^2f^2}{3}+\frac{2ad^3ef}{9}+\frac{bc^2df^2}{3}+\frac{2bcd^2ef}{3}+\frac{bd^3e^2}{9}\right)+x^7\left(\frac{3ac^2df^2}{7}+\frac{6acd^2ef}{7}+\frac{ad^3e^2}{7}+\frac{bc^3f^2}{7}+\frac{6bc^2def}{7}+\frac{3bcd^2e^2}{7}\right)+x^5\left(\frac{ac^3f^2}{5}+\frac{6ac^2def}{5}+\frac{3acd^2e^2}{5}+\frac{2bc^3ef}{5}+\frac{3bc^2de^2}{5}\right)+x^3\left(\frac{2ac^3ef}{3}+ac^2de^2+\frac{bc^3e^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**3*(f*x**2+e)**2,x)
```

```
[Out] a*c**3*e**2*x + b*d**3*f**2*x**13/13 + x**11*(a*d**3*f**2/11 + 3*
b*c*d**2*f**2/11 + 2*b*d**3*e*f/11) + x**9*(a*c*d**2*f**2/3 + 2*a
*d**3*e*f/9 + b*c**2*d*f**2/3 + 2*b*c*d**2*e*f/3 + b*d**3*e**2/9)
 + x**7*(3*a*c**2*d*f**2/7 + 6*a*c*d**2*e*f/7 + a*d**3*e**2/7 + b
*c**3*f**2/7 + 6*b*c**2*d*e*f/7 + 3*b*c*d**2*e**2/7) + x**5*(a*c*
*3*f**2/5 + 6*a*c**2*d*e*f/5 + 3*a*c*d**2*e**2/5 + 2*b*c**3*e*f/5
 + 3*b*c**2*d*e**2/5) + x**3*(2*a*c**3*e*f/3 + a*c**2*d*e**2 + b*
c**3*e**2/3)
```

GIAC/XCAS [A] time = 0.222756, size = 390, normalized size = 1.73

$$\frac{1}{13}\,bd^3f^2x^{13} + \frac{3}{11}\,bcd^2f^2x^{11} + \frac{1}{11}\,ad^3f^2x^{11} + \frac{2}{11}\,bd^3fx^{11}e + \frac{1}{3}\,bc^2df^2x^9 + \frac{1}{3}\,acd^2f^2x^9 + \frac{2}{3}\,bcd^2fx^9e + \frac{2}{9}\,ad^3fx^9e + \frac{1}{9}\,bd^3x^9e^2 + \frac{1}{7}\,bc^3f^2x^7 + \frac{3}{7}\,ac^2df^2x^7 + \frac{6}{7}\,bc^2dfx^7e + \frac{6}{7}\,acd^2fx^7e + \frac{3}{7}\,bcd^2x^7e^2 + \frac{1}{7}\,ad^3x^7e^2 + \frac{1}{5}\,ac^3f^2x^5 + \frac{2}{5}\,bc^3fx^5e + \frac{6}{5}\,ac^2dfx^5e + \frac{3}{5}\,bc^2dx^5e^2 + \frac{3}{5}\,acd^2x^5e^2 + \frac{2}{3}\,ac^3fx^3e + \frac{1}{3}\,bc^3x^3e^2 + ac^2dx^3e^2 + ac^3xe^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^3*(f*x^2 + e)^2,x, algorithm="giac")
```

```
[Out] 1/13*b*d^3*f^2*x^13 + 3/11*b*c*d^2*f^2*x^11 + 1/11*a*d^3*f^2*x^11
 + 2/11*b*d^3*f*x^11*e + 1/3*b*c^2*d*f^2*x^9 + 1/3*a*c*d^2*f^2*x^
9 + 2/3*b*c*d^2*f*x^9*e + 2/9*a*d^3*f*x^9*e + 1/9*b*d^3*x^9*e^2 +
 1/7*b*c^3*f^2*x^7 + 3/7*a*c^2*d*f^2*x^7 + 6/7*b*c^2*d*f*x^7*e +
6/7*a*c*d^2*f*x^7*e + 3/7*b*c*d^2*x^7*e^2 + 1/7*a*d^3*x^7*e^2 + 1
/5*a*c^3*f^2*x^5 + 2/5*b*c^3*f*x^5*e + 6/5*a*c^2*d*f*x^5*e + 3/5*
b*c^2*d*x^5*e^2 + 3/5*a*c*d^2*x^5*e^2 + 2/3*a*c^3*f*x^3*e + 1/3*b
*c^3*x^3*e^2 + a*c^2*d*x^3*e^2 + a*c^3*x*e^2
```

3.18 $\int \left(a+bx^2\right)\left(c+dx^2\right)^3\left(e+fx^2\right)\,dx$

Optimal. Leaf size=130

$$\frac{1}{3}c^2x^3(acf+3ade+bce)+\frac{1}{9}d^2x^9(adf+3bcf+bde)+\frac{1}{7}dx^7(ad(3cf+de)+3bc(cf+de)) + \frac{1}{5}cx^5(3ad(cf+de)+bc(cf+3de))+ac^3ex+\frac{1}{11}bd^3fx^{11}$$

```
[Out] a*c^3*e*x + (c^2*(b*c*e + 3*a*d*e + a*c*f)*x^3)/3 + (c*(3*a*d*(d*
e + c*f) + b*c*(3*d*e + c*f))*x^5)/5 + (d*(3*b*c*(d*e + c*f) + a*
d*(d*e + 3*c*f))*x^7)/7 + (d^2*(b*d*e + 3*b*c*f + a*d*f)*x^9)/9 +
 (b*d^3*f*x^11)/11
```

Rubi [A] time = 0.380114, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{1}{3}c^2x^3(acf+3ade+bce)+\frac{1}{9}d^2x^9(adf+3bcf+bde)+\frac{1}{7}dx^7(ad(3cf+de)+3bc(cf+de)) + \frac{1}{5}cx^5(3ad(cf+de)+bc(cf+3de))+ac^3ex+\frac{1}{11}bd^3fx^{11}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)*(c + d*x^2)^3*(e + f*x^2),x]
```

```
[Out] a*c^3*e*x + (c^2*(b*c*e + 3*a*d*e + a*c*f)*x^3)/3 + (c*(3*a*d*(d*
e + c*f) + b*c*(3*d*e + c*f))*x^5)/5 + (d*(3*b*c*(d*e + c*f) + a*
d*(d*e + 3*c*f))*x^7)/7 + (d^2*(b*d*e + 3*b*c*f + a*d*f)*x^9)/9 +
 (b*d^3*f*x^11)/11
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{bd^3fx^{11}}{11}+c^3e\int a\,dx+\frac{c^2x^3\left(acf+3ade+bce\right)}{3}+\frac{cx^5\left(3acdf+3ad^2e+bc^2f+3bcde\right)}{5} + \frac{d^2x^9\left(adf+3bcf+bde\right)}{9}+\frac{dx^7\left(3acdf+ad^2e+3bc^2f+3bcde\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)**3*(f*x**2+e),x)
```

```
[Out] b*d**3*f*x**11/11 + c**3*e*Integral(a, x) + c**2*x**3*(a*c*f + 3*
a*d*e + b*c*e)/3 + c*x**5*(3*a*c*d*f + 3*a*d**2*e + b*c**2*f + 3*
```

```
b*c*d*e)/5 + d**2*x**9*(a*d*f + 3*b*c*f + b*d*e)/9 + d*x**7*(3*a*
c*d*f + a*d**2*e + 3*b*c**2*f + 3*b*c*d*e)/7
```

Mathematica [A] time = 0.0966672, size = 130, normalized size = 1.

$$\frac{1}{3}c^2x^3(acf+3ade+bce)+\frac{1}{9}d^2x^9(adf+3bcf+bde)+\frac{1}{7}dx^7(ad(3cf+de)+3bc(cf+de))+\frac{1}{5}cx^5(3ad(cf+de)+bc(cf+3de))+ac^3ex+\frac{1}{11}bd^3fx^{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)*(c + d*x^2)^3*(e + f*x^2),x]
```

```
[Out] a*c^3*e*x + (c^2*(b*c*e + 3*a*d*e + a*c*f)*x^3)/3 + (c*(3*a*d*(d*
e + c*f) + b*c*(3*d*e + c*f))*x^5)/5 + (d*(3*b*c*(d*e + c*f) + a*
d*(d*e + 3*c*f))*x^7)/7 + (d^2*(b*d*e + 3*b*c*f + a*d*f)*x^9)/9 +
 (b*d^3*f*x^11)/11
```

Maple [A] time = 0., size = 149, normalized size = 1.2

$$\frac{bd^3fx^{11}}{11}+\frac{\left(\left(ad^3+3\,bcd^2\right)f+bd^3e\right)x^9}{9}+\frac{\left(\left(3\,acd^2+3\,bc^2d\right)f+\left(ad^3+3\,bcd^2\right)e\right)x^7}{7}+\frac{\left(\left(3\,ac^2d+bc^3\right)f+\left(3\,acd^2+3\,bc^2d\right)e\right)x^5}{5}+\frac{\left(ac^3f+\left(3\,ac^2d+bc^3\right)e\right)x^3}{3}+ac^3ex$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e),x)
```

```
[Out] 1/11*b*d^3*f*x^11+1/9*((a*d^3+3*b*c*d^2)*f+b*d^3*e)*x^9+1/7*((3*a
*c*d^2+3*b*c^2*d)*f+(a*d^3+3*b*c*d^2)*e)*x^7+1/5*((3*a*c^2*d+b*c^
3)*f+(3*a*c*d^2+3*b*c^2*d)*e)*x^5+1/3*(a*c^3*f+(3*a*c^2*d+b*c^3)*
e)*x^3+a*c^3*e*x
```

Maxima [A] time = 1.36694, size = 197, normalized size = 1.52

$$\frac{1}{11}\,bd^3fx^{11}+\frac{1}{9}\left(bd^3e+\left(3\,bcd^2+ad^3\right)f\right)x^9+\frac{1}{7}\left(\left(3\,bcd^2+ad^3\right)e+3\left(bc^2d+acd^2\right)f\right)x^7+ac^3ex+\frac{1}{5}\left(3\left(bc^2d+acd^2\right)e+\left(bc^3+3\,ac^2d\right)f\right)x^5+\frac{1}{3}\left(ac^3f+\left(bc^3+3\,ac^2d\right)e\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^3*(f*x^2 + e),x, algorithm="maxima")
```

```
[Out] 1/11*b*d^3*f*x^11 + 1/9*(b*d^3*e + (3*b*c*d^2 + a*d^3)*f)*x^9 + 1
/7*((3*b*c*d^2 + a*d^3)*e + 3*(b*c^2*d + a*c*d^2)*f)*x^7 + a*c^3*
e*x + 1/5*(3*(b*c^2*d + a*c*d^2)*e + (b*c^3 + 3*a*c^2*d)*f)*x^5 +
 1/3*(a*c^3*f + (b*c^3 + 3*a*c^2*d)*e)*x^3
```

Fricas [A] time = 0.182995, size = 1, normalized size = 0.01

$$\frac{1}{11}x^{11}fd^3b + \frac{1}{9}x^9ed^3b + \frac{1}{3}x^9fd^2cb + \frac{1}{9}x^9fd^3a + \frac{3}{7}x^7ed^2cb + \frac{3}{7}x^7fdc^2b + \frac{1}{7}x^7ed^3a + \frac{3}{7}x^7fd^2ca + \frac{3}{5}x^5edc^2b + \frac{1}{5}x^5fc^3b + \frac{3}{5}x^5ed^2ca + \frac{3}{5}x^5fdc^2a + \frac{1}{3}x^3ec^3b + x^3edc^2a + \frac{1}{3}x^3fc^3a + xec^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^3*(f*x^2 + e),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*f*d^3*b + 1/9*x^9*e*d^3*b + 1/3*x^9*f*d^2*c*b + 1/9*x^9
*f*d^3*a + 3/7*x^7*e*d^2*c*b + 3/7*x^7*f*d*c^2*b + 1/7*x^7*e*d^3*
a + 3/7*x^7*f*d^2*c*a + 3/5*x^5*e*d*c^2*b + 1/5*x^5*f*c^3*b + 3/5
*x^5*e*d^2*c*a + 3/5*x^5*f*d*c^2*a + 1/3*x^3*e*c^3*b + x^3*e*d*c^
2*a + 1/3*x^3*f*c^3*a + x*e*c^3*a
```

Sympy [A] time = 0.087388, size = 173, normalized size = 1.33

$$ac^3ex + \frac{bd^3fx^{11}}{11} + x^9\left(\frac{ad^3f}{9} + \frac{bcd^2f}{3} + \frac{bd^3e}{9}\right) + x^7\left(\frac{3acd^2f}{7} + \frac{ad^3e}{7} + \frac{3bc^2df}{7} + \frac{3bcd^2e}{7}\right) + x^5\left(\frac{3ac^2df}{5} + \frac{3acd^2e}{5} + \frac{bc^3f}{5} + \frac{3bc^2de}{5}\right) + x^3\left(\frac{ac^3f}{3} + ac^2de + \frac{bc^3e}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**3*(f*x**2+e),x)
```

```
[Out] a*c**3*e*x + b*d**3*f*x**11/11 + x**9*(a*d**3*f/9 + b*c*d**2*f/3
+ b*d**3*e/9) + x**7*(3*a*c*d**2*f/7 + a*d**3*e/7 + 3*b*c**2*d*f/
7 + 3*b*c*d**2*e/7) + x**5*(3*a*c**2*d*f/5 + 3*a*c*d**2*e/5 + b*c
**3*f/5 + 3*b*c**2*d*e/5) + x**3*(a*c**3*f/3 + a*c**2*d*e + b*c**
3*e/3)
```

GIAC/XCAS [A] time = 0.225356, size = 234, normalized size = 1.8

$$\frac{1}{11}\,bd^{3}fx^{11}+\frac{1}{3}\,bcd^{2}fx^{9}+\frac{1}{9}\,ad^{3}fx^{9}+\frac{1}{9}\,bd^{3}x^{9}e+\frac{3}{7}\,bc^{2}dfx^{7}+\frac{3}{7}\,acd^{2}fx^{7}+\frac{3}{7}\,bcd^{2}x^{7}e+\frac{1}{7}\,ad^{3}x^{7}e+\frac{1}{5}\,bc^{3}fx^{5}+\frac{3}{5}\,ac^{2}dfx^{5}+\frac{3}{5}\,bc^{2}dx^{5}e+\frac{3}{5}\,acd^{2}x^{5}e+\frac{1}{3}\,ac^{3}fx^{3}+\frac{1}{3}\,bc^{3}x^{3}e+ac^{2}dx^{3}e+ac^{3}xe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)^3*(f*x^2 + e),x, algorithm="giac")
```

```
[Out] 1/11*b*d^3*f*x^11 + 1/3*b*c*d^2*f*x^9 + 1/9*a*d^3*f*x^9 + 1/9*b*d
^3*x^9*e + 3/7*b*c^2*d*f*x^7 + 3/7*a*c*d^2*f*x^7 + 3/7*b*c*d^2*x^
7*e + 1/7*a*d^3*x^7*e + 1/5*b*c^3*f*x^5 + 3/5*a*c^2*d*f*x^5 + 3/5
*b*c^2*d*x^5*e + 3/5*a*c*d^2*x^5*e + 1/3*a*c^3*f*x^3 + 1/3*b*c^3*
x^3*e + a*c^2*d*x^3*e + a*c^3*x*e
```

3.19 $\int \frac{(a+bx^2)(c+dx^2)^3}{e+fx^2}\,dx$

Optimal. Leaf size=227

$$-\frac{x\left(c+dx^2\right)\left(7adf(5de-9cf)-b\left(24c^2f^2-63cdef+35d^2e^2\right)\right)}{105f^3}+\frac{x\left(7adf\left(33c^2f^2-40cdef+15d^2e^2\right)-b\left(-48c^3f^3+231c^2def^2-280cd^2e^2f+105d^3e^3\right)\right)}{105f^4}+\frac{(be-af)(de-cf)^3\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{9/2}}-\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{35f^2}+\frac{bx\left(c+dx^2\right)^3}{7f}$$

```
[Out] ((7*a*d*f*(15*d^2*e^2 - 40*c*d*e*f + 33*c^2*f^2) - b*(105*d^3*e^3
 - 280*c*d^2*e^2*f + 231*c^2*d*e*f^2 - 48*c^3*f^3))*x)/(105*f^4)
- ((7*a*d*f*(5*d*e - 9*c*f) - b*(35*d^2*e^2 - 63*c*d*e*f + 24*c^2
*f^2))*x*(c + d*x^2))/(105*f^3) - ((7*b*d*e - 6*b*c*f - 7*a*d*f)*
x*(c + d*x^2)^2)/(35*f^2) + (b*x*(c + d*x^2)^3)/(7*f) + ((b*e - a
*f)*(d*e - c*f)^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(9/2))
```

Rubi [A] time = 0.918329, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$-\frac{x\left(c+dx^2\right)\left(7adf(5de-9cf)-b\left(24c^2f^2-63cdef+35d^2e^2\right)\right)}{105f^3}+\frac{x\left(7adf\left(33c^2f^2-40cdef+15d^2e^2\right)-b\left(-48c^3f^3+231c^2def^2-280cd^2e^2f+105d^3e^3\right)\right)}{105f^4}+\frac{(be-af)(de-cf)^3\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{9/2}}-\frac{x\left(c+dx^2\right)^2(-7adf-6bcf+7bde)}{35f^2}+\frac{bx\left(c+dx^2\right)^3}{7f}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2),x]
```


```
[Out] ((7*a*d*f*(15*d^2*e^2 - 40*c*d*e*f + 33*c^2*f^2) - b*(105*d^3*e^3
 - 280*c*d^2*e^2*f + 231*c^2*d*e*f^2 - 48*c^3*f^3))*x)/(105*f^4)
- ((7*a*d*f*(5*d*e - 9*c*f) - b*(35*d^2*e^2 - 63*c*d*e*f + 24*c^2
*f^2))*x*(c + d*x^2))/(105*f^3) - ((7*b*d*e - 6*b*c*f - 7*a*d*f)*
x*(c + d*x^2)^2)/(35*f^2) + (b*x*(c + d*x^2)^3)/(7*f) + ((b*e - a
*f)*(d*e - c*f)^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(9/2))
```

Rubi in Sympy [A] time = 120.378, size = 279, normalized size = 1.23

$$\frac{bx(c+dx^2)^3}{7f}+\frac{dx\left(c\left(35acf^2-7adef-11bcef+7bde^2\right)+x^2\left(63acdf^2-35ad^2ef+24bc^2f^2-63bcdef+35bd^2e^2\right)\right)}{105f^3}+\frac{x(c+dx^2)^2(7adf+6bcf-7bde)}{35f^2}+\frac{x\left(259ac^2df^3-308acd^2ef^2+105ad^3e^2f+72bc^3f^3-283bc^2def^2+308bcd^2e^2f-105bd^3e^3\right)}{105f^4}+\frac{(af-be)(cf-de)^3\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)**3/(f*x**2+e),x)
```

```
[Out] b*x*(c + d*x**2)**3/(7*f) + d*x*(c*(35*a*c*f**2 - 7*a*d*e*f - 11*
b*c*e*f + 7*b*d*e**2) + x**2*(63*a*c*d*f**2 - 35*a*d**2*e*f + 24*
b*c**2*f**2 - 63*b*c*d*e*f + 35*b*d**2*e**2))/(105*f**3) + x*(c +
 d*x**2)**2*(7*a*d*f + 6*b*c*f - 7*b*d*e)/(35*f**2) + x*(259*a*c*
*2*d*f**3 - 308*a*c*d**2*e*f**2 + 105*a*d**3*e**2*f + 72*b*c**3*f
**3 - 283*b*c**2*d*e*f**2 + 308*b*c*d**2*e**2*f - 105*b*d**3*e**3
)/(105*f**4) + (a*f - b*e)*(c*f - d*e)**3*atan(sqrt(f)*x/sqrt(e))
/(sqrt(e)*f**(9/2))
```

Mathematica [A] time = 0.176666, size = 179, normalized size = 0.79

$$\frac{x\left(adf\left(3c^2f^2-3cdef+d^2e^2\right)-b(de-cf)^3\right)}{f^4}+\frac{dx^3\left(adf(3cf-de)+b\left(3c^2f^2-3cdef+d^2e^2\right)\right)}{3f^3}+\frac{d^2x^5(adf+3bcf-bde)}{5f^2}+\frac{(be-af)(de-cf)^3\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{9/2}}+\frac{bd^3x^7}{7f}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2),x]
```

```
[Out] ((-(b*(d*e - c*f)^3) + a*d*f*(d^2*e^2 - 3*c*d*e*f + 3*c^2*f^2))*x
)/f^4 + (d*(a*d*f*(-(d*e) + 3*c*f) + b*(d^2*e^2 - 3*c*d*e*f + 3*c
^2*f^2))*x^3)/(3*f^3) + (d^2*(-(b*d*e) + 3*b*c*f + a*d*f)*x^5)/(5
*f^2) + (b*d^3*x^7)/(7*f) + ((b*e - a*f)*(d*e - c*f)^3*ArcTan[(Sq
rt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(9/2))
```

Maple [A] time = 0.007, size = 401, normalized size = 1.8

$$\frac{bd^3x^7}{7f}+\frac{x^5ad^3}{5f}+\frac{3x^5bcd^2}{5f}-\frac{x^5bd^3e}{5f^2}+\frac{x^3acd^2}{f}-\frac{x^3ad^3e}{3f^2}+\frac{x^3bc^2d}{f}-\frac{x^3bcd^2e}{f^2}+\frac{x^3bd^3e^2}{3f^3}+3\frac{ac^2dx}{f}-3\frac{acd^2ex}{f^2}+\frac{ad^3e^2x}{f^3}+\frac{bc^3x}{f}-3\frac{bc^2dex}{f^2}+3\frac{bcd^2e^2x}{f^3}-\frac{bd^3e^3x}{f^4}+ac^3\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}-3\frac{ac^2de}{f\sqrt{ef}}\arctan\left(\frac{fx}{\sqrt{ef}}\right)+3\frac{acd^2e^2}{f^2\sqrt{ef}}\arctan\left(\frac{fx}{\sqrt{ef}}\right)-\frac{ad^3e^3}{f^3}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}-\frac{bc^3e}{f}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}+3\frac{bc^2de^2}{f^2\sqrt{ef}}\arctan\left(\frac{fx}{\sqrt{ef}}\right)-3\frac{bcd^2e^3}{f^3\sqrt{ef}}\arctan\left(\frac{fx}{\sqrt{ef}}\right)+\frac{bd^3e^4}{f^4}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e),x)
```

```
[Out] 1/7/f*b*d^3*x^7+1/5/f*x^5*a*d^3+3/5/f*x^5*b*c*d^2-1/5/f^2*x^5*b*d
^3*e+1/f*x^3*a*c*d^2-1/3/f^2*x^3*a*d^3*e+1/f*x^3*b*c^2*d-1/f^2*x^
3*b*c*d^2*e+1/3/f^3*x^3*b*d^3*e^2+3/f*a*c^2*d*x-3/f^2*a*c*d^2*e*x
+1/f^3*a*d^3*e^2*x+1/f*b*c^3*x-3/f^2*b*c^2*d*e*x+3/f^3*b*c*d^2*e^
2*x-1/f^4*b*d^3*e^3*x+1/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a*c^3
-3/f/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a*c^2*d*e+3/f^2/(e*f)^(1
/2)*arctan(x*f/(e*f)^(1/2))*a*c*d^2*e^2-1/f^3/(e*f)^(1/2)*arctan(
x*f/(e*f)^(1/2))*a*d^3*e^3-1/f/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2)
)*b*c^3*e+3/f^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*c^2*d*e^2-3
/f^3/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*c*d^2*e^3+1/f^4/(e*f)^
(1/2)*arctan(x*f/(e*f)^(1/2))*b*d^3*e^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^3/(f*x^2 + e),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.218182, size = 1, normalized size = 0.

$$\left[\frac{105\left(bd^3e^4+ac^3f^4-\left(3\,bcd^2+ad^3\right)e^3f+3\left(bc^2d+acd^2\right)e^2f^2-\left(bc^3+3\,ac^2d\right)ef^3\right)\log\left(\frac{2\,efx+\left(fx^2-e\right)\sqrt{-ef}}{fx^2+e}\right)+2\left(15\,bd^3f\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^3/(f*x^2 + e),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(b*d^3*e^4 + a*c^3*f^4 - (3*b*c*d^2 + a*d^3)*e^3*f +
3*(b*c^2*d + a*c*d^2)*e^2*f^2 - (b*c^3 + 3*a*c^2*d)*e*f^3)*log((2
*e*f*x + (f*x^2 - e)*sqrt(-e*f))/(f*x^2 + e)) + 2*(15*b*d^3*f^3*x
^7 - 21*(b*d^3*e*f^2 - (3*b*c*d^2 + a*d^3)*f^3)*x^5 + 35*(b*d^3*e
^2*f - (3*b*c*d^2 + a*d^3)*e*f^2 + 3*(b*c^2*d + a*c*d^2)*f^3)*x^3
 - 105*(b*d^3*e^3 - (3*b*c*d^2 + a*d^3)*e^2*f + 3*(b*c^2*d + a*c*
d^2)*e*f^2 - (b*c^3 + 3*a*c^2*d)*f^3)*x)*sqrt(-e*f))/(sqrt(-e*f)*
f^4), 1/105*(105*(b*d^3*e^4 + a*c^3*f^4 - (3*b*c*d^2 + a*d^3)*e^3
*f + 3*(b*c^2*d + a*c*d^2)*e^2*f^2 - (b*c^3 + 3*a*c^2*d)*e*f^3)*a
rctan(sqrt(e*f)*x/e) + (15*b*d^3*f^3*x^7 - 21*(b*d^3*e*f^2 - (3*b
*c*d^2 + a*d^3)*f^3)*x^5 + 35*(b*d^3*e^2*f - (3*b*c*d^2 + a*d^3)*
e*f^2 + 3*(b*c^2*d + a*c*d^2)*f^3)*x^3 - 105*(b*d^3*e^3 - (3*b*c*
d^2 + a*d^3)*e^2*f + 3*(b*c^2*d + a*c*d^2)*e*f^2 - (b*c^3 + 3*a*c
^2*d)*f^3)*x)*sqrt(e*f))/(sqrt(e*f)*f^4)]
```

Sympy [A] time = 3.70016, size = 508, normalized size = 2.24

$$\frac{bd^3x^7}{7f} - \frac{\sqrt{-\frac{1}{ef^9}}\left(af-be\right)\left(cf-de\right)^3\log\left(-\frac{ef^4\sqrt{-\frac{1}{ef^9}}(af-be)(cf-de)^3}{ac^3f^4-3ac^2def^3+3acd^2e^2f^2-ad^3e^3f-bc^3ef^3+3bc^2de^2f^2-3bcd^2e^3f+bd^3e^4}+x\right)}{2} + \frac{\sqrt{-\frac{1}{ef^9}}\left(af-be\right)\left(cf-de\right)^3\log\left(\frac{ef^4\sqrt{-\frac{1}{ef^9}}(af-be)(cf-de)^3}{ac^3f^4-3ac^2def^3+3acd^2e^2f^2-ad^3e^3f-bc^3ef^3+3bc^2de^2f^2-3bcd^2e^3f+bd^3e^4}+x\right)}{2} + \frac{x^5\left(ad^3f+3bcd^2f-bd^3e\right)}{5f^2} + \frac{x^3\left(3acd^2f^2-ad^3ef+3bc^2df^2-3bcd^2ef+bd^3e^2\right)}{3f^3} + \frac{x\left(3ac^2df^3-3acd^2ef^2+ad^3e^2f+bc^3f^3-3bc^2def^2+3bcd^2e^2f-bd^3e^3\right)}{f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**3/(f*x**2+e),x)
```

```
[Out] b*d**3*x**7/(7*f) - sqrt(-1/(e*f**9))*(a*f - b*e)*(c*f - d*e)**3*
log(-e*f**4*sqrt(-1/(e*f**9))*(a*f - b*e)*(c*f - d*e)**3/(a*c**3*
```

```
f**4 - 3*a*c**2*d*e*f**3 + 3*a*c*d**2*e**2*f**2 - a*d**3*e**3*f -
 b*c**3*e*f**3 + 3*b*c**2*d*e**2*f**2 - 3*b*c*d**2*e**3*f + b*d**
3*e**4) + x)/2 + sqrt(-1/(e*f**9))*(a*f - b*e)*(c*f - d*e)**3*log
(e*f**4*sqrt(-1/(e*f**9))*(a*f - b*e)*(c*f - d*e)**3/(a*c**3*f**4
 - 3*a*c**2*d*e*f**3 + 3*a*c*d**2*e**2*f**2 - a*d**3*e**3*f - b*c
**3*e*f**3 + 3*b*c**2*d*e**2*f**2 - 3*b*c*d**2*e**3*f + b*d**3*e*
*4) + x)/2 + x**5*(a*d**3*f + 3*b*c*d**2*f - b*d**3*e)/(5*f**2) +
 x**3*(3*a*c*d**2*f**2 - a*d**3*e*f + 3*b*c**2*d*f**2 - 3*b*c*d**
2*e*f + b*d**3*e**2)/(3*f**3) + x*(3*a*c**2*d*f**3 - 3*a*c*d**2*e
*f**2 + a*d**3*e**2*f + b*c**3*f**3 - 3*b*c**2*d*e*f**2 + 3*b*c*d
**2*e**2*f - b*d**3*e**3)/f**4
```

GIAC/XCAS [A] time = 0.229311, size = 414, normalized size = 1.82

$$\frac{\left(ac^3f^4 - bc^3f^3e - 3\,ac^2df^3e + 3\,bc^2df^2e^2 + 3\,acd^2f^2e^2 - 3\,bcd^2fe^3 - ad^3fe^3 + bd^3e^4\right)\arctan\left(\sqrt{f}xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{1}{2}\right)}}{f^{\frac{9}{2}}} + \frac{15\,bd^3f^6x^7 + 63\,bcd^2f^6x^5 + 21\,ad^3f^6x^5 - 21\,bd^3f^5x^5e + 105\,bc^2df^6x^3 + 105\,acd^2f^6x^3 - 105\,bcd^2f^5x^3e - 35\,ad^3f^5x^3e + \cdots}{10\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)^3/(f*x^2 + e),x, algorithm="giac")
```

```
[Out] (a*c^3*f^4 - b*c^3*f^3*e - 3*a*c^2*d*f^3*e + 3*b*c^2*d*f^2*e^2 +
3*a*c*d^2*f^2*e^2 - 3*b*c*d^2*f*e^3 - a*d^3*f*e^3 + b*d^3*e^4)*ar
ctan(sqrt(f)*x*e^(-1/2))*e^(-1/2)/f^(9/2) + 1/105*(15*b*d^3*f^6*x
^7 + 63*b*c*d^2*f^6*x^5 + 21*a*d^3*f^6*x^5 - 21*b*d^3*f^5*x^5*e +
 105*b*c^2*d*f^6*x^3 + 105*a*c*d^2*f^6*x^3 - 105*b*c*d^2*f^5*x^3*
e - 35*a*d^3*f^5*x^3*e + 35*b*d^3*f^4*x^3*e^2 + 105*b*c^3*f^6*x +
 315*a*c^2*d*f^6*x - 315*b*c^2*d*f^5*x*e - 315*a*c*d^2*f^5*x*e +
315*b*c*d^2*f^4*x*e^2 + 105*a*d^3*f^4*x*e^2 - 105*b*d^3*f^3*x*e^3
)/f^7
```

3.20 $\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^2}\,dx$

Optimal. Leaf size=242

$$-\frac{dx\left(5af\left(3c^2f^2-22cdef+15d^2e^2\right)-be\left(81c^2f^2-190cdef+105d^2e^2\right)\right)}{30ef^4}-\frac{(de-cf)^2\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(be(7de-cf)-af(cf+5de))}{2e^{3/2}f^{9/2}}-\frac{dx\left(c+dx^2\right)(be(35de-33cf)-5af(5de-3cf))}{30ef^3}+\frac{dx\left(c+dx^2\right)^2(7be-5af)}{10ef^2}-\frac{x\left(c+dx^2\right)^3(be-af)}{2ef\left(e+fx^2\right)}$$

```
[Out] -(d*(5*a*f*(15*d^2*e^2 - 22*c*d*e*f + 3*c^2*f^2) - b*e*(105*d^2*e
^2 - 190*c*d*e*f + 81*c^2*f^2))*x)/(30*e*f^4) - (d*(b*e*(35*d*e -
 33*c*f) - 5*a*f*(5*d*e - 3*c*f))*x*(c + d*x^2))/(30*e*f^3) + (d*
(7*b*e - 5*a*f)*x*(c + d*x^2)^2)/(10*e*f^2) - ((b*e - a*f)*x*(c +
 d*x^2)^3)/(2*e*f*(e + f*x^2)) - ((d*e - c*f)^2*(b*e*(7*d*e - c*f
) - a*f*(5*d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(2*e^(3/2)*f^
(9/2))
```

Rubi [A] time = 1.03289, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{dx\left(5af\left(3c^2f^2-22cdef+15d^2e^2\right)-be\left(81c^2f^2-190cdef+105d^2e^2\right)\right)}{30ef^4}-\frac{(de-cf)^2\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(be(7de-cf)-af(cf+5de))}{2e^{3/2}f^{9/2}}-\frac{dx\left(c+dx^2\right)(be(35de-33cf)-5af(5de-3cf))}{30ef^3}+\frac{dx\left(c+dx^2\right)^2(7be-5af)}{10ef^2}-\frac{x\left(c+dx^2\right)^3(be-af)}{2ef\left(e+fx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^2,x]
```

```
[Out] -(d*(5*a*f*(15*d^2*e^2 - 22*c*d*e*f + 3*c^2*f^2) - b*e*(105*d^2*e
^2 - 190*c*d*e*f + 81*c^2*f^2))*x)/(30*e*f^4) - (d*(b*e*(35*d*e -
 33*c*f) - 5*a*f*(5*d*e - 3*c*f))*x*(c + d*x^2))/(30*e*f^3) + (d*
(7*b*e - 5*a*f)*x*(c + d*x^2)^2)/(10*e*f^2) - ((b*e - a*f)*x*(c +
 d*x^2)^3)/(2*e*f*(e + f*x^2)) - ((d*e - c*f)^2*(b*e*(7*d*e - c*f
```

```
) - a*f*(5*d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(2*e^(3/2)*f^
(9/2))
```

Rubi in Sympy [A] time = 128.165, size = 275, normalized size = 1.14

$$-\frac{dx\left(c+dx^2\right)^2(5af-7be)}{10ef^2}+\frac{dx\left(c\left(5cf\left(af+be\right)+de\left(5af-7be\right)\right)-dx^2\left(15acf^2-25adef-33bcef+35bde^2\right)\right)}{30ef^3}-\frac{dx\left(35ac^2f^3-130acdef^2+75ad^2e^2f-109bc^2ef^2+218bcde^2f-105bd^2e^3\right)}{30ef^4}+\frac{x\left(c+dx^2\right)^3(af-be)}{2ef\left(e+fx^2\right)}+\frac{(cf-de)^2\left(acf^2+5adef+bcef-7bde^2\right)\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{\frac{3}{2}}f^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)**3/(f*x**2+e)**2,x)
```

```
[Out] -d*x*(c + d*x**2)**2*(5*a*f - 7*b*e)/(10*e*f**2) + d*x*(c*(5*c*f*
(a*f + b*e) + d*e*(5*a*f - 7*b*e)) - d*x**2*(15*a*c*f**2 - 25*a*d
*e*f - 33*b*c*e*f + 35*b*d*e**2))/(30*e*f**3) - d*x*(35*a*c**2*f*
*3 - 130*a*c*d*e*f**2 + 75*a*d**2*e**2*f - 109*b*c**2*e*f**2 + 21
8*b*c*d*e**2*f - 105*b*d**2*e**3)/(30*e*f**4) + x*(c + d*x**2)**3
*(a*f - b*e)/(2*e*f*(e + f*x**2)) + (c*f - d*e)**2*(a*c*f**2 + 5*
a*d*e*f + b*c*e*f - 7*b*d*e**2)*atan(sqrt(f)*x/sqrt(e))/(2*e**(3/
2)*f**(9/2))
```

Mathematica [A] time = 0.24015, size = 176, normalized size = 0.73

$$\frac{d^2x^3(adf+3bcf-2bde)}{3f^3}-\frac{(de-cf)^2\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(be(7de-cf)-af(cf+5de))}{2e^{3/2}f^{9/2}}+\frac{x(be-af)(de-cf)^3}{2ef^4\left(e+fx^2\right)}+\frac{dx\left(adf(3cf-2de)+3b(de-cf)^2\right)}{f^4}+\frac{bd^3x^5}{5f^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^2,x]
```

```
[Out] (d*(3*b*(d*e - c*f)^2 + a*d*f*(-2*d*e + 3*c*f))*x)/f^4 + (d^2*(-2
*b*d*e + 3*b*c*f + a*d*f)*x^3)/(3*f^3) + (b*d^3*x^5)/(5*f^2) + ((
b*e - a*f)*(d*e - c*f)^3*x)/(2*e*f^4*(e + f*x^2)) - ((d*e - c*f)^
2*(b*e*(7*d*e - c*f) - a*f*(5*d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt
```

```
[e]])/(2*e^(3/2)*f^(9/2))
```

Maple [B] time = 0.017, size = 475, normalized size = 2.

$$\frac{bc^3}{2f}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}+\frac{axc^3}{2e(fx^2+e)}+3\frac{acd^2x}{f^2}-2\frac{ad^3ex}{f^3}+3\frac{bc^2dx}{f^2}+\frac{d^2x^3bc}{f^2} -\frac{2d^3x^3be}{3f^3}+\frac{3ac^2d}{2f}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}+\frac{5ad^3e^2}{2f^3}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}} -\frac{7bd^3e^3}{2f^4}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}-6\frac{bcd^2ex}{f^3}-\frac{3axc^2d}{2f(fx^2+e)}-\frac{e^2xad^3}{2f^3(fx^2+e)} +\frac{e^3xbd^3}{2f^4(fx^2+e)}+3\frac{bd^3e^2x}{f^4}-\frac{bxc^3}{2f(fx^2+e)}+\frac{ac^3}{2e}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}+\frac{d^3x^3a}{3f^2} +\frac{bd^3x^5}{5f^2}+\frac{15bcd^2e^2}{2f^3}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}-\frac{9bc^2de}{2f^2}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}} -\frac{3e^2xbcd^2}{2f^3(fx^2+e)}+\frac{3acexd^2}{2f^2(fx^2+e)}+\frac{3exbc^2d}{2f^2(fx^2+e)}-\frac{9acd^2e}{2f^2}\arctan\left(fx\frac{1}{\sqrt{ef}}\right)\frac{1}{\sqrt{ef}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^2,x)
```

```
[Out] 1/2/f/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*c^3+1/2/e*x/(f*x^2+e)
*a*c^3+3*d^2/f^2*a*c*x-2*d^3/f^3*a*e*x+3*d/f^2*b*c^2*x+d^2/f^2*x^
3*b*c-2/3*d^3/f^3*x^3*b*e+3/2/f/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2
))*a*c^2*d+5/2/f^3*e^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a*d^3-
7/2/f^4*e^3/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*d^3-6*d^2/f^3*b
*c*e*x-3/2/f*x/(f*x^2+e)*a*c^2*d-1/2/f^3*e^2*x/(f*x^2+e)*a*d^3+1/
2/f^4*e^3*x/(f*x^2+e)*b*d^3+3*d^3/f^4*b*e^2*x-1/2/f*x/(f*x^2+e)*b
*c^3+1/2/e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a*c^3+1/3*d^3/f^2*
x^3*a+1/5*d^3/f^2*b*x^5+15/2/f^3*e^2/(e*f)^(1/2)*arctan(x*f/(e*f)
^(1/2))*b*c*d^2-9/2/f^2*e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*c
^2*d-3/2/f^3*e^2*x/(f*x^2+e)*b*c*d^2+3/2/f^2*e*x/(f*x^2+e)*a*c*d^
2+3/2/f^2*e*x/(f*x^2+e)*b*c^2*d-9/2/f^2*e/(e*f)^(1/2)*arctan(x*f/
(e*f)^(1/2))*a*c*d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^3/(f*x^2 + e)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.227947, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^3/(f*x^2 + e)^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(15*(7*b*d^3*e^5 - a*c^3*e*f^4 - 5*(3*b*c*d^2 + a*d^3)*e^4
*f + 9*(b*c^2*d + a*c*d^2)*e^3*f^2 - (b*c^3 + 3*a*c^2*d)*e^2*f^3
+ (7*b*d^3*e^4*f - a*c^3*f^5 - 5*(3*b*c*d^2 + a*d^3)*e^3*f^2 + 9*
(b*c^2*d + a*c*d^2)*e^2*f^3 - (b*c^3 + 3*a*c^2*d)*e*f^4)*x^2)*log
((2*e*f*x + (f*x^2 - e)*sqrt(-e*f))/(f*x^2 + e)) - 2*(6*b*d^3*e*f
^3*x^7 - 2*(7*b*d^3*e^2*f^2 - 5*(3*b*c*d^2 + a*d^3)*e*f^3)*x^5 +
10*(7*b*d^3*e^3*f - 5*(3*b*c*d^2 + a*d^3)*e^2*f^2 + 9*(b*c^2*d +
a*c*d^2)*e*f^3)*x^3 + 15*(7*b*d^3*e^4 + a*c^3*f^4 - 5*(3*b*c*d^2
+ a*d^3)*e^3*f + 9*(b*c^2*d + a*c*d^2)*e^2*f^2 - (b*c^3 + 3*a*c^2
*d)*e*f^3)*x)*sqrt(-e*f))/((e*f^5*x^2 + e^2*f^4)*sqrt(-e*f)), -1/
30*(15*(7*b*d^3*e^5 - a*c^3*e*f^4 - 5*(3*b*c*d^2 + a*d^3)*e^4*f +
 9*(b*c^2*d + a*c*d^2)*e^3*f^2 - (b*c^3 + 3*a*c^2*d)*e^2*f^3 + (7
*b*d^3*e^4*f - a*c^3*f^5 - 5*(3*b*c*d^2 + a*d^3)*e^3*f^2 + 9*(b*c
^2*d + a*c*d^2)*e^2*f^3 - (b*c^3 + 3*a*c^2*d)*e*f^4)*x^2)*arctan(
sqrt(e*f)*x/e) - (6*b*d^3*e*f^3*x^7 - 2*(7*b*d^3*e^2*f^2 - 5*(3*b
*c*d^2 + a*d^3)*e*f^3)*x^5 + 10*(7*b*d^3*e^3*f - 5*(3*b*c*d^2 + a
*d^3)*e^2*f^2 + 9*(b*c^2*d + a*c*d^2)*e*f^3)*x^3 + 15*(7*b*d^3*e^
4 + a*c^3*f^4 - 5*(3*b*c*d^2 + a*d^3)*e^3*f + 9*(b*c^2*d + a*c*d^
2)*e^2*f^2 - (b*c^3 + 3*a*c^2*d)*e*f^3)*x)*sqrt(e*f))/((e*f^5*x^2
 + e^2*f^4)*sqrt(e*f))]
```

Sympy [A] time = 11.5012, size = 654, normalized size = 2.7

$$\frac{bd^3x^5}{5f^2} + \frac{x\left(ac^3f^4 - 3ac^2def^3 + 3acd^2e^2f^2 - ad^3e^3f - bc^3ef^3 + 3bc^2de^2f^2 - 3bcd^2e^3f + bd^3e^4\right)}{2e^2f^4 + 2ef^5x^2}$$
$$- \frac{\sqrt{-\frac{1}{e^3f^9}}\left(cf - de\right)^2\left(acf^2 + 5adef + bcef - 7bde^2\right)\log\left(-\frac{e^2f^4\sqrt{-\frac{1}{e^3f^9}}(cf-de)^2\left(acf^2+5adef+bcef-7bde^2\right)}{ac^3f^4+3ac^2def^3-9acd^2e^2f^2+5ad^3e^3f+bc^3ef^3-9bc^2de^2f^2+15bcd^2e^3f-7bd^3e}\right.}{4}$$
$$+ \frac{\sqrt{-\frac{1}{e^3f^9}}\left(cf - de\right)^2\left(acf^2 + 5adef + bcef - 7bde^2\right)\log\left(\frac{e^2f^4\sqrt{-\frac{1}{e^3f^9}}(cf-de)^2\left(acf^2+5adef+bcef-7bde^2\right)}{ac^3f^4+3ac^2def^3-9acd^2e^2f^2+5ad^3e^3f+bc^3ef^3-9bc^2de^2f^2+15bcd^2e^3f-7bd^3e^4}\right.}{4}$$
$$+ \frac{x^3\left(ad^3f + 3bcd^2f - 2bd^3e\right)}{3f^3} + \frac{x\left(3acd^2f^2 - 2ad^3ef + 3bc^2df^2 - 6bcd^2ef + 3bd^3e^2\right)}{f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x**2+c)**3/(f*x**2+e)**2,x)
```

```
[Out] b*d**3*x**5/(5*f**2) + x*(a*c**3*f**4 - 3*a*c**2*d*e*f**3 + 3*a*c
*d**2*e**2*f**2 - a*d**3*e**3*f - b*c**3*e*f**3 + 3*b*c**2*d*e**2
*f**2 - 3*b*c*d**2*e**3*f + b*d**3*e**4)/(2*e**2*f**4 + 2*e*f**5*
x**2) - sqrt(-1/(e**3*f**9))*(c*f - d*e)**2*(a*c*f**2 + 5*a*d*e*f
 + b*c*e*f - 7*b*d*e**2)*log(-e**2*f**4*sqrt(-1/(e**3*f**9))*(c*f
 - d*e)**2*(a*c*f**2 + 5*a*d*e*f + b*c*e*f - 7*b*d*e**2)/(a*c**3*
f**4 + 3*a*c**2*d*e*f**3 - 9*a*c*d**2*e**2*f**2 + 5*a*d**3*e**3*f
 + b*c**3*e*f**3 - 9*b*c**2*d*e**2*f**2 + 15*b*c*d**2*e**3*f - 7*
b*d**3*e**4) + x)/4 + sqrt(-1/(e**3*f**9))*(c*f - d*e)**2*(a*c*f*
*2 + 5*a*d*e*f + b*c*e*f - 7*b*d*e**2)*log(e**2*f**4*sqrt(-1/(e**
3*f**9))*(c*f - d*e)**2*(a*c*f**2 + 5*a*d*e*f + b*c*e*f - 7*b*d*e
**2)/(a*c**3*f**4 + 3*a*c**2*d*e*f**3 - 9*a*c*d**2*e**2*f**2 + 5*
a*d**3*e**3*f + b*c**3*e*f**3 - 9*b*c**2*d*e**2*f**2 + 15*b*c*d**
2*e**3*f - 7*b*d**3*e**4) + x)/4 + x**3*(a*d**3*f + 3*b*c*d**2*f
- 2*b*d**3*e)/(3*f**3) + x*(3*a*c*d**2*f**2 - 2*a*d**3*e*f + 3*b*
c**2*d*f**2 - 6*b*c*d**2*e*f + 3*b*d**3*e**2)/f**4
```

GIAC/XCAS [A] time = 0.230292, size = 433, normalized size = 1.79

$$\frac{\left(ac^3f^4 + bc^3f^3e + 3\,ac^2df^3e - 9\,bc^2df^2e^2 - 9\,acd^2f^2e^2 + 15\,bcd^2fe^3 + 5\,ad^3fe^3 - 7\,bd^3e^4\right)\arctan\left(\sqrt{f}xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{3}{2}\right)}}{2\,f^{\frac{9}{2}}} + \frac{\left(ac^3f^4x - bc^3f^3xe - 3\,ac^2df^3xe + 3\,bc^2df^2xe^2 + 3\,acd^2f^2xe^2 - 3\,bcd^2fxe^3 - ad^3fxe^3 + bd^3xe^4\right)e^{(-1)}}{2\,(fx^2+e)f^4} + \frac{3\,bd^3f^8x^5 + 15\,bcd^2f^8x^3 + 5\,ad^3f^8x^3 - 10\,bd^3f^7x^3e + 45\,bc^2df^8x + 45\,acd^2f^8x - 90\,bcd^2f^7xe - 30\,ad^3f^7xe + 45\,bd^3f^6}{15\,f^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)^3/(f*x^2 + e)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a*c^3*f^4 + b*c^3*f^3*e + 3*a*c^2*d*f^3*e - 9*b*c^2*d*f^2*e^
2 - 9*a*c*d^2*f^2*e^2 + 15*b*c*d^2*f*e^3 + 5*a*d^3*f*e^3 - 7*b*d^
3*e^4)*arctan(sqrt(f)*x*e^(-1/2))*e^(-3/2)/f^(9/2) + 1/2*(a*c^3*f
^4*x - b*c^3*f^3*x*e - 3*a*c^2*d*f^3*x*e + 3*b*c^2*d*f^2*x*e^2 +
3*a*c*d^2*f^2*x*e^2 - 3*b*c*d^2*f*x*e^3 - a*d^3*f*x*e^3 + b*d^3*x
*e^4)*e^(-1)/((f*x^2 + e)*f^4) + 1/15*(3*b*d^3*f^8*x^5 + 15*b*c*d
^2*f^8*x^3 + 5*a*d^3*f^8*x^3 - 10*b*d^3*f^7*x^3*e + 45*b*c^2*d*f^
8*x + 45*a*c*d^2*f^8*x - 90*b*c*d^2*f^7*x*e - 30*a*d^3*f^7*x*e +
45*b*d^3*f^6*x*e^2)/f^10
```

3.21 $\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^3}\,dx$

Optimal. Leaf size=291

$$\frac{dx\left(3af\left(-3c^2f^2-4cdef+15d^2e^2\right)-be\left(3c^2f^2-100cdef+105d^2e^2\right)\right)}{24e^2f^4}+\frac{(de-cf)\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^2f^2-10cdef+35d^2e^2\right)-3af\left(c^2f^2+2cdef+5d^2e^2\right)\right)}{8e^{5/2}f^{9/2}}+\frac{dx\left(c+dx^2\right)(be(35de-3cf)-3af(3cf+5de))}{24e^2f^3}-\frac{x\left(c+dx^2\right)^2(be(7de-cf)-3af(cf+de))}{8e^2f^2\left(e+fx^2\right)}-\frac{x\left(c+dx^2\right)^3(be-af)}{4ef\left(e+fx^2\right)^2}$$

```
[Out] (d*(3*a*f*(15*d^2*e^2 - 4*c*d*e*f - 3*c^2*f^2) - b*e*(105*d^2*e^2
 - 100*c*d*e*f + 3*c^2*f^2))*x)/(24*e^2*f^4) + (d*(b*e*(35*d*e -
3*c*f) - 3*a*f*(5*d*e + 3*c*f))*x*(c + d*x^2))/(24*e^2*f^3) - ((b
*e - a*f)*x*(c + d*x^2)^3)/(4*e*f*(e + f*x^2)^2) - ((b*e*(7*d*e -
 c*f) - 3*a*f*(d*e + c*f))*x*(c + d*x^2)^2)/(8*e^2*f^2*(e + f*x^2
)) + ((d*e - c*f)*(b*e*(35*d^2*e^2 - 10*c*d*e*f - c^2*f^2) - 3*a*
f*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])
/(8*e^(5/2)*f^(9/2))
```

Rubi [A] time = 1.05878, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{dx\left(3af\left(-3c^2f^2-4cdef+15d^2e^2\right)-be\left(3c^2f^2-100cdef+105d^2e^2\right)\right)}{24e^2f^4}+\frac{(de-cf)\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^2f^2-10cdef+35d^2e^2\right)-3af\left(c^2f^2+2cdef+5d^2e^2\right)\right)}{8e^{5/2}f^{9/2}}+\frac{dx\left(c+dx^2\right)(be(35de-3cf)-3af(3cf+5de))}{24e^2f^3}-\frac{x\left(c+dx^2\right)^2(be(7de-cf)-3af(cf+de))}{8e^2f^2\left(e+fx^2\right)}-\frac{x\left(c+dx^2\right)^3(be-af)}{4ef\left(e+fx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^3,x]
```

```
[Out] (d*(3*a*f*(15*d^2*e^2 - 4*c*d*e*f - 3*c^2*f^2) - b*e*(105*d^2*e^2
 - 100*c*d*e*f + 3*c^2*f^2))*x)/(24*e^2*f^4) + (d*(b*e*(35*d*e -
3*c*f) - 3*a*f*(5*d*e + 3*c*f))*x*(c + d*x^2))/(24*e^2*f^3) - ((b
*e - a*f)*x*(c + d*x^2)^3)/(4*e*f*(e + f*x^2)^2) - ((b*e*(7*d*e -
```

```
c*f) - 3*a*f*(d*e + c*f))*x*(c + d*x^2)^2)/(8*e^2*f^2*(e + f*x^2
)) + ((d*e - c*f)*(b*e*(35*d^2*e^2 - 10*c*d*e*f - c^2*f^2) - 3*a*
f*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])
/(8*e^(5/2)*f^(9/2))
```

Rubi in Sympy [A] time = 129.74, size = 332, normalized size = 1.14

$$\frac{dx\left(c\left(cf\left(3af+be\right)-de\left(3af-7be\right)\right)-dx^2\left(3cf\left(3af+be\right)+5de\left(3af-7be\right)\right)\right)}{24e^2f^3} - \frac{dx\left(21ac^2f^3+24acdef^2-45ad^2e^2f+7bc^2ef^2-128bcde^2f+105bd^2e^3\right)}{24e^2f^4} + \frac{x\left(c+dx^2\right)^3\left(af-be\right)}{4ef\left(e+fx^2\right)^2} + \frac{x\left(c+dx^2\right)^2\left(cf\left(3af+be\right)+de\left(3af-7be\right)\right)}{8e^2f^2\left(e+fx^2\right)} + \frac{\left(cf-de\right)\left(3ac^2f^3+6acdef^2+15ad^2e^2f+bc^2ef^2+10bcde^2f-35bd^2e^3\right)\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{8e^{\frac{5}{2}}f^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)**3/(f*x**2+e)**3,x)
```

```
[Out] d*x*(c*(c*f*(3*a*f + b*e) - d*e*(3*a*f - 7*b*e)) - d*x**2*(3*c*f*
(3*a*f + b*e) + 5*d*e*(3*a*f - 7*b*e)))/(24*e**2*f**3) - d*x*(21*
a*c**2*f**3 + 24*a*c*d*e*f**2 - 45*a*d**2*e**2*f + 7*b*c**2*e*f**
2 - 128*b*c*d*e**2*f + 105*b*d**2*e**3)/(24*e**2*f**4) + x*(c + d
*x**2)**3*(a*f - b*e)/(4*e*f*(e + f*x**2)**2) + x*(c + d*x**2)**2
*(c*f*(3*a*f + b*e) + d*e*(3*a*f - 7*b*e))/(8*e**2*f**2*(e + f*x*
*2)) + (c*f - d*e)*(3*a*c**2*f**3 + 6*a*c*d*e*f**2 + 15*a*d**2*e*
*2*f + b*c**2*e*f**2 + 10*b*c*d*e**2*f - 35*b*d**2*e**3)*atan(sqr
t(f)*x/sqrt(e))/(8*e**(5/2)*f**(9/2))
```

Mathematica [A] time = 0.317139, size = 219, normalized size = 0.75

$$\frac{(de-cf)\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^2f^2-10cdef+35d^2e^2\right)-3af\left(c^2f^2+2cdef+5d^2e^2\right)\right)}{8e^{5/2}f^{9/2}} + \frac{d^2x(adf+3bcf-3bde)}{f^4} - \frac{x(de-cf)^2(be(13de-cf)-3af(cf+3de))}{8e^2f^4\left(e+fx^2\right)} + \frac{x(be-af)(de-cf)^3}{4ef^4\left(e+fx^2\right)^2} + \frac{bd^3x^3}{3f^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^3,x]
```

```
[Out] (d^2*(-3*b*d*e + 3*b*c*f + a*d*f)*x)/f^4 + (b*d^3*x^3)/(3*f^3) +
((b*e - a*f)*(d*e - c*f)^3*x)/(4*e*f^4*(e + f*x^2)^2) - ((d*e - c
*f)^2*(b*e*(13*d*e - c*f) - 3*a*f*(3*d*e + c*f))*x)/(8*e^2*f^4*(e
 + f*x^2)) + ((d*e - c*f)*(b*e*(35*d^2*e^2 - 10*c*d*e*f - c^2*f^2
) - 3*a*f*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2))*ArcTan[(Sqrt[f]*x)/S
qrt[e]])/(8*e^(5/2)*f^(9/2))
```

Maple [B] time = 0.018, size = 589, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^3,x)
```

```
[Out] 5/8/(f*x^2+e)^2/e*x*a*c^3-3*d^3/f^4*x*b*e+3*d^2/f^3*x*b*c-15/8/f/
(f*x^2+e)^2*x^3*a*c*d^2+9/8/f^2/(f*x^2+e)^2*x^3*a*d^3*e-15/8/f/(f
*x^2+e)^2*x^3*b*c^2*d-13/8/f^3/(f*x^2+e)^2*x^3*b*d^3*e^2-3/8/f/(f
*x^2+e)^2*a*c^2*d*x+7/8/f^3/(f*x^2+e)^2*a*d^3*e^2*x-11/8/f^4/(f*x
^2+e)^2*b*d^3*e^3*x+9/8/f^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a
*c*d^2-15/8/f^3*e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a*d^3+1/8/f
/e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*c^3+9/8/f^2/(e*f)^(1/2)*
arctan(x*f/(e*f)^(1/2))*b*c^2*d+35/8/f^4*e^2/(e*f)^(1/2)*arctan(x
*f/(e*f)^(1/2))*b*d^3-45/8/f^3*e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/
2))*b*c*d^2+27/8/f^2/(f*x^2+e)^2*x^3*b*c*d^2*e+21/8/f^3/(f*x^2+e)
^2*b*c*d^2*e^2*x+3/8/f/e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a*c^
2*d-9/8/f^2/(f*x^2+e)^2*a*c*d^2*e*x-9/8/f^2/(f*x^2+e)^2*b*c^2*d*e
*x+3/8/(f*x^2+e)^2/e*x^3*a*c^2*d+3/8*f/(f*x^2+e)^2/e^2*x^3*a*c^3-
1/8/f/(f*x^2+e)^2*b*c^3*x+3/8/e^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1
/2))*a*c^3+1/8/(f*x^2+e)^2/e*x^3*b*c^3+d^3/f^3*x*a+1/3*d^3/f^3*x^
3*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^3/(f*x^2 + e)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.226608, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^3/(f*x^2 + e)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(35*b*d^3*e^6 + 3*a*c^3*e^2*f^4 - 15*(3*b*c*d^2 + a*d^3)
*e^5*f + 9*(b*c^2*d + a*c*d^2)*e^4*f^2 + (b*c^3 + 3*a*c^2*d)*e^3*
f^3 + (35*b*d^3*e^4*f^2 + 3*a*c^3*f^6 - 15*(3*b*c*d^2 + a*d^3)*e^
3*f^3 + 9*(b*c^2*d + a*c*d^2)*e^2*f^4 + (b*c^3 + 3*a*c^2*d)*e*f^5
)*x^4 + 2*(35*b*d^3*e^5*f + 3*a*c^3*e*f^5 - 15*(3*b*c*d^2 + a*d^3
)*e^4*f^2 + 9*(b*c^2*d + a*c*d^2)*e^3*f^3 + (b*c^3 + 3*a*c^2*d)*e
^2*f^4)*x^2)*log((2*e*f*x + (f*x^2 - e)*sqrt(-e*f))/(f*x^2 + e))
+ 2*(8*b*d^3*e^2*f^3*x^7 - 8*(7*b*d^3*e^3*f^2 - 3*(3*b*c*d^2 + a*
d^3)*e^2*f^3)*x^5 - (175*b*d^3*e^4*f - 9*a*c^3*f^5 - 75*(3*b*c*d^
2 + a*d^3)*e^3*f^2 + 45*(b*c^2*d + a*c*d^2)*e^2*f^3 - 3*(b*c^3 +
3*a*c^2*d)*e*f^4)*x^3 - 3*(35*b*d^3*e^5 - 5*a*c^3*e*f^4 - 15*(3*b
*c*d^2 + a*d^3)*e^4*f + 9*(b*c^2*d + a*c*d^2)*e^3*f^2 + (b*c^3 +
3*a*c^2*d)*e^2*f^3)*x)*sqrt(-e*f))/((e^2*f^6*x^4 + 2*e^3*f^5*x^2
+ e^4*f^4)*sqrt(-e*f)), 1/24*(3*(35*b*d^3*e^6 + 3*a*c^3*e^2*f^4 -
 15*(3*b*c*d^2 + a*d^3)*e^5*f + 9*(b*c^2*d + a*c*d^2)*e^4*f^2 + (
b*c^3 + 3*a*c^2*d)*e^3*f^3 + (35*b*d^3*e^4*f^2 + 3*a*c^3*f^6 - 15
*(3*b*c*d^2 + a*d^3)*e^3*f^3 + 9*(b*c^2*d + a*c*d^2)*e^2*f^4 + (b
*c^3 + 3*a*c^2*d)*e*f^5)*x^4 + 2*(35*b*d^3*e^5*f + 3*a*c^3*e*f^5
- 15*(3*b*c*d^2 + a*d^3)*e^4*f^2 + 9*(b*c^2*d + a*c*d^2)*e^3*f^3
+ (b*c^3 + 3*a*c^2*d)*e^2*f^4)*x^2)*arctan(sqrt(e*f)*x/e) + (8*b*
d^3*e^2*f^3*x^7 - 8*(7*b*d^3*e^3*f^2 - 3*(3*b*c*d^2 + a*d^3)*e^2*
f^3)*x^5 - (175*b*d^3*e^4*f - 9*a*c^3*f^5 - 75*(3*b*c*d^2 + a*d^3
)*e^3*f^2 + 45*(b*c^2*d + a*c*d^2)*e^2*f^3 - 3*(b*c^3 + 3*a*c^2*d
)*e*f^4)*x^3 - 3*(35*b*d^3*e^5 - 5*a*c^3*e*f^4 - 15*(3*b*c*d^2 +
a*d^3)*e^4*f + 9*(b*c^2*d + a*c*d^2)*e^3*f^2 + (b*c^3 + 3*a*c^2*d
)*e^2*f^3)*x)*sqrt(e*f))/((e^2*f^6*x^4 + 2*e^3*f^5*x^2 + e^4*f^4)
*sqrt(e*f))]
```

Sympy [A] time = 74.199, size = 862, normalized size = 2.96

$$\frac{bd^3x^3}{3f^3} - \frac{\sqrt{-\frac{1}{e^5f^9}}\,(cf-de)\left(3ac^2f^3+6acdef^2+15ad^2e^2f+bc^2ef^2+10bcde^2f-35bd^2e^3\right)\log\left(-\frac{e^3f^4\sqrt{-\frac{1}{e^5f^9}}(cf-de)(3ac^2f^3+6acde\ldots}{3ac^3f^4+3ac^2def^3+9acd^2e^2f^2-15ad^3e\ldots}\right.}{16}$$

$$+\frac{\sqrt{-\frac{1}{e^5f^9}}\,(cf-de)\left(3ac^2f^3+6acdef^2+15ad^2e^2f+bc^2ef^2+10bcde^2f-35bd^2e^3\right)\log\left(\frac{e^3f^4\sqrt{-\frac{1}{e^5f^9}}(cf-de)(3ac^2f^3+6acdef^2+\ldots}{3ac^3f^4+3ac^2def^3+9acd^2e^2f^2-15ad^3e^3f\ldots}\right.}{16}$$

$$+\frac{x^3\left(3ac^3f^5+3ac^2def^4-15acd^2e^2f^3+9ad^3e^3f^2+bc^3ef^4-15bc^2de^2f^3+27bcd^2e^3f^2-13bd^3e^4f\right)+x\left(5ac^3ef^4-3ac^2\ldots\right.}{8e^4f^4+16e^3f^5x^2+8e^2f^6x^4}$$

$$+\frac{x\left(ad^3f+3bcd^2f-3bd^3e\right)}{f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x**2+c)**3/(f*x**2+e)**3,x)
```

```
[Out] b*d**3*x**3/(3*f**3) - sqrt(-1/(e**5*f**9))*(c*f - d*e)*(3*a*c**2
*f**3 + 6*a*c*d*e*f**2 + 15*a*d**2*e**2*f + b*c**2*e*f**2 + 10*b*
c*d*e**2*f - 35*b*d**2*e**3)*log(-e**3*f**4*sqrt(-1/(e**5*f**9))*
(c*f - d*e)*(3*a*c**2*f**3 + 6*a*c*d*e*f**2 + 15*a*d**2*e**2*f +
b*c**2*e*f**2 + 10*b*c*d*e**2*f - 35*b*d**2*e**3)/(3*a*c**3*f**4
+ 3*a*c**2*d*e*f**3 + 9*a*c*d**2*e**2*f**2 - 15*a*d**3*e**3*f + b
*c**3*e*f**3 + 9*b*c**2*d*e**2*f**2 - 45*b*c*d**2*e**3*f + 35*b*d
**3*e**4) + x)/16 + sqrt(-1/(e**5*f**9))*(c*f - d*e)*(3*a*c**2*f*
*3 + 6*a*c*d*e*f**2 + 15*a*d**2*e**2*f + b*c**2*e*f**2 + 10*b*c*d
*e**2*f - 35*b*d**2*e**3)*log(e**3*f**4*sqrt(-1/(e**5*f**9))*(c*f
 - d*e)*(3*a*c**2*f**3 + 6*a*c*d*e*f**2 + 15*a*d**2*e**2*f + b*c*
*2*e*f**2 + 10*b*c*d*e**2*f - 35*b*d**2*e**3)/(3*a*c**3*f**4 + 3*
a*c**2*d*e*f**3 + 9*a*c*d**2*e**2*f**2 - 15*a*d**3*e**3*f + b*c**
3*e*f**3 + 9*b*c**2*d*e**2*f**2 - 45*b*c*d**2*e**3*f + 35*b*d**3*
e**4) + x)/16 + (x**3*(3*a*c**3*f**5 + 3*a*c**2*d*e*f**4 - 15*a*c
*d**2*e**2*f**3 + 9*a*d**3*e**3*f**2 + b*c**3*e*f**4 - 15*b*c**2*
d*e**2*f**3 + 27*b*c*d**2*e**3*f**2 - 13*b*d**3*e**4*f) + x*(5*a*
c**3*e*f**4 - 3*a*c**2*d*e**2*f**3 - 9*a*c*d**2*e**3*f**2 + 7*a*d
**3*e**4*f - b*c**3*e**2*f**3 - 9*b*c**2*d*e**3*f**2 + 21*b*c*d**
2*e**4*f - 11*b*d**3*e**5))/(8*e**4*f**4 + 16*e**3*f**5*x**2 + 8*
e**2*f**6*x**4) + x*(a*d**3*f + 3*b*c*d**2*f - 3*b*d**3*e)/f**4
```

GIAC/XCAS [A] time = 0.228202, size = 501, normalized size = 1.72

$$\frac{\left(3\,ac^3f^4 + bc^3f^3e + 3\,ac^2df^3e + 9\,bc^2df^2e^2 + 9\,acd^2f^2e^2 - 45\,bcd^2fe^3 - 15\,ad^3fe^3 + 35\,bd^3e^4\right)\arctan\left(\sqrt{f}xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{5}{2}\right)}}{8\,f^{\frac{9}{2}}} + \frac{\left(3\,ac^3f^5x^3 + bc^3f^4x^3e + 3\,ac^2df^4x^3e - 15\,bc^2df^3x^3e^2 - 15\,acd^2f^3x^3e^2 + 27\,bcd^2f^2x^3e^3 + 9\,ad^3f^2x^3e^3 + 5\,ac^3f^4xe - 13}{8\,(fx^2+e)^2} + \frac{bd^3f^6x^3 + 9\,bcd^2f^6x + 3\,ad^3f^6x - 9\,bd^3f^5xe}{3\,f^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)^3/(f*x^2 + e)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*a*c^3*f^4 + b*c^3*f^3*e + 3*a*c^2*d*f^3*e + 9*b*c^2*d*f^2*
e^2 + 9*a*c*d^2*f^2*e^2 - 45*b*c*d^2*f*e^3 - 15*a*d^3*f*e^3 + 35*
b*d^3*e^4)*arctan(sqrt(f)*x*e^(-1/2))*e^(-5/2)/f^(9/2) + 1/8*(3*a
*c^3*f^5*x^3 + b*c^3*f^4*x^3*e + 3*a*c^2*d*f^4*x^3*e - 15*b*c^2*d
*f^3*x^3*e^2 - 15*a*c*d^2*f^3*x^3*e^2 + 27*b*c*d^2*f^2*x^3*e^3 +
9*a*d^3*f^2*x^3*e^3 + 5*a*c^3*f^4*x*e - 13*b*d^3*f*x^3*e^4 - b*c^
3*f^3*x*e^2 - 3*a*c^2*d*f^3*x*e^2 - 9*b*c^2*d*f^2*x*e^3 - 9*a*c*d
^2*f^2*x*e^3 + 21*b*c*d^2*f*x*e^4 + 7*a*d^3*f*x*e^4 - 11*b*d^3*x*
e^5)*e^(-2)/((f*x^2 + e)^2*f^4) + 1/3*(b*d^3*f^6*x^3 + 9*b*c*d^2*
f^6*x + 3*a*d^3*f^6*x - 9*b*d^3*f^5*x*e)/f^9
```

3.22 $\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^4}\,dx$

Optimal. Leaf size=348

$$\frac{dx\left(be\left(-3c^2f^2-10cdef+105d^2e^2\right)-af\left(15c^2f^2+14cdef+15d^2e^2\right)\right)}{48e^3f^4}-\frac{x\left(c+dx^2\right)\left(be\left(-3c^2f^2-8cdef+35d^2e^2\right)-af\left(15c^2f^2+4cdef+5d^2e^2\right)\right)}{48e^3f^3\left(e+fx^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^3f^3-3c^2def^2-15cd^2e^2f+35d^3e^3\right)-af\left(5c^3f^3+3c^2def^2+3cd^2e^2f+5d^3e^3\right)\right)}{16e^{7/2}f^{9/2}}-\frac{x\left(c+dx^2\right)^2\left(be(7de-cf)-af(5cf+de)\right)}{24e^2f^2\left(e+fx^2\right)^2}-\frac{x\left(c+dx^2\right)^3(be-af)}{6ef\left(e+fx^2\right)^3}$$

```
[Out] (d*(b*e*(105*d^2*e^2 - 10*c*d*e*f - 3*c^2*f^2) - a*f*(15*d^2*e^2
+ 14*c*d*e*f + 15*c^2*f^2))*x)/(48*e^3*f^4) - ((b*e - a*f)*x*(c +
 d*x^2)^3)/(6*e*f*(e + f*x^2)^3) - ((b*e*(7*d*e - c*f) - a*f*(d*e
 + 5*c*f))*x*(c + d*x^2)^2)/(24*e^2*f^2*(e + f*x^2)^2) - ((b*e*(3
5*d^2*e^2 - 8*c*d*e*f - 3*c^2*f^2) - a*f*(5*d^2*e^2 + 4*c*d*e*f +
 15*c^2*f^2))*x*(c + d*x^2))/(48*e^3*f^3*(e + f*x^2)) - ((b*e*(35
*d^3*e^3 - 15*c*d^2*e^2*f - 3*c^2*d*e*f^2 - c^3*f^3) - a*f*(5*d^3
*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3))*ArcTan[(Sqrt[f
]*x)/Sqrt[e]])/(16*e^(7/2)*f^(9/2))
```

Rubi [A] time = 1.17798, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$\frac{dx\left(be\left(-3c^2f^2-10cdef+105d^2e^2\right)-af\left(15c^2f^2+14cdef+15d^2e^2\right)\right)}{48e^3f^4}-\frac{x\left(c+dx^2\right)\left(be\left(-3c^2f^2-8cdef+35d^2e^2\right)-af\left(15c^2f^2+4cdef+5d^2e^2\right)\right)}{48e^3f^3\left(e+fx^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^3f^3-3c^2def^2-15cd^2e^2f+35d^3e^3\right)-af\left(5c^3f^3+3c^2def^2+3cd^2e^2f+5d^3e^3\right)\right)}{16e^{7/2}f^{9/2}}-\frac{x\left(c+dx^2\right)^2\left(be(7de-cf)-af(5cf+de)\right)}{24e^2f^2\left(e+fx^2\right)^2}-\frac{x\left(c+dx^2\right)^3(be-af)}{6ef\left(e+fx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^4,x]
```



```
[Out] (d*(b*e*(105*d^2*e^2 - 10*c*d*e*f - 3*c^2*f^2) - a*f*(15*d^2*e^2
+ 14*c*d*e*f + 15*c^2*f^2))*x)/(48*e^3*f^4) - ((b*e - a*f)*x*(c +
 d*x^2)^3)/(6*e*f*(e + f*x^2)^3) - ((b*e*(7*d*e - c*f) - a*f*(d*e
```

```
 + 5*c*f))*x*(c + d*x^2)^2)/(24*e^2*f^2*(e + f*x^2)^2) - ((b*e*(3
5*d^2*e^2 - 8*c*d*e*f - 3*c^2*f^2) - a*f*(5*d^2*e^2 + 4*c*d*e*f +
 15*c^2*f^2))*x*(c + d*x^2))/(48*e^3*f^3*(e + f*x^2)) - ((b*e*(35
*d^3*e^3 - 15*c*d^2*e^2*f - 3*c^2*d*e*f^2 - c^3*f^3) - a*f*(5*d^3
*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3))*ArcTan[(Sqrt[f
]*x)/Sqrt[e]])/(16*e^(7/2)*f^(9/2))
```

Rubi in Sympy [A] time = 148.443, size = 359, normalized size = 1.03

$$-\frac{dx\left(cf\left(3cf\left(5af+be\right)-de\left(af-7be\right)\right)+3de\left(cf\left(5af+be\right)+5de\left(af-7be\right)\right)\right)}{48e^3f^4}+\frac{x\left(c+dx^2\right)^3\left(af-be\right)}{6ef\left(e+fx^2\right)^3}+\frac{x\left(c+dx^2\right)^2\left(cf\left(5af+be\right)+de\left(af-7be\right)\right)}{24e^2f^2\left(e+fx^2\right)^2}+\frac{x\left(c+dx^2\right)\left(cf\left(3cf\left(5af+be\right)-de\left(af-7be\right)\right)+de\left(cf\left(5af+be\right)+5de\left(af-7be\right)\right)\right)}{48e^3f^3\left(e+fx^2\right)}+\frac{\left(5ac^3f^4+3ac^2def^3+3acd^2e^2f^2+5ad^3e^3f+bc^3ef^3+3bc^2de^2f^2+15bcd^2e^3f-35bd^3e^4\right)\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{16e^{\frac{7}{2}}f^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)**3/(f*x**2+e)**4,x)
```

```
[Out] -d*x*(c*f*(3*c*f*(5*a*f + b*e) - d*e*(a*f - 7*b*e)) + 3*d*e*(c*f*
(5*a*f + b*e) + 5*d*e*(a*f - 7*b*e)))/(48*e**3*f**4) + x*(c + d*x
**2)**3*(a*f - b*e)/(6*e*f*(e + f*x**2)**3) + x*(c + d*x**2)**2*(
c*f*(5*a*f + b*e) + d*e*(a*f - 7*b*e))/(24*e**2*f**2*(e + f*x**2)
**2) + x*(c + d*x**2)*(c*f*(3*c*f*(5*a*f + b*e) - d*e*(a*f - 7*b*
e)) + d*e*(c*f*(5*a*f + b*e) + 5*d*e*(a*f - 7*b*e)))/(48*e**3*f**
3*(e + f*x**2)) + (5*a*c**3*f**4 + 3*a*c**2*d*e*f**3 + 3*a*c*d**2
*e**2*f**2 + 5*a*d**3*e**3*f + b*c**3*e*f**3 + 3*b*c**2*d*e**2*f*
*2 + 15*b*c*d**2*e**3*f - 35*b*d**3*e**4)*atan(sqrt(f)*x/sqrt(e))
/(16*e**(7/2)*f**(9/2))
```

Mathematica [A] time = 0.42651, size = 295, normalized size = 0.85

$$\frac{x(de-cf)\left(be\left(-c^2f^2-4cdef+29d^2e^2\right)-af\left(5c^2f^2+8cdef+11d^2e^2\right)\right)}{16e^3f^4\left(e+fx^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be\left(-c^3f^3-3c^2def^2-15cd^2e^2f+35d^3e^3\right)-af\left(5c^3f^3+3c^2def^2+3cd^2e^2f+5d^3e^3\right)\right)}{16e^{7/2}f^{9/2}}-\frac{x(de-cf)^2(be(19de-cf)-af(5cf+13de))}{24e^2f^4\left(e+fx^2\right)^2}+\frac{x(be-af)(de-cf)^3}{6ef^4\left(e+fx^2\right)^3}+\frac{bd^3x}{f^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^4,x]
```

```
[Out] (b*d^3*x)/f^4 + ((b*e - a*f)*(d*e - c*f)^3*x)/(6*e*f^4*(e + f*x^2
)^3) - ((d*e - c*f)^2*(b*e*(19*d*e - c*f) - a*f*(13*d*e + 5*c*f))
*x)/(24*e^2*f^4*(e + f*x^2)^2) + ((d*e - c*f)*(b*e*(29*d^2*e^2 -
4*c*d*e*f - c^2*f^2) - a*f*(11*d^2*e^2 + 8*c*d*e*f + 5*c^2*f^2))*
x)/(16*e^3*f^4*(e + f*x^2)) - ((b*e*(35*d^3*e^3 - 15*c*d^2*e^2*f
- 3*c^2*d*e*f^2 - c^3*f^3) - a*f*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c
^2*d*e*f^2 + 5*c^3*f^3))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(16*e^(7/2)
*f^(9/2))
```

Maple [B] time = 0.021, size = 735, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^4,x)
```

```
[Out] 11/16/(f*x^2+e)^3/e*x*a*c^3-3/16/f/(f*x^2+e)^3*a*c^2*d*x-5/16/f^3
/(f*x^2+e)^3*a*d^3*e^2*x+15/16/f^3/(e*f)^(1/2)*arctan(x*f/(e*f)^(
1/2))*b*c*d^2+1/16/f/e^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*c^
3-35/16/f^4*e/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*b*d^3+19/16/f^4
/(f*x^2+e)^3*b*d^3*e^3*x+1/2/(f*x^2+e)^3/e*x^3*a*c^2*d+3/16/(f*x^
2+e)^3/e*x^5*b*c^2*d-1/2/f/(f*x^2+e)^3*x^3*b*c^2*d+17/6/f^3/(f*x^
2+e)^3*x^3*b*d^3*e^2+3/16/(f*x^2+e)^3/e*x^5*a*c*d^2+5/16*f^2/(f*x
^2+e)^3/e^3*x^5*a*c^3+1/16*f/(f*x^2+e)^3/e^2*x^5*b*c^3-33/16/f/(f
*x^2+e)^3*x^5*b*c*d^2+29/16/f^2/(f*x^2+e)^3*x^5*b*d^3*e+5/6*f/(f*
x^2+e)^3/e^2*x^3*a*c^3-1/2/f/(f*x^2+e)^3*x^3*a*c*d^2-5/6/f^2/(f*x
^2+e)^3*x^3*a*d^3*e+3/16/f/e^2/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2)
)*a*c^2*d-5/2/f^2/(f*x^2+e)^3*x^3*b*c*d^2*e+b*d^3/f^4*x-3/16/f^2/
(f*x^2+e)^3*a*c*d^2*e*x-3/16/f^2/(f*x^2+e)^3*b*c^2*d*e*x+5/16/e^3
/(e*f)^(1/2)*arctan(x*f/(e*f)^(1/2))*a*c^3+5/16/f^3/(e*f)^(1/2)*a
rctan(x*f/(e*f)^(1/2))*a*d^3+1/6/(f*x^2+e)^3/e*x^3*b*c^3-11/16/f/
(f*x^2+e)^3*x^5*a*d^3-1/16/f/(f*x^2+e)^3*b*c^3*x+3/16/f^2/e/(e*f)
^(1/2)*arctan(x*f/(e*f)^(1/2))*b*c^2*d-15/16/f^3/(f*x^2+e)^3*b*c*
d^2*e^2*x+3/16*f/(f*x^2+e)^3/e^2*x^5*a*c^2*d+3/16/f^2/e/(e*f)^(1/
2)*arctan(x*f/(e*f)^(1/2))*a*c*d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^3/(f*x^2 + e)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.229092, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^3/(f*x^2 + e)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(35*b*d^3*e^7 - 5*a*c^3*e^3*f^4 - 5*(3*b*c*d^2 + a*d^3)
*e^6*f - 3*(b*c^2*d + a*c*d^2)*e^5*f^2 - (b*c^3 + 3*a*c^2*d)*e^4*
f^3 + (35*b*d^3*e^4*f^3 - 5*a*c^3*f^7 - 5*(3*b*c*d^2 + a*d^3)*e^3
*f^4 - 3*(b*c^2*d + a*c*d^2)*e^2*f^5 - (b*c^3 + 3*a*c^2*d)*e*f^6)
*x^6 + 3*(35*b*d^3*e^5*f^2 - 5*a*c^3*e*f^6 - 5*(3*b*c*d^2 + a*d^3
)*e^4*f^3 - 3*(b*c^2*d + a*c*d^2)*e^3*f^4 - (b*c^3 + 3*a*c^2*d)*e
^2*f^5)*x^4 + 3*(35*b*d^3*e^6*f - 5*a*c^3*e^2*f^5 - 5*(3*b*c*d^2
+ a*d^3)*e^5*f^2 - 3*(b*c^2*d + a*c*d^2)*e^4*f^3 - (b*c^3 + 3*a*c
^2*d)*e^3*f^4)*x^2)*log((2*e*f*x + (f*x^2 - e)*sqrt(-e*f))/(f*x^2
 + e)) - 2*(48*b*d^3*e^3*f^3*x^7 + 3*(77*b*d^3*e^4*f^2 + 5*a*c^3*
f^6 - 11*(3*b*c*d^2 + a*d^3)*e^3*f^3 + 3*(b*c^2*d + a*c*d^2)*e^2*
f^4 + (b*c^3 + 3*a*c^2*d)*e*f^5)*x^5 + 8*(35*b*d^3*e^5*f + 5*a*c^
3*e*f^5 - 5*(3*b*c*d^2 + a*d^3)*e^4*f^2 - 3*(b*c^2*d + a*c*d^2)*e
^3*f^3 + (b*c^3 + 3*a*c^2*d)*e^2*f^4)*x^3 + 3*(35*b*d^3*e^6 + 11*
a*c^3*e^2*f^4 - 5*(3*b*c*d^2 + a*d^3)*e^5*f - 3*(b*c^2*d + a*c*d^
2)*e^4*f^2 - (b*c^3 + 3*a*c^2*d)*e^3*f^3)*x)*sqrt(-e*f))/((e^3*f^
7*x^6 + 3*e^4*f^6*x^4 + 3*e^5*f^5*x^2 + e^6*f^4)*sqrt(-e*f)), -1/
48*(3*(35*b*d^3*e^7 - 5*a*c^3*e^3*f^4 - 5*(3*b*c*d^2 + a*d^3)*e^6
*f - 3*(b*c^2*d + a*c*d^2)*e^5*f^2 - (b*c^3 + 3*a*c^2*d)*e^4*f^3
+ (35*b*d^3*e^4*f^3 - 5*a*c^3*f^7 - 5*(3*b*c*d^2 + a*d^3)*e^3*f^4
 - 3*(b*c^2*d + a*c*d^2)*e^2*f^5 - (b*c^3 + 3*a*c^2*d)*e*f^6)*x^6
 + 3*(35*b*d^3*e^5*f^2 - 5*a*c^3*e*f^6 - 5*(3*b*c*d^2 + a*d^3)*e^
4*f^3 - 3*(b*c^2*d + a*c*d^2)*e^3*f^4 - (b*c^3 + 3*a*c^2*d)*e^2*f
^5)*x^4 + 3*(35*b*d^3*e^6*f - 5*a*c^3*e^2*f^5 - 5*(3*b*c*d^2 + a*
d^3)*e^5*f^2 - 3*(b*c^2*d + a*c*d^2)*e^4*f^3 - (b*c^3 + 3*a*c^2*d
)*e^3*f^4)*x^2)*arctan(sqrt(e*f)*x/e) - (48*b*d^3*e^3*f^3*x^7 + 3
*(77*b*d^3*e^4*f^2 + 5*a*c^3*f^6 - 11*(3*b*c*d^2 + a*d^3)*e^3*f^3
 + 3*(b*c^2*d + a*c*d^2)*e^2*f^4 + (b*c^3 + 3*a*c^2*d)*e*f^5)*x^5
 + 8*(35*b*d^3*e^5*f + 5*a*c^3*e*f^5 - 5*(3*b*c*d^2 + a*d^3)*e^4*
f^2 - 3*(b*c^2*d + a*c*d^2)*e^3*f^3 + (b*c^3 + 3*a*c^2*d)*e^2*f^4
)*x^3 + 3*(35*b*d^3*e^6 + 11*a*c^3*e^2*f^4 - 5*(3*b*c*d^2 + a*d^3
)*e^5*f - 3*(b*c^2*d + a*c*d^2)*e^4*f^2 - (b*c^3 + 3*a*c^2*d)*e^3
*f^3)*x)*sqrt(e*f))/((e^3*f^7*x^6 + 3*e^4*f^6*x^4 + 3*e^5*f^5*x^2
 + e^6*f^4)*sqrt(e*f))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x**2+c)**3/(f*x**2+e)**4,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.223699, size = 603, normalized size = 1.73

$$\frac{bd^3x}{f^4} + \frac{\left(5\,ac^3f^4 + bc^3f^3e + 3\,ac^2df^3e + 3\,bc^2df^2e^2 + 3\,acd^2f^2e^2 + 15\,bcd^2fe^3 + 5\,ad^3fe^3 - 35\,bd^3e^4\right)\arctan\left(\sqrt{f}xe^{\left(-\frac{1}{2}\right)}\right)e^{\left(-\frac{7}{2}\right)}}{16\,f^{\frac{9}{2}}} + \frac{\left(15\,ac^3f^6x^5 + 3\,bc^3f^5x^5e + 9\,ac^2df^5x^5e + 9\,bc^2df^4x^5e^2 + 9\,acd^2f^4x^5e^2 - 99\,bcd^2f^3x^5e^3 - 33\,ad^3f^3x^5e^3 + 40\,ac^3f^5x^3e + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^3/(f*x^2 + e)^4,x, algorithm="giac")
```

```
[Out] b*d^3*x/f^4 + 1/16*(5*a*c^3*f^4 + b*c^3*f^3*e + 3*a*c^2*d*f^3*e +
 3*b*c^2*d*f^2*e^2 + 3*a*c*d^2*f^2*e^2 + 15*b*c*d^2*f*e^3 + 5*a*d
^3*f*e^3 - 35*b*d^3*e^4)*arctan(sqrt(f)*x*e^(-1/2))*e^(-7/2)/f^(9
/2) + 1/48*(15*a*c^3*f^6*x^5 + 3*b*c^3*f^5*x^5*e + 9*a*c^2*d*f^5*
x^5*e + 9*b*c^2*d*f^4*x^5*e^2 + 9*a*c*d^2*f^4*x^5*e^2 - 99*b*c*d^
2*f^3*x^5*e^3 - 33*a*d^3*f^3*x^5*e^3 + 40*a*c^3*f^5*x^3*e + 87*b*
d^3*f^2*x^5*e^4 + 8*b*c^3*f^4*x^3*e^2 + 24*a*c^2*d*f^4*x^3*e^2 -
24*b*c^2*d*f^3*x^3*e^3 - 24*a*c*d^2*f^3*x^3*e^3 - 120*b*c*d^2*f^2
*x^3*e^4 - 40*a*d^3*f^2*x^3*e^4 + 33*a*c^3*f^4*x*e^2 + 136*b*d^3*
f*x^3*e^5 - 3*b*c^3*f^3*x*e^3 - 9*a*c^2*d*f^3*x*e^3 - 9*b*c^2*d*f
^2*x*e^4 - 9*a*c*d^2*f^2*x*e^4 - 45*b*c*d^2*f*x*e^5 - 15*a*d^3*f*
x*e^5 + 57*b*d^3*x*e^6)*e^(-3)/((f*x^2 + e)^3*f^4)
```

3.23 $\int \left(a+bx^2\right)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}\,dx$

Optimal. Leaf size=544

$$\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}\left(7adf(3cf+de)-b\left(6c^2f^2-6cdef+4d^2e^2\right)\right)}{105df^2}$$
$$-\frac{e^{3/2}\sqrt{c+dx^2}\left(7adf(de-9cf)-b\left(-3c^2f^2-9cdef+4d^2e^2\right)\right)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{105df^{5/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{x\sqrt{c+dx^2}\left(7adf\left(-3c^2f^2-7cdef+2d^2e^2\right)-b\left(-6c^3f^3+9c^2def^2-19cd^2e^2f+8d^3e^3\right)\right)}{105d^2f^2\sqrt{e+fx^2}}$$
$$+\frac{\sqrt{e}\sqrt{c+dx^2}\left(7adf\left(-3c^2f^2-7cdef+2d^2e^2\right)-b\left(-6c^3f^3+9c^2def^2-19cd^2e^2f+8d^3e^3\right)\right)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{105d^2f^{5/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{x\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}(7adf-2bcf+bde)}{35df}+\frac{bx\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}{7d}$$

```
[Out] -((7*a*d*f*(2*d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2) - b*(8*d^3*e^3 - 1
9*c*d^2*e^2*f + 9*c^2*d*e*f^2 - 6*c^3*f^3))*x*Sqrt[c + d*x^2])/(1
05*d^2*f^2*Sqrt[e + f*x^2]) + ((7*a*d*f*(d*e + 3*c*f) - b*(4*d^2*
e^2 - 6*c*d*e*f + 6*c^2*f^2))*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/
(105*d*f^2) + ((b*d*e - 2*b*c*f + 7*a*d*f)*x*(c + d*x^2)^(3/2)*Sq
rt[e + f*x^2])/(35*d*f) + (b*x*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2])
/(7*d) + (Sqrt[e]*(7*a*d*f*(2*d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2) -
b*(8*d^3*e^3 - 19*c*d^2*e^2*f + 9*c^2*d*e*f^2 - 6*c^3*f^3))*Sqrt[
c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)
])/(105*d^2*f^(5/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
+ f*x^2]) - (e^(3/2)*(7*a*d*f*(d*e - 9*c*f) - b*(4*d^2*e^2 - 9*c*
d*e*f - 3*c^2*f^2))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/
Sqrt[e]], 1 - (d*e)/(c*f)])/(105*d*f^(5/2)*Sqrt[(e*(c + d*x^2))/(
c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 1.89573, antiderivative size = 544, normalized size of antiderivative = 1., number of

steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}\left(7adf(3cf+de)-b\left(6c^2f^2-6cdef+4d^2e^2\right)\right)}{105df^2} - \frac{e^{3/2}\sqrt{c+dx^2}\left(7adf(de-9cf)-b\left(-3c^2f^2-9cdef+4d^2e^2\right)\right)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{105df^{5/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} - \frac{x\sqrt{c+dx^2}\left(7adf\left(-3c^2f^2-7cdef+2d^2e^2\right)-b\left(-6c^3f^3+9c^2def^2-19cd^2e^2f+8d^3e^3\right)\right)}{105d^2f^2\sqrt{e+fx^2}} + \frac{\sqrt{e}\sqrt{c+dx^2}\left(7adf\left(-3c^2f^2-7cdef+2d^2e^2\right)-b\left(-6c^3f^3+9c^2def^2-19cd^2e^2f+8d^3e^3\right)\right)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{105d^2f^{5/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{x\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}(7adf-2bcf+bde)}{35df} + \frac{bx\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}{7d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2],x]
```

```
[Out] -((7*a*d*f*(2*d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2) - b*(8*d^3*e^3 - 1
9*c*d^2*e^2*f + 9*c^2*d*e*f^2 - 6*c^3*f^3))*x*Sqrt[c + d*x^2])/(1
05*d^2*f^2*Sqrt[e + f*x^2]) + ((7*a*d*f*(d*e + 3*c*f) - b*(4*d^2*
e^2 - 6*c*d*e*f + 6*c^2*f^2))*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/
(105*d*f^2) + ((b*d*e - 2*b*c*f + 7*a*d*f)*x*(c + d*x^2)^(3/2)*Sq
rt[e + f*x^2])/(35*d*f) + (b*x*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2])
/(7*d) + (Sqrt[e]*(7*a*d*f*(2*d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2) -
b*(8*d^3*e^3 - 19*c*d^2*e^2*f + 9*c^2*d*e*f^2 - 6*c^3*f^3))*Sqrt[
c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)
])/(105*d^2*f^(5/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
+ f*x^2]) - (e^(3/2)*(7*a*d*f*(d*e - 9*c*f) - b*(4*d^2*e^2 - 9*c*
d*e*f - 3*c^2*f^2))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/
Sqrt[e]], 1 - (d*e)/(c*f)])/(105*d*f^(5/2)*Sqrt[(e*(c + d*x^2))/(
c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)**(3/2)*(f*x**2+e)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.06531, size = 373, normalized size = 0.69

$$\frac{-ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(cf-de)\left(b\left(3c^2f^2-15cdef+8d^2e^2\right)-14adf(de-3cf)\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+fx\sqrt{\frac{d}{c}}\left(c+d\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2],x]
```

```
[Out] (Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2)*(7*a*d*f*(6*c*f + d*(e + 3
*f*x^2)) + b*(3*c^2*f^2 + 3*c*d*f*(3*e + 8*f*x^2) + d^2*(-4*e^2 +
 3*e*f*x^2 + 15*f^2*x^4))) + I*e*(7*a*d*f*(2*d^2*e^2 - 7*c*d*e*f
- 3*c^2*f^2) + b*(-8*d^3*e^3 + 19*c*d^2*e^2*f - 9*c^2*d*e*f^2 + 6
*c^3*f^3))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*Ar
cSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*e*(-(d*e) + c*f)*(-14*a*d*f*
(d*e - 3*c*f) + b*(8*d^2*e^2 - 15*c*d*e*f + 3*c^2*f^2))*Sqrt[1 +
(d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x],
(c*f)/(d*e)])/(105*d*Sqrt[d/c]*f^3*Sqrt[c + d*x^2]*Sqrt[e + f*x^2
])
```

Maple [B] time = 0.06, size = 1332, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2),x)
```

```
[Out] 1/105*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*(-19*((d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d
^2*e^3*f-8*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-
d/c)^(1/2),(c*f/d/e)^(1/2))*b*d^3*e^4+8*((d*x^2+c)/c)^(1/2)*((f*x
^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d^3*e^
4+39*(-d/c)^(1/2)*x^7*b*c*d^2*f^4+18*(-d/c)^(1/2)*x^7*b*d^3*e*f^3
+63*(-d/c)^(1/2)*x^5*a*c*d^2*f^4+28*(-d/c)^(1/2)*x^5*a*d^3*e*f^3+
27*(-d/c)^(1/2)*x^5*b*c^2*d*f^4-(-d/c)^(1/2)*x^5*b*d^3*e^2*f^2+42
*(-d/c)^(1/2)*x^3*a*c^2*d*f^4+7*(-d/c)^(1/2)*x^3*a*d^3*e^2*f^2-4*
(-d/c)^(1/2)*x^3*b*d^3*e^3*f+3*(-d/c)^(1/2)*x*b*c^3*e*f^3+3*(-d/c
)^(1/2)*x^3*b*c^3*f^4+15*(-d/c)^(1/2)*x^9*b*d^3*f^4+42*(-d/c)^(1/
2)*x*a*c^2*d*e*f^3+7*(-d/c)^(1/2)*x*a*c*d^2*e^2*f^2+14*((d*x^2+c)
/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^
(1/2))*a*d^3*e^3*f+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elli
pticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*e*f^3+9*(-d/c)^(1/2)*
x*b*c^2*d*e^2*f^2-4*(-d/c)^(1/2)*x*b*c*d^2*e^3*f-14*((d*x^2+c)/c)
^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/
2))*a*d^3*e^3*f-6*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellipti
```

```
cE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*e*f^3+21*(-d/c)^(1/2)*x^
7*a*d^3*f^4+42*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(
x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d*e*f^3+9*((d*x^2+c)/c)^(1/
2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*
b*c^2*d*e^2*f^2+21*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellipt
icE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d*e*f^3+49*((d*x^2+c)/c
)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1
/2))*a*c*d^2*e^2*f^2-56*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*E
llipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d^2*e^2*f^2-18*((d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/
d/e)^(1/2))*b*c^2*d*e^2*f^2+23*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^
(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d^2*e^3*f+51*
(-d/c)^(1/2)*x^5*b*c*d^2*e*f^3+70*(-d/c)^(1/2)*x^3*a*c*d^2*e*f^3+
36*(-d/c)^(1/2)*x^3*b*c^2*d*e*f^3+8*(-d/c)^(1/2)*x^3*b*c*d^2*e^2*
f^2)/d/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)/f^3/(-d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)\left(dx^2+c\right)^{\frac{3}{2}}\sqrt{fx^2+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bdx^4+(bc+ad)x^2+ac\right)\sqrt{dx^2+c}\sqrt{fx^2+e},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e),x, algorithm="fricas")
```

```
[Out] integral((b*d*x^4 + (b*c + a*d)*x^2 + a*c)*sqrt(d*x^2 + c)*sqrt(f
*x^2 + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^{\frac{3}{2}}\sqrt{e+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x**2+c)**(3/2)*(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)*(c + d*x**2)**(3/2)*sqrt(e + f*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + a\right)\left(dx^2 + c\right)^{\frac{3}{2}}\sqrt{fx^2 + e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e), x)
```

3.24 $\int \left(a+bx^2\right) \sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$

Optimal. Leaf size=381

$$\frac{x\sqrt{c+dx^2}\left(5adf(cf+de)-2b\left(c^2f^2-cdef+d^2e^2\right)\right)}{15d^2f\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}\left(5adf(cf+de)-2b\left(c^2f^2-cdef+d^2e^2\right)\right)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15d^2f^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{e^{3/2}\sqrt{c+dx^2}(-10adf+bcf+bde)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15df^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-2bcf+bde)}{15df}+\frac{bx\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{5d}$$

```
[Out] ((5*a*d*f*(d*e + c*f) - 2*b*(d^2*e^2 - c*d*e*f + c^2*f^2))*x*Sqrt
[c + d*x^2])/(15*d^2*f*Sqrt[e + f*x^2]) + ((b*d*e - 2*b*c*f + 5*a
*d*f)*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(15*d*f) + (b*x*(c + d*x
^2)^(3/2)*Sqrt[e + f*x^2])/(5*d) - (Sqrt[e]*(5*a*d*f*(d*e + c*f)
- 2*b*(d^2*e^2 - c*d*e*f + c^2*f^2))*Sqrt[c + d*x^2]*EllipticE[Ar
cTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*d^2*f^(3/2)*Sqrt
[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (e^(3/2)*(b*
d*e + b*c*f - 10*a*d*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]
*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*d*f^(3/2)*Sqrt[(e*(c + d*x^2)
)/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 1.10216, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{x\sqrt{c+dx^2}\left(5adf(cf+de)-2b\left(c^2f^2-cdef+d^2e^2\right)\right)}{15d^2f\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}\left(5adf(cf+de)-2b\left(c^2f^2-cdef+d^2e^2\right)\right)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15d^2f^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{e^{3/2}\sqrt{c+dx^2}(-10adf+bcf+bde)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15df^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-2bcf+bde)}{15df}+\frac{bx\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{5d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2],x]
```

```
[Out] ((5*a*d*f*(d*e + c*f) - 2*b*(d^2*e^2 - c*d*e*f + c^2*f^2))*x*Sqrt
[c + d*x^2])/(15*d^2*f*Sqrt[e + f*x^2]) + ((b*d*e - 2*b*c*f + 5*a
*d*f)*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(15*d*f) + (b*x*(c + d*x
^2)^(3/2)*Sqrt[e + f*x^2])/(5*d) - (Sqrt[e]*(5*a*d*f*(d*e + c*f)
- 2*b*(d^2*e^2 - c*d*e*f + c^2*f^2))*Sqrt[c + d*x^2]*EllipticE[Ar
cTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*d^2*f^(3/2)*Sqrt
[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (e^(3/2)*(b*
d*e + b*c*f - 10*a*d*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]
*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*d*f^(3/2)*Sqrt[(e*(c + d*x^2)
)/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi in Sympy [A] time = 115.453, size = 374, normalized size = 0.98

$$\frac{bx\left(c+dx^2\right)^{\frac{3}{2}}\sqrt{e+fx^2}}{5d}+\frac{c^{\frac{3}{2}}\sqrt{e+fx^2}\,(10adf-bcf-bde)\,F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{15d^{\frac{3}{2}}f\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}\,(5adf-2bcf+bde)}{15df}+\frac{\sqrt{e}\sqrt{c+dx^2}\left(-5acdf^2-5ad^2ef+2bc^2f^2-2bcdef+2bd^2e^2\right)E\left(\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{15d^2f^{\frac{3}{2}}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}-\frac{x\sqrt{c+dx^2}\left(-5acdf^2-5ad^2ef+2bc^2f^2-2bcdef+2bd^2e^2\right)}{15d^2f\sqrt{e+fx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)**(1/2)*(f*x**2+e)**(1/2),x)
```

```
[Out] b*x*(c + d*x**2)**(3/2)*sqrt(e + f*x**2)/(5*d) + c**(3/2)*sqrt(e
+ f*x**2)*(10*a*d*f - b*c*f - b*d*e)*elliptic_f(atan(sqrt(d)*x/sq
rt(c)), -c*f/(d*e) + 1)/(15*d**(3/2)*f*sqrt(c*(e + f*x**2)/(e*(c
+ d*x**2)))*sqrt(c + d*x**2)) + x*sqrt(c + d*x**2)*sqrt(e + f*x**
2)*(5*a*d*f - 2*b*c*f + b*d*e)/(15*d*f) + sqrt(e)*sqrt(c + d*x**2
)*(-5*a*c*d*f**2 - 5*a*d**2*e*f + 2*b*c**2*f**2 - 2*b*c*d*e*f + 2
*b*d**2*e**2)*elliptic_e(atan(sqrt(f)*x/sqrt(e)), 1 - d*e/(c*f))/
(15*d**2*f**(3/2)*sqrt(e*(c + d*x**2)/(c*(e + f*x**2)))*sqrt(e +
f*x**2)) - x*sqrt(c + d*x**2)*(-5*a*c*d*f**2 - 5*a*d**2*e*f + 2*b
*c**2*f**2 - 2*b*c*d*e*f + 2*b*d**2*e**2)/(15*d**2*f*sqrt(e + f*x
**2))
```

Mathematica [C] time = 1.3225, size = 267, normalized size = 0.7

$$\frac{ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(2b\left(c^2f^2-cdef+d^2e^2\right)-5adf(cf+de)\right)E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+fx\sqrt{\frac{d}{c}}\left(c+dx^2\right)\left(e+fx^2\right)(5c}{15df^2\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2],x]
```

```
[Out] (Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2)*(b*c*f + 5*a*d*f + b*d*(e
+ 3*f*x^2)) + I*e*(-5*a*d*f*(d*e + c*f) + 2*b*(d^2*e^2 - c*d*e*f
+ c^2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*A
rcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*e*(-(d*e) + c*f)*(-2*b*d*e
+ b*c*f + 5*a*d*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Ellipt
icF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(15*d*Sqrt[d/c]*f^2*Sqr
t[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [B] time = 0.023, size = 865, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x)
```

```
[Out] 1/15*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*(3*(-d/c)^(1/2)*x^7*b*d^2*f^
3+5*(-d/c)^(1/2)*x^5*a*d^2*f^3+4*(-d/c)^(1/2)*x^5*b*c*d*f^3+4*(-d
/c)^(1/2)*x^5*b*d^2*e*f^2+5*(-d/c)^(1/2)*x^3*a*c*d*f^3+5*(-d/c)^(
1/2)*x^3*a*d^2*e*f^2+(-d/c)^(1/2)*x^3*b*c^2*f^3+5*(-d/c)^(1/2)*x^
3*b*c*d*e*f^2+(-d/c)^(1/2)*x^3*b*d^2*e^2*f+5*((d*x^2+c)/c)^(1/2)*
((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c
*d*e*f^2-5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-
d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^2*e^2*f+((d*x^2+c)/c)^(1/2)*((f*x
^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*e*
f^2-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^
(1/2),(c*f/d/e)^(1/2))*b*c*d*e^2*f+2*((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d^2*e^3+5
*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2)
,(c*f/d/e)^(1/2))*a*c*d*e*f^2+5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^2*e^2*f-2*((
d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c
*f/d/e)^(1/2))*b*c^2*e*f^2+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1
/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d*e^2*f-2*((d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/
d/e)^(1/2))*b*d^2*e^3+5*(-d/c)^(1/2)*x*a*c*d*e*f^2+(-d/c)^(1/2)*x
*b*c^2*e*f^2+(-d/c)^(1/2)*x*b*c*d*e^2*f)/(d*f*x^4+c*f*x^2+d*e*x^2
+c*e)/f^2/d/(-d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)\sqrt{dx^2 + c}\sqrt{fx^2 + e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^2 + a)\sqrt{dx^2 + c}\sqrt{fx^2 + e}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + bx^2)\sqrt{c + dx^2}\sqrt{e + fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(1/2)*(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)\sqrt{dx^2 + c}\sqrt{fx^2 + e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e),x, algorithm="giac")

[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e), x)
```

3.25 $\int \frac{(a+bx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=283

$$\frac{x\sqrt{c+dx^2}(3adf-2bcf+bde)}{3d^2\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}(3adf-2bcf+bde)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3d^2\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{e^{3/2}\sqrt{c+dx^2}(bc-3ad)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3cd\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3d}$$

```
[Out] ((b*d*e - 2*b*c*f + 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*d^2*Sqrt[e + f
*x^2]) + (b*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*d) - (Sqrt[e]*(
b*d*e - 2*b*c*f + 3*a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt
[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*d^2*Sqrt[f]*Sqrt[(e*(c + d*
x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - ((b*c - 3*a*d)*e^(3/2)*
Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/
(c*f)])/(3*c*d*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt
[e + f*x^2])
```

Rubi [A] time = 0.611093, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{x\sqrt{c+dx^2}(3adf-2bcf+bde)}{3d^2\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}(3adf-2bcf+bde)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3d^2\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{e^{3/2}\sqrt{c+dx^2}(bc-3ad)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3cd\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3d}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*Sqrt[e + f*x^2])/Sqrt[c + d*x^2],x]
```


```
[Out] ((b*d*e - 2*b*c*f + 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*d^2*Sqrt[e + f
*x^2]) + (b*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*d) - (Sqrt[e]*(
b*d*e - 2*b*c*f + 3*a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt
[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*d^2*Sqrt[f]*Sqrt[(e*(c + d*
x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - ((b*c - 3*a*d)*e^(3/2)*
Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/
(c*f)])/(3*c*d*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt
[e + f*x^2])
```

Rubi in Sympy [A] time = 69.5829, size = 252, normalized size = 0.89

$$\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3d}+\frac{\sqrt{c}\sqrt{e+fx^2}\,(3ad-bc)\,F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{3d^{\frac{3}{2}}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}\,(3adf-2bcf+bde)\,E\left(\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{3d^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}+\frac{x\sqrt{c+dx^2}\,(3adf-2bcf+bde)}{3d^2\sqrt{e+fx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(f*x**2+e)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] b*x*sqrt(c + d*x**2)*sqrt(e + f*x**2)/(3*d) + sqrt(c)*sqrt(e + f*
x**2)*(3*a*d - b*c)*elliptic_f(atan(sqrt(d)*x/sqrt(c)), -c*f/(d*e
) + 1)/(3*d**(3/2)*sqrt(c*(e + f*x**2)/(e*(c + d*x**2)))*sqrt(c +
 d*x**2)) - sqrt(e)*sqrt(c + d*x**2)*(3*a*d*f - 2*b*c*f + b*d*e)*
elliptic_e(atan(sqrt(f)*x/sqrt(e)), 1 - d*e/(c*f))/(3*d**2*sqrt(f
)*sqrt(e*(c + d*x**2)/(c*(e + f*x**2)))*sqrt(e + f*x**2)) + x*sqr
t(c + d*x**2)*(3*a*d*f - 2*b*c*f + b*d*e)/(3*d**2*sqrt(e + f*x**2
))
```

Mathematica [C] time = 0.637933, size = 212, normalized size = 0.75

$$\frac{ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(-3adf+2bcf-bde)E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)\middle|\frac{cf}{de}\right)+bfx\sqrt{\frac{d}{c}}\left(c+dx^2\right)\left(e+fx^2\right)-ibe\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\cdots}{3df\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*Sqrt[e + f*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] (b*Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2) + I*e*(-(b*d*e) + 2*b*c*
f - 3*a*d*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*
ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*b*e*(-(d*e) + c*f)*Sqrt[1
+ (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x]
, (c*f)/(d*e)])/(3*d*Sqrt[d/c]*f*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.029, size = 394, normalized size = 1.4

$$\frac{1}{(3dfx^4+3cfx^2+3dex^2+3ce)df}\sqrt{fx^2+e}\sqrt{dx^2+c}\left(\sqrt{-\frac{d}{c}}x^5bdf^2+\sqrt{-\frac{d}{c}}x^3bcf^2+\sqrt{-\frac{d}{c}}x^3bdef+\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{fx^2+\cdots}{e}}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/3*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)*((-d/c)^(1/2)*x^5*b*d*f^2+(-d
/c)^(1/2)*x^3*b*c*f^2+(-d/c)^(1/2)*x^3*b*d*e*f+((d*x^2+c)/c)^(1/2
)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b
*c*e*f-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)
^(1/2),(c*f/d/e)^(1/2))*b*d*e^2+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/
e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e*f-2*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f
/d/e)^(1/2))*b*c*e*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elli
pticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d*e^2+(-d/c)^(1/2)*x*b*c*
e*f)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)/d/(-d/c)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)\sqrt{fx^2+e}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2+a)\sqrt{fx^2+e}}{\sqrt{dx^2+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)*sqrt(f*x^2 + e)/sqrt(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(f*x**2+e)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(e + f*x**2)/sqrt(c + d*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)\sqrt{fx^2+e}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*sqrt(f*x^2 + e)/sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/sqrt(d*x^2 + c), x)
```

3.26 $\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=271

$$\frac{fx\sqrt{c+dx^2}(2bc-ad)}{cd^2\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(2bc-ad)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{cd^2\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x\sqrt{e+fx^2}(bc-ad)}{cd\sqrt{c+dx^2}}+\frac{be^{3/2}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{cd\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

```
[Out] ((2*b*c - a*d)*f*x*Sqrt[c + d*x^2])/(c*d^2*Sqrt[e + f*x^2]) - ((b
*c - a*d)*x*Sqrt[e + f*x^2])/(c*d*Sqrt[c + d*x^2]) - ((2*b*c - a*
d)*Sqrt[e]*Sqrt[f]*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/S
qrt[e]], 1 - (d*e)/(c*f)])/(c*d^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*
x^2))]*Sqrt[e + f*x^2]) + (b*e^(3/2)*Sqrt[c + d*x^2]*EllipticF[Ar
cTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*d*Sqrt[f]*Sqrt[(e
*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.58857, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{fx\sqrt{c+dx^2}(2bc-ad)}{cd^2\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(2bc-ad)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{cd^2\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x\sqrt{e+fx^2}(bc-ad)}{cd\sqrt{c+dx^2}}+\frac{be^{3/2}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{cd\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*Sqrt[e + f*x^2])/(c + d*x^2)^(3/2),x]
```

```
[Out] ((2*b*c - a*d)*f*x*Sqrt[c + d*x^2])/(c*d^2*Sqrt[e + f*x^2]) - ((b
*c - a*d)*x*Sqrt[e + f*x^2])/(c*d*Sqrt[c + d*x^2]) - ((2*b*c - a*
d)*Sqrt[e]*Sqrt[f]*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/S
qrt[e]], 1 - (d*e)/(c*f)])/(c*d^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*
x^2))]*Sqrt[e + f*x^2]) + (b*e^(3/2)*Sqrt[c + d*x^2]*EllipticF[Ar
cTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*d*Sqrt[f]*Sqrt[(e
*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi in Sympy [A] time = 70.3996, size = 224, normalized size = 0.83

$$\frac{be^{\frac{3}{2}}\sqrt{c+dx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{cd\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}-\frac{x\sqrt{e+fx^2}\,(ad-2bc)}{cd\sqrt{c+dx^2}}+\frac{x\sqrt{e+fx^2}\,(ad-bc)}{cd\sqrt{c+dx^2}}+\frac{\sqrt{e+fx^2}\,(ad-2bc)\,E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{\sqrt{c}d^{\frac{3}{2}}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(f*x**2+e)**(1/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] b*e**(3/2)*sqrt(c + d*x**2)*elliptic_f(atan(sqrt(f)*x/sqrt(e)), 1
 - d*e/(c*f))/(c*d*sqrt(f)*sqrt(e*(c + d*x**2)/(c*(e + f*x**2)))*
sqrt(e + f*x**2)) - x*sqrt(e + f*x**2)*(a*d - 2*b*c)/(c*d*sqrt(c
+ d*x**2)) + x*sqrt(e + f*x**2)*(a*d - b*c)/(c*d*sqrt(c + d*x**2)
) + sqrt(e + f*x**2)*(a*d - 2*b*c)*elliptic_e(atan(sqrt(d)*x/sqrt
(c)), -c*f/(d*e) + 1)/(sqrt(c)*d**(3/2)*sqrt(c*(e + f*x**2)/(e*(c
 + d*x**2)))*sqrt(c + d*x**2))
```

Mathematica [C] time = 0.561719, size = 192, normalized size = 0.71

$$\frac{-(bc-ad)\left(x\sqrt{\frac{d}{c}}\left(e+fx^2\right)-ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)\right)-ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(2bc-ad)E\left(i\sinh^{-1}\right.}{c^2\left(\frac{d}{c}\right)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*Sqrt[e + f*x^2])/(c + d*x^2)^(3/2),x]
```

```
[Out] ((-I)*(2*b*c - a*d)*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Ell
ipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (b*c - a*d)*(Sqrt[d
/c]*x*(e + f*x^2) - I*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*E
llipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(c^2*(d/c)^(3/2)*
Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.058, size = 328, normalized size = 1.2

$$\frac{1}{d\left(dfx^4+cfx^2+dex^2+ce\right)c}\sqrt{fx^2+e}\sqrt{dx^2+c}\left(x^3adf\sqrt{-\frac{d}{c}}-x^3bcf\sqrt{-\frac{d}{c}}+EllipticF\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)ade\sqrt{\frac{dx^2+c}{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x)
```

```
[Out] (f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)*(x^3*a*d*f*(-d/c)^(1/2)-x^3*b*c*f
*(-d/c)^(1/2)+EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e*((d
*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticF(x*(-d/c)^(1/2),(c*
f/d/e)^(1/2))*b*c*e*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-Ellip
ticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e*((d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)+2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c
*e*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+x*a*d*e*(-d/c)^(1/2)-x
*b*c*e*(-d/c)^(1/2))/d/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)/c/(-d/c)^(1/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)\sqrt{fx^2 + e}}{(dx^2 + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)\sqrt{fx^2 + e}}{(dx^2 + c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^2)\sqrt{e + fx^2}}{(c + dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(f*x**2+e)**(1/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(e + f*x**2)/(c + d*x**2)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)\sqrt{fx^2+e}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(3/2), x)
```

3.27 $\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=274

$$\frac{\sqrt{e+fx^2}(de(2ad+bc)-cf(ad+2bc))E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}d^{3/2}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(bc-ad)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2d\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x\sqrt{e+fx^2}(bc-ad)}{3cd\left(c+dx^2\right)^{3/2}}$$

```
[Out] -((b*c - a*d)*x*Sqrt[e + f*x^2])/(3*c*d*(c + d*x^2)^(3/2)) + ((d*
(b*c + 2*a*d)*e - c*(2*b*c + a*d)*f)*Sqrt[e + f*x^2]*EllipticE[Ar
cTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(3*c^(3/2)*d^(3/2)*(
d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
 + ((b*c - a*d)*e^(3/2)*Sqrt[f]*Sqrt[c + d*x^2]*EllipticF[ArcTan[
(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*d*(d*e - c*f)*Sqrt
[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.618998, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{\sqrt{e+fx^2}(de(2ad+bc)-cf(ad+2bc))E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}d^{3/2}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(bc-ad)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2d\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x\sqrt{e+fx^2}(bc-ad)}{3cd\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*Sqrt[e + f*x^2])/(c + d*x^2)^(5/2),x]
```


```
[Out] -((b*c - a*d)*x*Sqrt[e + f*x^2])/(3*c*d*(c + d*x^2)^(3/2)) + ((d*
(b*c + 2*a*d)*e - c*(2*b*c + a*d)*f)*Sqrt[e + f*x^2]*EllipticE[Ar
cTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(3*c^(3/2)*d^(3/2)*(
d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
 + ((b*c - a*d)*e^(3/2)*Sqrt[f]*Sqrt[c + d*x^2]*EllipticF[ArcTan[
(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*d*(d*e - c*f)*Sqrt
[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi in Sympy [A] time = 66.2034, size = 231, normalized size = 0.84

$$\frac{x\sqrt{e+fx^2}\,(ad-bc)}{3cd\,(c+dx^2)^{\frac{3}{2}}}+\frac{e^{\frac{3}{2}}\sqrt{f}\sqrt{c+dx^2}\,(ad-bc)\,F\left(\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{3c^2d\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}\,(cf-de)}+\frac{\sqrt{e+fx^2}\,(cf\,(ad+2bc)-de\,(2ad+bc))\,E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{3c^{\frac{3}{2}}d^{\frac{3}{2}}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}\,(cf-de)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(f*x**2+e)**(1/2)/(d*x**2+c)**(5/2),x)
```

```
[Out] x*sqrt(e + f*x**2)*(a*d - b*c)/(3*c*d*(c + d*x**2)**(3/2)) + e**(
3/2)*sqrt(f)*sqrt(c + d*x**2)*(a*d - b*c)*elliptic_f(atan(sqrt(f)
*x/sqrt(e)), 1 - d*e/(c*f))/(3*c**2*d*sqrt(e*(c + d*x**2)/(c*(e +
 f*x**2)))*sqrt(e + f*x**2)*(c*f - d*e)) + sqrt(e + f*x**2)*(c*f*
(a*d + 2*b*c) - d*e*(2*a*d + b*c))*elliptic_e(atan(sqrt(d)*x/sqrt
(c)), -c*f/(d*e) + 1)/(3*c**(3/2)*d**(3/2)*sqrt(c*(e + f*x**2)/(e
*(c + d*x**2)))*sqrt(c + d*x**2)*(c*f - d*e))
```

Mathematica [C] time = 1.73948, size = 297, normalized size = 1.08

$$\frac{x\sqrt{\frac{d}{c}}\left(e+fx^2\right)\left(ad\left(2c^2f-3cde+cdfx^2-2d^2ex^2\right)+bc\left(c^2f+2cdfx^2-d^2ex^2\right)\right)-ie\left(c+dx^2\right)\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(2ad}{3c^3\left(\frac{d}{c}\right)^{3/2}\left(c+dx^2\right.}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*Sqrt[e + f*x^2])/(c + d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d/c]*x*(e + f*x^2)*(a*d*(-3*c*d*e + 2*c^2*f - 2*d^2*e*x^2 +
 c*d*f*x^2) + b*c*(c^2*f - d^2*e*x^2 + 2*c*d*f*x^2)) + I*e*(a*d*(
-2*d*e + c*f) + b*c*(-(d*e) + 2*c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2
)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/
(d*e)] - I*(b*c + 2*a*d)*e*(-(d*e) + c*f)*(c + d*x^2)*Sqrt[1 + (d
*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c
*f)/(d*e)])/(3*c^3*(d/c)^(3/2)*(-(d*e) + c*f)*(c + d*x^2)^(3/2)*S
qrt[e + f*x^2])
```

Maple [B] time = 0.067, size = 1236, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/3*(x^5*a*c*d^2*f^2*(-d/c)^(1/2)+x^3*b*c^3*f^2*(-d/c)^(1/2)+Elli
pticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c*d^2*e^2*((d*x^2+c)/
c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)
^(1/2))*a*c^2*d*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-Ellip
ticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d*e*f*((d*x^2+c)/c)^(1
/2)*((f*x^2+e)/e)^(1/2)-x^5*b*c*d^2*e*f*(-d/c)^(1/2)-2*x^3*a*c*d^
2*e*f*(-d/c)^(1/2)+2*x^3*b*c^2*d*e*f*(-d/c)^(1/2)+2*x*a*c^2*d*e*f
*(-d/c)^(1/2)-2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*d
^3*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticE(x*(-d/
c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*d^3*e^2*((d*x^2+c)/c)^(1/2)*((f*x
^2+e)/e)^(1/2)-2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d^
2*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticF(x*(-d/c)^
(1/2),(c*f/d/e)^(1/2))*b*c^3*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e
)^(1/2)-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*d*e^2*((d
*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticE(x*(-d/c)^(1/2),(
c*f/d/e)^(1/2))*a*c*d^2*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/
2)-2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*e*f*((d*x^2+
c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticE(x*(-d/c)^(1/2),(c*f/d/e
)^(1/2))*b*c^2*d*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-Elli
pticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c*d^2*e^2*((d*x^2+c)/
c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*x^3*a*d^3*e^2*(-d/c)^(1/2)-2*x^5*a
*d^3*e*f*(-d/c)^(1/2)+2*x^5*b*c^2*d*f^2*(-d/c)^(1/2)+2*x^3*a*c^2*
d*f^2*(-d/c)^(1/2)-x^3*b*c*d^2*e^2*(-d/c)^(1/2)-3*x*a*c*d^2*e^2*(
-d/c)^(1/2)+x*b*c^3*e*f*(-d/c)^(1/2)+2*EllipticF(x*(-d/c)^(1/2),(
c*f/d/e)^(1/2))*x^2*a*c*d^2*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)+EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^2*d*e*f*
((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticE(x*(-d/c)^(1/2),
(c*f/d/e)^(1/2))*x^2*a*c*d^2*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e
)^(1/2)-2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^2*d*e
*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2))/(f*x^2+e)^(1/2)/(-d/c
)^(1/2)/(c*f-d*e)/c^2/d/(d*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)\sqrt{fx^2+e}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)\sqrt{fx^2+e}}{\left(d^2x^4+2\,cdx^2+c^2\right)\sqrt{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)*sqrt(f*x^2 + e)/((d^2*x^4 + 2*c*d*x^2 + c^2)
*sqrt(d*x^2 + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(f*x**2+e)**(1/2)/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^2+a\right)\sqrt{fx^2+e}}{\left(dx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(5/2), x)
```

3.28 $\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx$

Optimal. Leaf size=385

$$-\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(2ad(2de-3cf)+bc(cf+de))F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15c^3d\sqrt{e+fx^2}(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x\sqrt{e+fx^2}(ad(4de-3cf)+bc(de-2cf))}{15c^2d\left(c+dx^2\right)^{3/2}(de-cf)}+\frac{\sqrt{e+fx^2}\left(ad\left(3c^2f^2-13cdef+8d^2e^2\right)+2bc\left(c^2f^2-cdef+d^2e^2\right)\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}d^{3/2}\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{x\sqrt{e+fx^2}(bc-ad)}{5cd\left(c+dx^2\right)^{5/2}}$$

```
[Out] -((b*c - a*d)*x*Sqrt[e + f*x^2])/(5*c*d*(c + d*x^2)^(5/2)) + ((a*
d*(4*d*e - 3*c*f) + b*c*(d*e - 2*c*f))*x*Sqrt[e + f*x^2])/(15*c^2
*d*(d*e - c*f)*(c + d*x^2)^(3/2)) + ((2*b*c*(d^2*e^2 - c*d*e*f +
c^2*f^2) + a*d*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2))*Sqrt[e + f*x
^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(15*
c^(5/2)*d^(3/2)*(d*e - c*f)^2*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2)
)/(e*(c + d*x^2))]) - (e^(3/2)*Sqrt[f]*(2*a*d*(2*d*e - 3*c*f) + b
*c*(d*e + c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt
[e]], 1 - (d*e)/(c*f)])/(15*c^3*d*(d*e - c*f)^2*Sqrt[(e*(c + d*x^
2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 1.12132, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(2ad(2de-3cf)+bc(cf+de))F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15c^3d\sqrt{e+fx^2}(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x\sqrt{e+fx^2}(ad(4de-3cf)+bc(de-2cf))}{15c^2d\left(c+dx^2\right)^{3/2}(de-cf)}+\frac{\sqrt{e+fx^2}\left(ad\left(3c^2f^2-13cdef+8d^2e^2\right)+2bc\left(c^2f^2-cdef+d^2e^2\right)\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}d^{3/2}\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{x\sqrt{e+fx^2}(bc-ad)}{5cd\left(c+dx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*Sqrt[e + f*x^2])/(c + d*x^2)^(7/2),x]
```

```
[Out] -((b*c - a*d)*x*Sqrt[e + f*x^2])/(5*c*d*(c + d*x^2)^(5/2)) + ((a*
d*(4*d*e - 3*c*f) + b*c*(d*e - 2*c*f))*x*Sqrt[e + f*x^2])/(15*c^2
*d*(d*e - c*f)*(c + d*x^2)^(3/2)) + ((2*b*c*(d^2*e^2 - c*d*e*f +
c^2*f^2) + a*d*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2))*Sqrt[e + f*x
^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(15*
c^(5/2)*d^(3/2)*(d*e - c*f)^2*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2)
)/(e*(c + d*x^2))]) - (e^(3/2)*Sqrt[f]*(2*a*d*(2*d*e - 3*c*f) + b
*c*(d*e + c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt
[e]], 1 - (d*e)/(c*f)])/(15*c^3*d*(d*e - c*f)^2*Sqrt[(e*(c + d*x^
2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(f*x**2+e)**(1/2)/(d*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.30643, size = 379, normalized size = 0.98

$$-x\sqrt{\frac{d}{c}}\left(e+fx^2\right)\left(-\left(c+dx^2\right)^2\left(ad\left(3c^2f^2-13cdef+8d^2e^2\right)+2bc\left(c^2f^2-cdef+d^2e^2\right)\right)+3c^2(bc-ad)(de-cf)^2-c\,(c$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*Sqrt[e + f*x^2])/(c + d*x^2)^(7/2),x]
```

```
[Out] (-(Sqrt[d/c]*x*(e + f*x^2)*(3*c^2*(b*c - a*d)*(d*e - c*f)^2 - c*(
d*e - c*f)*(a*d*(4*d*e - 3*c*f) + b*c*(d*e - 2*c*f))*(c + d*x^2)
- (2*b*c*(d^2*e^2 - c*d*e*f + c^2*f^2) + a*d*(8*d^2*e^2 - 13*c*d*
e*f + 3*c^2*f^2))*(c + d*x^2)^2)) + I*e*(c + d*x^2)^2*Sqrt[1 + (d
*x^2)/c]*Sqrt[1 + (f*x^2)/e]*((2*b*c*(d^2*e^2 - c*d*e*f + c^2*f^2
) + a*d*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2))*EllipticE[I*ArcSinh
[Sqrt[d/c]*x], (c*f)/(d*e)] - (-(d*e) + c*f)*(b*c*(-2*d*e + c*f)
+ a*d*(-8*d*e + 9*c*f))*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(
d*e)]))/(15*c^4*(d/c)^(3/2)*(d*e - c*f)^2*(c + d*x^2)^(5/2)*Sqrt[
e + f*x^2])
```

Maple [B] time = 0.08, size = 2856, normalized size = 7.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x)
```

```
[Out] 1/15*(x^3*b*c^5*f^3*(-d/c)^(1/2)+2*x^5*b*c*d^4*e^3*(-d/c)^(1/2)+9
*x^3*a*c^4*d*f^3*(-d/c)^(1/2)+20*x^3*a*c*d^4*e^3*(-d/c)^(1/2)+5*x
^3*b*c^2*d^3*e^3*(-d/c)^(1/2)+15*x*a*c^2*d^3*e^3*(-d/c)^(1/2)+x*b
*c^5*e*f^2*(-d/c)^(1/2)+3*x^7*a*c^2*d^3*f^3*(-d/c)^(1/2)-13*x^7*a
*c*d^4*e*f^2*(-d/c)^(1/2)-2*x^7*b*c^2*d^3*e*f^2*(-d/c)^(1/2)+2*x^
7*b*c*d^4*e^2*f*(-d/c)^(1/2)-30*x^5*a*c^2*d^3*e*f^2*(-d/c)^(1/2)+
7*x^5*a*c*d^4*e^2*f*(-d/c)^(1/2)-5*x^5*b*c^3*d^2*e*f^2*(-d/c)^(1/
2)+3*x^5*b*c^2*d^3*e^2*f*(-d/c)^(1/2)-17*x^3*a*c^3*d^2*e*f^2*(-d/
c)^(1/2)-18*x^3*a*c^2*d^3*e^2*f*(-d/c)^(1/2)+7*x^3*b*c^4*d*e*f^2*
(-d/c)^(1/2)+2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*b*c*
d^4*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-7*x^3*b*c^3*d^2*e
^2*f*(-d/c)^(1/2)+9*x*a*c^4*d*e*f^2*(-d/c)^(1/2)-26*x*a*c^3*d^2*e
^2*f*(-d/c)^(1/2)+x*b*c^4*d*e^2*f*(-d/c)^(1/2)+8*EllipticF(x*(-d/
c)^(1/2),(c*f/d/e)^(1/2))*x^4*a*d^5*e^3*((d*x^2+c)/c)^(1/2)*((f*x
^2+e)/e)^(1/2)-8*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*a*
d^5*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+8*EllipticF(x*(-d
/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d^3*e^3*((d*x^2+c)/c)^(1/2)*((f*
x^2+e)/e)^(1/2)+EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^5*e
*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticF(x*(-d/c)
^(1/2),(c*f/d/e)^(1/2))*b*c^3*d^2*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2
+e)/e)^(1/2)-8*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d^
3*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*EllipticE(x*(-d/c
)^(1/2),(c*f/d/e)^(1/2))*b*c^5*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)-2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*d^2
*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+8*x^7*a*d^5*e^2*f*(-
d/c)^(1/2)+2*x^7*b*c^3*d^2*f^3*(-d/c)^(1/2)+9*x^5*a*c^3*d^2*f^3*(
-d/c)^(1/2)+6*x^5*b*c^4*d*f^3*(-d/c)^(1/2)-3*EllipticE(x*(-d/c)^(
1/2),(c*f/d/e)^(1/2))*x^4*a*c^2*d^3*e*f^2*((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)+13*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4
*a*c*d^4*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*Elliptic
E(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*b*c^3*d^2*e*f^2*((d*x^2+c)/
c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)
^(1/2))*x^4*b*c^2*d^3*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/
2)+18*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c^3*d^2*e*f
^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-6*EllipticE(x*(-d/c)^(
1/2),(c*f/d/e)^(1/2))*x^2*a*c^3*d^2*e*f^2*((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)+26*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2
*a*c^2*d^3*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-4*Ellipt
icE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^4*d*e*f^2*((d*x^2+c)/
c)^(1/2)*((f*x^2+e)/e)^(1/2)+4*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)
^(1/2))*x^2*b*c^3*d^2*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/
2)-34*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c^2*d^3*e^2
*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticF(x*(-d/c)^(
1/2),(c*f/d/e)^(1/2))*x^2*b*c^4*d*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x
^2+e)/e)^(1/2)-2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*b*
c*d^4*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+16*EllipticF(x*
```

```
(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c*d^4*e^3*((d*x^2+c)/c)^(1/2)
*((f*x^2+e)/e)^(1/2)+4*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*
x^2*b*c^2*d^3*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-16*Elli
pticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c*d^4*e^3*((d*x^2+c)/
c)^(1/2)*((f*x^2+e)/e)^(1/2)-4*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)
^(1/2))*x^2*b*c^2*d^3*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)
+9*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^4*d*e*f^2*((d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-17*EllipticF(x*(-d/c)^(1/2),(c*
f/d/e)^(1/2))*a*c^3*d^2*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)-3*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^4*d*e^2*f*((
d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-3*EllipticE(x*(-d/c)^(1/2),
(c*f/d/e)^(1/2))*a*c^4*d*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^
(1/2)+13*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^3*d^2*e^2*
f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticE(x*(-d/c)^(1
/2),(c*f/d/e)^(1/2))*b*c^4*d*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)
/e)^(1/2)-6*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^3*d
^2*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+9*EllipticF(x*(-
d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*a*c^2*d^3*e*f^2*((d*x^2+c)/c)^(1/
2)*((f*x^2+e)/e)^(1/2)-17*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2
))*x^4*a*c*d^4*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+Elli
pticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*b*c^3*d^2*e*f^2*((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-3*EllipticF(x*(-d/c)^(1/2),(c*f/
d/e)^(1/2))*x^4*b*c^2*d^3*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)+8*x^5*a*d^5*e^3*(-d/c)^(1/2))/(f*x^2+e)^(1/2)/(-d/c)^(1/2)
/(c*f-d*e)^2/c^3/d/(d*x^2+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)\sqrt{fx^2+e}}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2+a)\sqrt{fx^2+e}}{(d^3x^6+3\,cd^2x^4+3\,c^2dx^2+c^3)\sqrt{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)*sqrt(f*x^2 + e)/((d^3*x^6 + 3*c*d^2*x^4 + 3*
c^2*d*x^2 + c^3)*sqrt(d*x^2 + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(f*x**2+e)**(1/2)/(d*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)\sqrt{fx^2 + e}}{\left(dx^2 + c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(7/2), x)
```

3.29 $\int \left(a+bx^2\right) \sqrt{c+dx^2} \left(e+fx^2\right)^{3/2} \, dx$

Optimal. Leaf size=543

$$\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}\left(14adf(3de-cf)+b\left(8c^2f^2-15cdef+3d^2e^2\right)\right)}{105d^2f}$$
$$+\frac{e^{3/2}\sqrt{c+dx^2}\left(7adf(9de-cf)-b\left(-4c^2f^2+9cdef+3d^2e^2\right)\right)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{105d^2f^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{x\sqrt{c+dx^2}\left(7adf\left(-2c^2f^2+7cdef+3d^2e^2\right)-b\left(-8c^3f^3+19c^2def^2-9cd^2e^2f+6d^3e^3\right)\right)}{105d^3f\sqrt{e+fx^2}}$$
$$-\frac{\sqrt{e}\sqrt{c+dx^2}\left(7adf\left(-2c^2f^2+7cdef+3d^2e^2\right)-b\left(-8c^3f^3+19c^2def^2-9cd^2e^2f+6d^3e^3\right)\right)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{105d^3f^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{x\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}(7adf-4bcf+3bde)}{35d^2}+\frac{bx\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{7d}$$

```
[Out] ((7*a*d*f*(3*d^2*e^2 + 7*c*d*e*f - 2*c^2*f^2) - b*(6*d^3*e^3 - 9*
c*d^2*e^2*f + 19*c^2*d*e*f^2 - 8*c^3*f^3))*x*Sqrt[c + d*x^2])/(10
5*d^3*f*Sqrt[e + f*x^2]) + ((14*a*d*f*(3*d*e - c*f) + b*(3*d^2*e^
2 - 15*c*d*e*f + 8*c^2*f^2))*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(
105*d^2*f) + ((3*b*d*e - 4*b*c*f + 7*a*d*f)*x*(c + d*x^2)^(3/2)*S
qrt[e + f*x^2])/(35*d^2) + (b*x*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/
2))/(7*d) - (Sqrt[e]*(7*a*d*f*(3*d^2*e^2 + 7*c*d*e*f - 2*c^2*f^2)
 - b*(6*d^3*e^3 - 9*c*d^2*e^2*f + 19*c^2*d*e*f^2 - 8*c^3*f^3))*Sq
rt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c
*f)])/(105*d^3*f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt
[e + f*x^2]) + (e^(3/2)*(7*a*d*f*(9*d*e - c*f) - b*(3*d^2*e^2 + 9
*c*d*e*f - 4*c^2*f^2))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*
x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(105*d^2*f^(3/2)*Sqrt[(e*(c + d*x^
2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 1.71365, antiderivative size = 543, normalized size of antiderivative = 1., number of

steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}\left(14adf(3de-cf)+b\left(8c^2f^2-15cdef+3d^2e^2\right)\right)}{105d^2f}$$
$$+\frac{e^{3/2}\sqrt{c+dx^2}\left(7adf(9de-cf)-b\left(-4c^2f^2+9cdef+3d^2e^2\right)\right)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{105d^2f^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{x\sqrt{c+dx^2}\left(7adf\left(-2c^2f^2+7cdef+3d^2e^2\right)-b\left(-8c^3f^3+19c^2def^2-9cd^2e^2f+6d^3e^3\right)\right)}{105d^3f\sqrt{e+fx^2}}$$
$$-\frac{\sqrt{e}\sqrt{c+dx^2}\left(7adf\left(-2c^2f^2+7cdef+3d^2e^2\right)-b\left(-8c^3f^3+19c^2def^2-9cd^2e^2f+6d^3e^3\right)\right)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{105d^3f^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{x\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}(7adf-4bcf+3bde)}{35d^2}+\frac{bx\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}{7d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2),x]
```

```
[Out] ((7*a*d*f*(3*d^2*e^2 + 7*c*d*e*f - 2*c^2*f^2) - b*(6*d^3*e^3 - 9*
c*d^2*e^2*f + 19*c^2*d*e*f^2 - 8*c^3*f^3))*x*Sqrt[c + d*x^2])/(10
5*d^3*f*Sqrt[e + f*x^2]) + ((14*a*d*f*(3*d*e - c*f) + b*(3*d^2*e^
2 - 15*c*d*e*f + 8*c^2*f^2))*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(
105*d^2*f) + ((3*b*d*e - 4*b*c*f + 7*a*d*f)*x*(c + d*x^2)^(3/2)*S
qrt[e + f*x^2])/(35*d^2) + (b*x*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/
2))/(7*d) - (Sqrt[e]*(7*a*d*f*(3*d^2*e^2 + 7*c*d*e*f - 2*c^2*f^2)
 - b*(6*d^3*e^3 - 9*c*d^2*e^2*f + 19*c^2*d*e*f^2 - 8*c^3*f^3))*Sq
rt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c
*f)])/(105*d^3*f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt
[e + f*x^2]) + (e^(3/2)*(7*a*d*f*(9*d*e - c*f) - b*(3*d^2*e^2 + 9
*c*d*e*f - 4*c^2*f^2))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*
x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(105*d^2*f^(3/2)*Sqrt[(e*(c + d*x^
2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)**(1/2)*(f*x**2+e)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.1212, size = 372, normalized size = 0.69

$$ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(cf-de)\left(b\left(4c^2f^2-6cdef+6d^2e^2\right)-7adf(cf+3de)\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+fx\left(-\sqrt{\frac{d}{c}}\right)(c+d$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2),x]
```

```
[Out] (-(Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2)*(4*b*c^2*f^2 - 3*b*c*d*f
*(3*e + f*x^2) - 7*a*d*f*(6*d*e + c*f + 3*d*f*x^2) - 3*b*d^2*(e^2
 + 8*e*f*x^2 + 5*f^2*x^4))) - I*e*(7*a*d*f*(3*d^2*e^2 + 7*c*d*e*f
 - 2*c^2*f^2) + b*(-6*d^3*e^3 + 9*c*d^2*e^2*f - 19*c^2*d*e*f^2 +
8*c^3*f^3))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*A
rcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*e*(-(d*e) + c*f)*(-7*a*d*f*
(3*d*e + c*f) + b*(6*d^2*e^2 - 6*c*d*e*f + 4*c^2*f^2))*Sqrt[1 + (
d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (
c*f)/(d*e)])/(105*c^2*(d/c)^(5/2)*f^2*Sqrt[c + d*x^2]*Sqrt[e + f*
x^2])
```

Maple [B] time = 0.028, size = 1332, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x)
```

```
[Out] 1/105*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*(9*((d*x^2+c)/c)^(1/2)*((f*
x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d^2
*e^3*f+6*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/
c)^(1/2),(c*f/d/e)^(1/2))*b*d^3*e^4-6*((d*x^2+c)/c)^(1/2)*((f*x^2
+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d^3*e^4+
18*(-d/c)^(1/2)*x^7*b*c*d^2*f^4+39*(-d/c)^(1/2)*x^7*b*d^3*e*f^3+2
8*(-d/c)^(1/2)*x^5*a*c*d^2*f^4+63*(-d/c)^(1/2)*x^5*a*d^3*e*f^3-(-
d/c)^(1/2)*x^5*b*c^2*d*f^4+27*(-d/c)^(1/2)*x^5*b*d^3*e^2*f^2+7*(-
d/c)^(1/2)*x^3*a*c^2*d*f^4+42*(-d/c)^(1/2)*x^3*a*d^3*e^2*f^2+3*(-
d/c)^(1/2)*x^3*b*d^3*e^3*f-4*(-d/c)^(1/2)*x*b*c^3*e*f^3-4*(-d/c)^
(1/2)*x^3*b*c^3*f^4+15*(-d/c)^(1/2)*x^9*b*d^3*f^4+7*(-d/c)^(1/2)*
x*a*c^2*d*e*f^3+42*(-d/c)^(1/2)*x*a*c*d^2*e^2*f^2-21*((d*x^2+c)/c
)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1
/2))*a*d^3*e^3*f-4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellipt
icF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*e*f^3+9*(-d/c)^(1/2)*x*
b*c^2*d*e^2*f^2+3*(-d/c)^(1/2)*x*b*c*d^2*e^3*f+21*((d*x^2+c)/c)^(
1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2)
```

```
)*a*d^3*e^3*f+8*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*e*f^3+21*(-d/c)^(1/2)*x^7*
a*d^3*f^4+7*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(
-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d*e*f^3-19*((d*x^2+c)/c)^(1/2)
*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*
c^2*d*e^2*f^2-14*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elliptic
E(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d*e*f^3+49*((d*x^2+c)/c)^
(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2
))*a*c*d^2*e^2*f^2+14*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ell
ipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d^2*e^2*f^2+10*((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/
e)^(1/2))*b*c^2*d*e^2*f^2-12*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1
/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d^2*e^3*f+51*(-
d/c)^(1/2)*x^5*b*c*d^2*e*f^3+70*(-d/c)^(1/2)*x^3*a*c*d^2*e*f^3+8*
(-d/c)^(1/2)*x^3*b*c^2*d*e*f^3+36*(-d/c)^(1/2)*x^3*b*c*d^2*e^2*f^
2)/f^2/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)/d^2/(-d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)\sqrt{dx^2+c}\left(fx^2+e\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bfx^4+(be+af)x^2+ae\right)\sqrt{dx^2+c}\sqrt{fx^2+e},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*f*x^4 + (b*e + a*f)*x^2 + a*e)*sqrt(d*x^2 + c)*sqrt(f
*x^2 + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+bx^2\right)\sqrt{c+dx^2}\left(e+fx^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x**2+c)**(1/2)*(f*x**2+e)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2+a\right)\sqrt{dx^2+c}\left(fx^2+e\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2), x)
```

3.30 $\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=400

$$\frac{e^{3/2}\sqrt{c+dx^2}\left(5ad(3de-cf)-b\left(6cde-4c^2f\right)\right)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15cd^2\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x\sqrt{c+dx^2}\left(10adf(2de-cf)+b\left(8c^2f^2-13cdef+3d^2e^2\right)\right)}{15d^3\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}\left(10adf(2de-cf)+b\left(8c^2f^2-13cdef+3d^2e^2\right)\right)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15d^3\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-4bcf+3bde)}{15d^2}+\frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5d}$$

```
[Out] ((10*a*d*f*(2*d*e - c*f) + b*(3*d^2*e^2 - 13*c*d*e*f + 8*c^2*f^2)
)*x*Sqrt[c + d*x^2])/(15*d^3*Sqrt[e + f*x^2]) + ((3*b*d*e - 4*b*c
*f + 5*a*d*f)*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(15*d^2) + (b*x*
Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/(5*d) - (Sqrt[e]*(10*a*d*f*(2*
d*e - c*f) + b*(3*d^2*e^2 - 13*c*d*e*f + 8*c^2*f^2))*Sqrt[c + d*x
^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*
d^3*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]
) + (e^(3/2)*(5*a*d*(3*d*e - c*f) - b*(6*c*d*e - 4*c^2*f))*Sqrt[c
 + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)]
)/(15*c*d^2*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
+ f*x^2])
```

Rubi [A] time = 1.21625, antiderivative size = 400, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{x\sqrt{c+dx^2}\left(10adf(2de-cf)+b\left(8c^2f^2-13cdef+3d^2e^2\right)\right)}{15d^3\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}\left(10adf(2de-cf)+b\left(8c^2f^2-13cdef+3d^2e^2\right)\right)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15d^3\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{e^{3/2}\sqrt{c+dx^2}(2bc(3de-2cf)-5ad(3de-cf))F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15cd^2\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-4bcf+3bde)}{15d^2}+\frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*(e + f*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] ((10*a*d*f*(2*d*e - c*f) + b*(3*d^2*e^2 - 13*c*d*e*f + 8*c^2*f^2)
)*x*Sqrt[c + d*x^2])/(15*d^3*Sqrt[e + f*x^2]) + ((3*b*d*e - 4*b*c
*f + 5*a*d*f)*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(15*d^2) + (b*x*
Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/(5*d) - (Sqrt[e]*(10*a*d*f*(2*
d*e - c*f) + b*(3*d^2*e^2 - 13*c*d*e*f + 8*c^2*f^2))*Sqrt[c + d*x
^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*
d^3*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]
) - (e^(3/2)*(2*b*c*(3*d*e - 2*c*f) - 5*a*d*(3*d*e - c*f))*Sqrt[c
 + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)]
)/(15*c*d^2*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
+ f*x^2])
```

Rubi in Sympy [A] time = 114.798, size = 389, normalized size = 0.97

$$\frac{bx\sqrt{c+dx^2}\left(e+fx^2\right)^{\frac{3}{2}}}{5d} + \frac{\sqrt{c}\sqrt{e+fx^2}\left(-5acdf+15ad^2e+4bc^2f-6bcde\right)F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{15d^{\frac{5}{2}}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}} + \frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}\left(5adf-4bcf+3bde\right)}{15d^2} - \frac{\sqrt{e}\sqrt{c+dx^2}\left(-10acdf^2+20ad^2ef+8bc^2f^2-13bcdef+3bd^2e^2\right)E\left(\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{15d^3\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{x\sqrt{c+dx^2}\left(-10acdf^2+20ad^2ef+8bc^2f^2-13bcdef+3bd^2e^2\right)}{15d^3\sqrt{e+fx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(f*x**2+e)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] b*x*sqrt(c + d*x**2)*(e + f*x**2)**(3/2)/(5*d) + sqrt(c)*sqrt(e +
 f*x**2)*(-5*a*c*d*f + 15*a*d**2*e + 4*b*c**2*f - 6*b*c*d*e)*elli
ptic_f(atan(sqrt(d)*x/sqrt(c)), -c*f/(d*e) + 1)/(15*d**(5/2)*sqrt
(c*(e + f*x**2)/(e*(c + d*x**2)))*sqrt(c + d*x**2)) + x*sqrt(c +
d*x**2)*sqrt(e + f*x**2)*(5*a*d*f - 4*b*c*f + 3*b*d*e)/(15*d**2)
- sqrt(e)*sqrt(c + d*x**2)*(-10*a*c*d*f**2 + 20*a*d**2*e*f + 8*b*
c**2*f**2 - 13*b*c*d*e*f + 3*b*d**2*e**2)*elliptic_e(atan(sqrt(f)
*x/sqrt(e)), 1 - d*e/(c*f))/(15*d**3*sqrt(f)*sqrt(e*(c + d*x**2)/
(c*(e + f*x**2)))*sqrt(e + f*x**2)) + x*sqrt(c + d*x**2)*(-10*a*c
*d*f**2 + 20*a*d**2*e*f + 8*b*c**2*f**2 - 13*b*c*d*e*f + 3*b*d**2
*e**2)/(15*d**3*sqrt(e + f*x**2))
```

Mathematica [C] time = 1.51848, size = 275, normalized size = 0.69

$$\frac{-ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(10adf(2de-cf)+b\left(8c^2f^2-13cdef+3d^2e^2\right)\right)E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+fx\left(-\sqrt{\frac{d}{c}}\right)\left(c+dx^2\right)(}{15c^2f\left(\frac{d}{c}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*(e + f*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (-(Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2)*(4*b*c*f - 5*a*d*f - 3*b
*d*(2*e + f*x^2))) - I*e*(10*a*d*f*(2*d*e - c*f) + b*(3*d^2*e^2 -
 13*c*d*e*f + 8*c^2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]
*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*e*(-(d*e) + c
*f)*(-3*b*d*e + 4*b*c*f - 5*a*d*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (
f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(15*c^2
*(d/c)^(5/2)*f*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [B] time = 0.027, size = 870, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/15*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)*(3*(-d/c)^(1/2)*x^7*b*d^2*f^
3+5*(-d/c)^(1/2)*x^5*a*d^2*f^3-(-d/c)^(1/2)*x^5*b*c*d*f^3+9*(-d/c
)^(1/2)*x^5*b*d^2*e*f^2+5*(-d/c)^(1/2)*x^3*a*c*d*f^3+5*(-d/c)^(1/
2)*x^3*a*d^2*e*f^2-4*(-d/c)^(1/2)*x^3*b*c^2*f^3+5*(-d/c)^(1/2)*x^
3*b*c*d*e*f^2+6*(-d/c)^(1/2)*x^3*b*d^2*e^2*f+5*((d*x^2+c)/c)^(1/2
)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a
*c*d*e*f^2-5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*
(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^2*e^2*f-4*((d*x^2+c)/c)^(1/2)*(
(f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^
2*e*f^2+7*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d
/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d*e^2*f-3*((d*x^2+c)/c)^(1/2)*((f*
x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d^2*e
^3-10*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^
(1/2),(c*f/d/e)^(1/2))*a*c*d*e*f^2+20*((d*x^2+c)/c)^(1/2)*((f*x^2
+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^2*e^2*
f+8*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1
/2),(c*f/d/e)^(1/2))*b*c^2*e*f^2-13*((d*x^2+c)/c)^(1/2)*((f*x^2+e
)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d*e^2*f+
3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2
),(c*f/d/e)^(1/2))*b*d^2*e^3+5*(-d/c)^(1/2)*x*a*c*d*e*f^2-4*(-d/c
)^(1/2)*x*b*c^2*e*f^2+6*(-d/c)^(1/2)*x*b*c*d*e^2*f)/f/d^2/(d*f*x^
4+c*f*x^2+d*e*x^2+c*e)/(-d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/sqrt(d*x^2 + c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/sqrt(d*x^2 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bfx^4+(be+af)x^2+ae)\sqrt{fx^2+e}}{\sqrt{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/sqrt(d*x^2 + c),x, algorithm="fricas")
```

```
[Out] integral((b*f*x^4 + (b*e + a*f)*x^2 + a*e)*sqrt(f*x^2 + e)/sqrt(d
*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx^2)(e+fx^2)^{\frac{3}{2}}}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(f*x**2+e)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)*(e + f*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/sqrt(d*x^2 + c),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/sqrt(d*x^2 + c), x)
```

3.31 $\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=369

$$\frac{fx\sqrt{c+dx^2}(bc(7de-8cf)-3ad(de-2cf))}{3cd^3\sqrt{e+fx^2}} - \frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(bc(7de-8cf)-3ad(de-2cf))E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3cd^3\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{e^{3/2}\sqrt{c+dx^2}(3adf-4bcf+3bde)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3cd^2\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{fx\sqrt{c+dx^2}\sqrt{e+fx^2}(4bc-3ad)}{3cd^2} - \frac{x\left(e+fx^2\right)^{3/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

```
[Out] (f*(b*c*(7*d*e - 8*c*f) - 3*a*d*(d*e - 2*c*f))*x*Sqrt[c + d*x^2])
/(3*c*d^3*Sqrt[e + f*x^2]) + ((4*b*c - 3*a*d)*f*x*Sqrt[c + d*x^2]
*Sqrt[e + f*x^2])/(3*c*d^2) - ((b*c - a*d)*x*(e + f*x^2)^(3/2))/(
c*d*Sqrt[c + d*x^2]) - (Sqrt[e]*Sqrt[f]*(b*c*(7*d*e - 8*c*f) - 3*
a*d*(d*e - 2*c*f))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/S
qrt[e]], 1 - (d*e)/(c*f)])/(3*c*d^3*Sqrt[(e*(c + d*x^2))/(c*(e +
f*x^2))]*Sqrt[e + f*x^2]) + (e^(3/2)*(3*b*d*e - 4*b*c*f + 3*a*d*f
)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e
)/(c*f)])/(3*c*d^2*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*
Sqrt[e + f*x^2])
```

Rubi [A] time = 1.12367, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{fx\sqrt{c+dx^2}(bc(7de-8cf)-3ad(de-2cf))}{3cd^3\sqrt{e+fx^2}} - \frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(bc(7de-8cf)-3ad(de-2cf))E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3cd^3\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{e^{3/2}\sqrt{c+dx^2}(3adf-4bcf+3bde)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3cd^2\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{fx\sqrt{c+dx^2}\sqrt{e+fx^2}(4bc-3ad)}{3cd^2} - \frac{x\left(e+fx^2\right)^{3/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(3/2),x]
```

```
[Out] (f*(b*c*(7*d*e - 8*c*f) - 3*a*d*(d*e - 2*c*f))*x*Sqrt[c + d*x^2])
/(3*c*d^3*Sqrt[e + f*x^2]) + ((4*b*c - 3*a*d)*f*x*Sqrt[c + d*x^2]
*Sqrt[e + f*x^2])/(3*c*d^2) - ((b*c - a*d)*x*(e + f*x^2)^(3/2))/(
c*d*Sqrt[c + d*x^2]) - (Sqrt[e]*Sqrt[f]*(b*c*(7*d*e - 8*c*f) - 3*
a*d*(d*e - 2*c*f))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/S
qrt[e]], 1 - (d*e)/(c*f)])/(3*c*d^3*Sqrt[(e*(c + d*x^2))/(c*(e +
f*x^2))]*Sqrt[e + f*x^2]) + (e^(3/2)*(3*b*d*e - 4*b*c*f + 3*a*d*f
)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e
)/(c*f)])/(3*c*d^2*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*
Sqrt[e + f*x^2])
```

Rubi in Sympy [A] time = 114.324, size = 345, normalized size = 0.93

$$\frac{\sqrt{c}\sqrt{e+fx^2}\,(3adf-4bcf+3bde)\,F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{3d^{\frac{5}{2}}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}}+\frac{x\left(e+fx^2\right)^{\frac{3}{2}}(ad-bc)}{cd\sqrt{c+dx^2}}-\frac{fx\sqrt{c+dx^2}\sqrt{e+fx^2}\,(3ad-4bc)}{3cd^2}+\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}\left(-6acdf+3ad^2e+8bc^2f-7bcde\right)E\left(\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{3cd^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}-\frac{fx\sqrt{c+dx^2}\left(-6acdf+3ad^2e+8bc^2f-7bcde\right)}{3cd^3\sqrt{e+fx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(f*x**2+e)**(3/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] sqrt(c)*sqrt(e + f*x**2)*(3*a*d*f - 4*b*c*f + 3*b*d*e)*elliptic_f
(atan(sqrt(d)*x/sqrt(c)), -c*f/(d*e) + 1)/(3*d**(5/2)*sqrt(c*(e +
 f*x**2)/(e*(c + d*x**2)))*sqrt(c + d*x**2)) + x*(e + f*x**2)**(3
/2)*(a*d - b*c)/(c*d*sqrt(c + d*x**2)) - f*x*sqrt(c + d*x**2)*sqr
t(e + f*x**2)*(3*a*d - 4*b*c)/(3*c*d**2) + sqrt(e)*sqrt(f)*sqrt(c
 + d*x**2)*(-6*a*c*d*f + 3*a*d**2*e + 8*b*c**2*f - 7*b*c*d*e)*ell
iptic_e(atan(sqrt(f)*x/sqrt(e)), 1 - d*e/(c*f))/(3*c*d**3*sqrt(e*
(c + d*x**2)/(c*(e + f*x**2)))*sqrt(e + f*x**2)) - f*x*sqrt(c + d
*x**2)*(-6*a*c*d*f + 3*a*d**2*e + 8*b*c**2*f - 7*b*c*d*e)/(3*c*d*
*3*sqrt(e + f*x**2))
```

Mathematica [C] time = 1.19675, size = 248, normalized size = 0.67

$$\frac{\sqrt{\frac{d}{c}}\left(x\sqrt{\frac{d}{c}}\left(e+fx^2\right)\left(3ad(de-cf)+bc\left(4cf-3de+dfx^2\right)\right)-ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(4bc-3ad)(cf-de)F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}\right.\right.\right.}{3d^3\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d/c]*(Sqrt[d/c]*x*(e + f*x^2)*(3*a*d*(d*e - c*f) + b*c*(-3*
d*e + 4*c*f + d*f*x^2)) + I*e*(3*a*d*(d*e - 2*c*f) + b*c*(-7*d*e
+ 8*c*f))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*Arc
Sinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*(4*b*c - 3*a*d)*e*(-(d*e) + c
*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[S
qrt[d/c]*x], (c*f)/(d*e)]))/(3*d^3*Sqrt[c + d*x^2]*Sqrt[e + f*x^2
])
```

Maple [A] time = 0.038, size = 671, normalized size = 1.8

$$-\frac{1}{3\,d^2\left(dfx^4+cfx^2+dex^2+ce\right)c}\sqrt{fx^2+e}\sqrt{dx^2+c}\left(-x^5bcdf^2\sqrt{-\frac{d}{c}}+3\,x^3acdf^2\sqrt{-\frac{d}{c}}-3\,x^3ad^2ef\sqrt{-\frac{d}{c}}-4\,x^3bc^2f^2\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x)
```

```
[Out] -1/3*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)*(-x^5*b*c*d*f^2*(-d/c)^(1/2)
+3*x^3*a*c*d*f^2*(-d/c)^(1/2)-3*x^3*a*d^2*e*f*(-d/c)^(1/2)-4*x^3*
b*c^2*f^2*(-d/c)^(1/2)+2*x^3*b*c*d*e*f*(-d/c)^(1/2)+3*EllipticF(x
*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d*e*f*((d*x^2+c)/c)^(1/2)*((f*
x^2+e)/e)^(1/2)-3*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^2
*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-4*EllipticF(x*(-d/c)
^(1/2),(c*f/d/e)^(1/2))*b*c^2*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/
e)^(1/2)+4*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d*e^2*((
d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-6*EllipticE(x*(-d/c)^(1/2),
(c*f/d/e)^(1/2))*a*c*d*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2
)+3*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^2*e^2*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+8*EllipticE(x*(-d/c)^(1/2),(c*f/d/
e)^(1/2))*b*c^2*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-7*Ell
ipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d*e^2*((d*x^2+c)/c)^(1
/2)*((f*x^2+e)/e)^(1/2)+3*x*a*c*d*e*f*(-d/c)^(1/2)-3*x*a*d^2*e^2*
(-d/c)^(1/2)-4*x*b*c^2*e*f*(-d/c)^(1/2)+3*x*b*c*d*e^2*(-d/c)^(1/2
))/d^2/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)/(-d/c)^(1/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)(fx^2 + e)^{\frac{3}{2}}}{(dx^2 + c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bfx^4 + (be + af)x^2 + ae)\sqrt{fx^2 + e}}{(dx^2 + c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*f*x^4 + (b*e + a*f)*x^2 + a*e)*sqrt(f*x^2 + e)/(d*x^2
 + c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^2)(e + fx^2)^{\frac{3}{2}}}{(c + dx^2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(f*x**2+e)**(3/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)*(e + f*x**2)**(3/2)/(c + d*x**2)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)(fx^2 + e)^{\frac{3}{2}}}{(dx^2 + c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(3/2), x)
```

3.32 $\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=373

$$-\frac{fx\sqrt{c+dx^2}(2ad(cf+de)+bc(de-8cf))}{3c^2d^3\sqrt{e+fx^2}}+\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(2ad(cf+de)+bc(de-8cf))E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2d^3\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(4bc-ad)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2d^2\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x\sqrt{e+fx^2}(ad(cf+2de)+bc(de-4cf))}{3c^2d^2\sqrt{c+dx^2}}-\frac{x\left(e+fx^2\right)^{3/2}(bc-ad)}{3cd\left(c+dx^2\right)^{3/2}}$$

```
[Out] -(f*(b*c*(d*e - 8*c*f) + 2*a*d*(d*e + c*f))*x*Sqrt[c + d*x^2])/(3
*c^2*d^3*Sqrt[e + f*x^2]) + ((b*c*(d*e - 4*c*f) + a*d*(2*d*e + c*
f))*x*Sqrt[e + f*x^2])/(3*c^2*d^2*Sqrt[c + d*x^2]) - ((b*c - a*d)
*x*(e + f*x^2)^(3/2))/(3*c*d*(c + d*x^2)^(3/2)) + (Sqrt[e]*Sqrt[f
]*(b*c*(d*e - 8*c*f) + 2*a*d*(d*e + c*f))*Sqrt[c + d*x^2]*Ellipti
cE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*d^3*Sqrt
[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + ((4*b*c - a*
d)*e^(3/2)*Sqrt[f]*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/S
qrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*d^2*Sqrt[(e*(c + d*x^2))/(c*(e
+ f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 1.14391, antiderivative size = 373, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{fx\sqrt{c+dx^2}(2ad(cf+de)+bc(de-8cf))}{3c^2d^3\sqrt{e+fx^2}}+\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(2ad(cf+de)+bc(de-8cf))E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2d^3\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(4bc-ad)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2d^2\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x\sqrt{e+fx^2}(ad(cf+2de)+bc(de-4cf))}{3c^2d^2\sqrt{c+dx^2}}-\frac{x\left(e+fx^2\right)^{3/2}(bc-ad)}{3cd\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(5/2),x]
```

```
[Out] -(f*(b*c*(d*e - 8*c*f) + 2*a*d*(d*e + c*f))*x*Sqrt[c + d*x^2])/(3
*c^2*d^3*Sqrt[e + f*x^2]) + ((b*c*(d*e - 4*c*f) + a*d*(2*d*e + c*
f))*x*Sqrt[e + f*x^2])/(3*c^2*d^2*Sqrt[c + d*x^2]) - ((b*c - a*d)
*x*(e + f*x^2)^(3/2))/(3*c*d*(c + d*x^2)^(3/2)) + (Sqrt[e]*Sqrt[f
]*(b*c*(d*e - 8*c*f) + 2*a*d*(d*e + c*f))*Sqrt[c + d*x^2]*Ellipti
cE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*d^3*Sqrt
[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + ((4*b*c - a*
d)*e^(3/2)*Sqrt[f]*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/S
qrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*d^2*Sqrt[(e*(c + d*x^2))/(c*(e
+ f*x^2))]*Sqrt[e + f*x^2])
```

Rubi in Sympy [A] time = 118.492, size = 342, normalized size = 0.92

$$\frac{x\left(e+fx^2\right)^{\frac{3}{2}}(ad-bc)}{3cd\left(c+dx^2\right)^{\frac{3}{2}}}-\frac{e^{\frac{3}{2}}\sqrt{f}\sqrt{c+dx^2}\,(ad-4bc)\,F\left(\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{3c^2d^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}+\frac{x\sqrt{e+fx^2}\,(cf(ad-4bc)+de(2ad+bc))}{3c^2d^2\sqrt{c+dx^2}}+\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}\,(2cf(ad-4bc)+de(2ad+bc))\,E\left(\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{3c^2d^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}-\frac{fx\sqrt{c+dx^2}\,(2cf(ad-4bc)+de(2ad+bc))}{3c^2d^3\sqrt{e+fx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(f*x**2+e)**(3/2)/(d*x**2+c)**(5/2),x)
```

```
[Out] x*(e + f*x**2)**(3/2)*(a*d - b*c)/(3*c*d*(c + d*x**2)**(3/2)) - e
**(3/2)*sqrt(f)*sqrt(c + d*x**2)*(a*d - 4*b*c)*elliptic_f(atan(sq
rt(f)*x/sqrt(e)), 1 - d*e/(c*f))/(3*c**2*d**2*sqrt(e*(c + d*x**2)
/(c*(e + f*x**2)))*sqrt(e + f*x**2)) + x*sqrt(e + f*x**2)*(c*f*(a
*d - 4*b*c) + d*e*(2*a*d + b*c))/(3*c**2*d**2*sqrt(c + d*x**2)) +
 sqrt(e)*sqrt(f)*sqrt(c + d*x**2)*(2*c*f*(a*d - 4*b*c) + d*e*(2*a
*d + b*c))*elliptic_e(atan(sqrt(f)*x/sqrt(e)), 1 - d*e/(c*f))/(3*
c**2*d**3*sqrt(e*(c + d*x**2)/(c*(e + f*x**2)))*sqrt(e + f*x**2))
 - f*x*sqrt(c + d*x**2)*(2*c*f*(a*d - 4*b*c) + d*e*(2*a*d + b*c))
/(3*c**2*d**3*sqrt(e + f*x**2))
```

Mathematica [C] time = 1.8114, size = 296, normalized size = 0.79

$$\left(\frac{d}{c}\right)^{3/2}\left(x\sqrt{\frac{d}{c}}\left(e+fx^2\right)\left(ad\left(c^2f+cd\left(3e+2fx^2\right)+2d^2ex^2\right)+bc\left(-4c^2f-5cdfx^2+d^2ex^2\right)\right)+ie\left(c+dx^2\right)\sqrt{\frac{dx^2}{c}+1}\sqrt{f$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(5/2),x]
```

```
[Out] ((d/c)^(3/2)*(Sqrt[d/c]*x*(e + f*x^2)*(b*c*(-4*c^2*f + d^2*e*x^2
- 5*c*d*f*x^2) + a*d*(c^2*f + 2*d^2*e*x^2 + c*d*(3*e + 2*f*x^2)))
 - I*e*(-2*a*d*(d*e + c*f) + b*c*(-(d*e) + 8*c*f))*(c + d*x^2)*Sq
rt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/
c]*x], (c*f)/(d*e)] + I*e*(-(a*d*(2*d*e + c*f)) + b*c*(-(d*e) + 4
*c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Ellipt
icF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(3*d^4*(c + d*x^2)^(3/
2)*Sqrt[e + f*x^2])
```

Maple [B] time = 0.043, size = 1225, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/3*(2*x^5*a*c*d^2*f^2*(-d/c)^(1/2)-4*x^3*b*c^3*f^2*(-d/c)^(1/2)-
EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c*d^2*e^2*((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticF(x*(-d/c)^(1/2),(c*f/d/
e)^(1/2))*a*c^2*d*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*E
llipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d*e*f*((d*x^2+c)/c
)^(1/2)*((f*x^2+e)/e)^(1/2)+x^5*b*c*d^2*e*f*(-d/c)^(1/2)+5*x^3*a*
c*d^2*e*f*(-d/c)^(1/2)-5*x^3*b*c^2*d*e*f*(-d/c)^(1/2)+x*a*c^2*d*e
*f*(-d/c)^(1/2)+2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a
*d^3*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*EllipticE(x*(-
d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*d^3*e^2*((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)+2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*
d^2*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-4*EllipticF(x*(-d
/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)+EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*d*e^2
*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*EllipticE(x*(-d/c)^(1/
2),(c*f/d/e)^(1/2))*a*c*d^2*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)+8*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*e*f*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticE(x*(-d/c)^(1/2),(c*f
/d/e)^(1/2))*b*c^2*d*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+
EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c*d^2*e^2*((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*x^3*a*d^3*e^2*(-d/c)^(1/2)+2*x
^5*a*d^3*e*f*(-d/c)^(1/2)-5*x^5*b*c^2*d*f^2*(-d/c)^(1/2)+x^3*a*c^
```

```
2*d*f^2*(-d/c)^(1/2)+x^3*b*c*d^2*e^2*(-d/c)^(1/2)+3*x*a*c*d^2*e^2
*(-d/c)^(1/2)-4*x*b*c^3*e*f*(-d/c)^(1/2)+EllipticF(x*(-d/c)^(1/2)
,(c*f/d/e)^(1/2))*x^2*a*c*d^2*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/
e)^(1/2)-4*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^2*d*
e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*EllipticE(x*(-d/c)^
(1/2),(c*f/d/e)^(1/2))*x^2*a*c*d^2*e*f*((d*x^2+c)/c)^(1/2)*((f*x^
2+e)/e)^(1/2)+8*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c
^2*d*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2))/(f*x^2+e)^(1/2)
/c^2/(-d/c)^(1/2)/(d*x^2+c)^(3/2)/d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bfx^4+(be+af)x^2+ae\right)\sqrt{fx^2+e}}{(d^2x^4+2\,cdx^2+c^2)\sqrt{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*f*x^4 + (b*e + a*f)*x^2 + a*e)*sqrt(f*x^2 + e)/((d^2*
x^4 + 2*c*d*x^2 + c^2)*sqrt(d*x^2 + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(f*x**2+e)**(3/2)/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)\,(fx^2 + e)^{\frac{3}{2}}}{(dx^2 + c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(5/2), x)
```

3.33 $\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx$

Optimal. Leaf size=376

$$-\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(ad(4de-cf)+bc(de-4cf))F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15c^3d^2\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x\sqrt{e+fx^2}(de(4ad+bc)-cf(ad+4bc))}{15c^2d^2(c+dx^2)^{3/2}}+\frac{\sqrt{e+fx^2}\left(ad\left(-2c^2f^2-3cdef+8d^2e^2\right)+bc\left(-8c^2f^2+3cdef+2d^2e^2\right)\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}d^{5/2}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{x\left(e+fx^2\right)^{3/2}(bc-ad)}{5cd\left(c+dx^2\right)^{5/2}}$$

```
[Out] ((d*(b*c + 4*a*d)*e - c*(4*b*c + a*d)*f)*x*Sqrt[e + f*x^2])/(15*c
^2*d^2*(c + d*x^2)^(3/2)) - ((b*c - a*d)*x*(e + f*x^2)^(3/2))/(5*
c*d*(c + d*x^2)^(5/2)) + ((b*c*(2*d^2*e^2 + 3*c*d*e*f - 8*c^2*f^2
) + a*d*(8*d^2*e^2 - 3*c*d*e*f - 2*c^2*f^2))*Sqrt[e + f*x^2]*Elli
pticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(15*c^(5/2)*
d^(5/2)*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c +
d*x^2))]) - (e^(3/2)*Sqrt[f]*(b*c*(d*e - 4*c*f) + a*d*(4*d*e - c*
f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d
*e)/(c*f)])/(15*c^3*d^2*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e +
f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 1.13699, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(ad(4de-cf)+bc(de-4cf))F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15c^3d^2\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x\sqrt{e+fx^2}(de(4ad+bc)-cf(ad+4bc))}{15c^2d^2(c+dx^2)^{3/2}}+\frac{\sqrt{e+fx^2}\left(ad\left(-2c^2f^2-3cdef+8d^2e^2\right)+bc\left(-8c^2f^2+3cdef+2d^2e^2\right)\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}d^{5/2}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{x\left(e+fx^2\right)^{3/2}(bc-ad)}{5cd\left(c+dx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(7/2),x]
```

```
[Out] ((d*(b*c + 4*a*d)*e - c*(4*b*c + a*d)*f)*x*Sqrt[e + f*x^2])/(15*c
^2*d^2*(c + d*x^2)^(3/2)) - ((b*c - a*d)*x*(e + f*x^2)^(3/2))/(5*
c*d*(c + d*x^2)^(5/2)) + ((b*c*(2*d^2*e^2 + 3*c*d*e*f - 8*c^2*f^2
) + a*d*(8*d^2*e^2 - 3*c*d*e*f - 2*c^2*f^2))*Sqrt[e + f*x^2]*Elli
pticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(15*c^(5/2)*
d^(5/2)*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c +
d*x^2))]) - (e^(3/2)*Sqrt[f]*(b*c*(d*e - 4*c*f) + a*d*(4*d*e - c*
f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d
*e)/(c*f)])/(15*c^3*d^2*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e +
f*x^2))]*Sqrt[e + f*x^2])
```

Rubi in Sympy [A] time = 108.873, size = 333, normalized size = 0.89

$$\frac{x\left(e+fx^2\right)^{\frac{3}{2}}(ad-bc)}{5cd\left(c+dx^2\right)^{\frac{5}{2}}}-\frac{x\sqrt{e+fx^2}\,(cf(ad+4bc)-de(4ad+bc))}{15c^2d^2\left(c+dx^2\right)^{\frac{3}{2}}}-\frac{f\sqrt{e+fx^2}\,(cf(ad+4bc)-de(4ad+bc))\,F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{15c^{\frac{3}{2}}d^{\frac{5}{2}}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}\,(cf-de)}+\frac{\sqrt{e+fx^2}\,(cf(2cf(ad+4bc)+de(4ad+bc))-de(cf(ad+4bc)+2de(4ad+bc)))\,E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{15c^{\frac{5}{2}}d^{\frac{5}{2}}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}\,(cf-de)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(f*x**2+e)**(3/2)/(d*x**2+c)**(7/2),x)
```

```
[Out] x*(e + f*x**2)**(3/2)*(a*d - b*c)/(5*c*d*(c + d*x**2)**(5/2)) - x
*sqrt(e + f*x**2)*(c*f*(a*d + 4*b*c) - d*e*(4*a*d + b*c))/(15*c**
2*d**2*(c + d*x**2)**(3/2)) - f*sqrt(e + f*x**2)*(c*f*(a*d + 4*b*
c) - d*e*(4*a*d + b*c))*elliptic_f(atan(sqrt(d)*x/sqrt(c)), -c*f/
(d*e) + 1)/(15*c**(3/2)*d**(5/2)*sqrt(c*(e + f*x**2)/(e*(c + d*x*
*2)))*sqrt(c + d*x**2)*(c*f - d*e)) + sqrt(e + f*x**2)*(c*f*(2*c*
f*(a*d + 4*b*c) + d*e*(4*a*d + b*c)) - d*e*(c*f*(a*d + 4*b*c) + 2
*d*e*(4*a*d + b*c)))*elliptic_e(atan(sqrt(d)*x/sqrt(c)), -c*f/(d*
e) + 1)/(15*c**(5/2)*d**(5/2)*sqrt(c*(e + f*x**2)/(e*(c + d*x**2)
))*sqrt(c + d*x**2)*(c*f - d*e))
```

Mathematica [C] time = 2.35337, size = 382, normalized size = 1.02

$$\sqrt{\frac{d}{c}}\left(-x\sqrt{\frac{d}{c}}\left(e+fx^2\right)\left(\left(c+dx^2\right)^2\left(ad\left(2c^2f^2+3cdef-8d^2e^2\right)+bc\left(8c^2f^2-3cdef-2d^2e^2\right)\right)+3c^2(bc-ad)(de-cf)^2\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(7/2),x]
```

```
[Out] (Sqrt[d/c]*(-(Sqrt[d/c]*x*(e + f*x^2)*(3*c^2*(b*c - a*d)*(d*e - c
*f)^2 - c*(d*e - c*f)*(b*c*(d*e - 7*c*f) + 2*a*d*(2*d*e + c*f))*(
c + d*x^2) + (a*d*(-8*d^2*e^2 + 3*c*d*e*f + 2*c^2*f^2) + b*c*(-2*
d^2*e^2 - 3*c*d*e*f + 8*c^2*f^2))*(c + d*x^2)^2)) - I*e*(c + d*x^
2)^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*((a*d*(-8*d^2*e^2 +
3*c*d*e*f + 2*c^2*f^2) + b*c*(-2*d^2*e^2 - 3*c*d*e*f + 8*c^2*f^2)
)*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (d*e - c*f)*(a
*d*(8*d*e + c*f) + 2*b*c*(d*e + 2*c*f))*EllipticF[I*ArcSinh[Sqrt[
d/c]*x], (c*f)/(d*e)])))/(15*c^2*d^3*(d*e - c*f)*(c + d*x^2)^(5/2
)*Sqrt[e + f*x^2])
```

Maple [B] time = 0.055, size = 2860, normalized size = 7.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x)
```

```
[Out] -1/15*(-4*x^3*b*c^5*f^3*(-d/c)^(1/2)+2*x^5*b*c*d^4*e^3*(-d/c)^(1/
2)-x^3*a*c^4*d*f^3*(-d/c)^(1/2)+20*x^3*a*c*d^4*e^3*(-d/c)^(1/2)+5
*x^3*b*c^2*d^3*e^3*(-d/c)^(1/2)+15*x*a*c^2*d^3*e^3*(-d/c)^(1/2)-4
*x*b*c^5*e*f^2*(-d/c)^(1/2)-2*x^7*a*c^2*d^3*f^3*(-d/c)^(1/2)-3*x^
7*a*c*d^4*e*f^2*(-d/c)^(1/2)+3*x^7*b*c^2*d^3*e*f^2*(-d/c)^(1/2)+2
*x^7*b*c*d^4*e^2*f*(-d/c)^(1/2)-10*x^5*a*c^2*d^3*e*f^2*(-d/c)^(1/
2)+17*x^5*a*c*d^4*e^2*f*(-d/c)^(1/2)-10*x^5*b*c^3*d^2*e*f^2*(-d/c
)^(1/2)+8*x^5*b*c^2*d^3*e^2*f*(-d/c)^(1/2)-17*x^3*a*c^3*d^2*e*f^2
*(-d/c)^(1/2)+7*x^3*a*c^2*d^3*e^2*f*(-d/c)^(1/2)-8*x^3*b*c^4*d*e*
f^2*(-d/c)^(1/2)+2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*
b*c*d^4*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*x^3*b*c^3*d
^2*e^2*f*(-d/c)^(1/2)-x*a*c^4*d*e*f^2*(-d/c)^(1/2)-11*x*a*c^3*d^2
*e^2*f*(-d/c)^(1/2)+x*b*c^4*d*e^2*f*(-d/c)^(1/2)+8*EllipticF(x*(-
d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*a*d^5*e^3*((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)-8*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*
a*d^5*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+8*EllipticF(x*(
-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d^3*e^3*((d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)-4*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c
^5*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticF(x*(-
d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*d^2*e^3*((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)-8*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^
2*d^3*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+8*EllipticE(x*(
-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^5*e*f^2*((d*x^2+c)/c)^(1/2)*((f*
x^2+e)/e)^(1/2)-2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3
*d^2*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+8*x^7*a*d^5*e^2*
f*(-d/c)^(1/2)-8*x^7*b*c^3*d^2*f^3*(-d/c)^(1/2)-6*x^5*a*c^3*d^2*f
^3*(-d/c)^(1/2)-9*x^5*b*c^4*d*f^3*(-d/c)^(1/2)+2*EllipticE(x*(-d/
c)^(1/2),(c*f/d/e)^(1/2))*x^4*a*c^2*d^3*e*f^2*((d*x^2+c)/c)^(1/2)
```

```
*((f*x^2+e)/e)^(1/2)+3*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*
x^4*a*c*d^4*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+8*Ellip
ticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*b*c^3*d^2*e*f^2*((d*x^2+
c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-3*EllipticE(x*(-d/c)^(1/2),(c*f/d
/e)^(1/2))*x^4*b*c^2*d^3*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^
(1/2)-2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c^3*d^2*e
*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+4*EllipticE(x*(-d/c)
^(1/2),(c*f/d/e)^(1/2))*x^2*a*c^3*d^2*e*f^2*((d*x^2+c)/c)^(1/2)*(
(f*x^2+e)/e)^(1/2)+6*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^
2*a*c^2*d^3*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+16*Elli
pticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^4*d*e*f^2*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-6*EllipticE(x*(-d/c)^(1/2),(c*f/d/
e)^(1/2))*x^2*b*c^3*d^2*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)-14*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c^2*d^3*e
^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-8*EllipticF(x*(-d/c)
^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^4*d*e*f^2*((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)-2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*
b*c*d^4*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+16*EllipticF(
x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c*d^4*e^3*((d*x^2+c)/c)^(1/
2)*((f*x^2+e)/e)^(1/2)+4*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2)
)*x^2*b*c^2*d^3*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-16*El
lipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c*d^4*e^3*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-4*EllipticE(x*(-d/c)^(1/2),(c*f/d/
e)^(1/2))*x^2*b*c^2*d^3*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/
2)-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^4*d*e*f^2*((d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-7*EllipticF(x*(-d/c)^(1/2),(c*f
/d/e)^(1/2))*a*c^3*d^2*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1
/2)+2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^4*d*e^2*f*((d
*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticE(x*(-d/c)^(1/2),(
c*f/d/e)^(1/2))*a*c^4*d*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)+3*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^3*d^2*e^2*f*
((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-3*EllipticE(x*(-d/c)^(1/2
),(c*f/d/e)^(1/2))*b*c^4*d*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e
)^(1/2)+4*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^3*d^2
*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticF(x*(-d/c)
^(1/2),(c*f/d/e)^(1/2))*x^4*a*c^2*d^3*e*f^2*((d*x^2+c)/c)^(1/2)*(
(f*x^2+e)/e)^(1/2)-7*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^
4*a*c*d^4*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-4*Ellipti
cF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*b*c^3*d^2*e*f^2*((d*x^2+c)
/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e
)^(1/2))*x^4*b*c^2*d^3*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1
/2)+8*x^5*a*d^5*e^3*(-d/c)^(1/2))/(f*x^2+e)^(1/2)/c^3/(c*f-d*e)/(
-d/c)^(1/2)/(d*x^2+c)^(5/2)/d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bfx^4 + (be + af)x^2 + ae\right)\sqrt{fx^2 + e}}{(d^3x^6 + 3\,cd^2x^4 + 3\,c^2dx^2 + c^3)\sqrt{dx^2 + c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b*f*x^4 + (b*e + a*f)*x^2 + a*e)*sqrt(f*x^2 + e)/((d^3*
x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3)*sqrt(d*x^2 + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(f*x**2+e)**(3/2)/(d*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)\left(fx^2 + e\right)^{\frac{3}{2}}}{(dx^2 + c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(7/2), x)
```

3.34 $\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx$

Optimal. Leaf size=531

$$\frac{x\sqrt{e+fx^2}(de(6ad+bc)-cf(3ad+4bc))}{35c^2d^2(c+dx^2)^{5/2}} - \frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}\left(3ad\left(c^2f^2-11cdef+8d^2e^2\right)+2bc\left(2c^2f^2-cdef+2d^2e^2\right)\right)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{105c^4d^2\sqrt{e+fx^2}(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{x\sqrt{e+fx^2}\left(3ad\left(-2c^2f^2-5cdef+8d^2e^2\right)+bc\left(-8c^2f^2+cdef+4d^2e^2\right)\right)}{105c^3d^2(c+dx^2)^{3/2}(de-cf)} + \frac{\sqrt{e+fx^2}\left(6ad\left(c^3f^3+2c^2def^2-12cd^2e^2f+8d^3e^3\right)+bc\left(8c^3f^3-5c^2def^2-5cd^2e^2f+8d^3e^3\right)\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{105c^{7/2}d^{5/2}\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} - \frac{x\left(e+fx^2\right)^{3/2}(bc-ad)}{7cd\left(c+dx^2\right)^{7/2}}$$

```
[Out] ((d*(b*c + 6*a*d)*e - c*(4*b*c + 3*a*d)*f)*x*Sqrt[e + f*x^2])/(35
*c^2*d^2*(c + d*x^2)^(5/2)) + ((b*c*(4*d^2*e^2 + c*d*e*f - 8*c^2*
f^2) + 3*a*d*(8*d^2*e^2 - 5*c*d*e*f - 2*c^2*f^2))*x*Sqrt[e + f*x^
2])/(105*c^3*d^2*(d*e - c*f)*(c + d*x^2)^(3/2)) - ((b*c - a*d)*x*
(e + f*x^2)^(3/2))/(7*c*d*(c + d*x^2)^(7/2)) + ((6*a*d*(8*d^3*e^3
 - 12*c*d^2*e^2*f + 2*c^2*d*e*f^2 + c^3*f^3) + b*c*(8*d^3*e^3 - 5
*c*d^2*e^2*f - 5*c^2*d*e*f^2 + 8*c^3*f^3))*Sqrt[e + f*x^2]*Ellipt
icE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(105*c^(7/2)*d
^(5/2)*(d*e - c*f)^2*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c +
 d*x^2))]) - (e^(3/2)*Sqrt[f]*(3*a*d*(8*d^2*e^2 - 11*c*d*e*f + c^
2*f^2) + 2*b*c*(2*d^2*e^2 - c*d*e*f + 2*c^2*f^2))*Sqrt[c + d*x^2]
*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(105*c^
4*d^2*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
+ f*x^2])
```

Rubi [A] time = 1.72548, antiderivative size = 531, normalized size of antiderivative = 1., number of

steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{x\sqrt{e+fx^2}(de(6ad+bc)-cf(3ad+4bc))}{35c^2d^2(c+dx^2)^{5/2}} - \frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}\left(3ad\left(c^2f^2-11cdef+8d^2e^2\right)+2bc\left(2c^2f^2-cdef+2d^2e^2\right)\right)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{105c^4d^2\sqrt{e+fx^2}(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{x\sqrt{e+fx^2}\left(3ad\left(-2c^2f^2-5cdef+8d^2e^2\right)+bc\left(-8c^2f^2+cdef+4d^2e^2\right)\right)}{105c^3d^2(c+dx^2)^{3/2}(de-cf)} + \frac{\sqrt{e+fx^2}\left(6ad\left(c^3f^3+2c^2def^2-12cd^2e^2f+8d^3e^3\right)+bc\left(8c^3f^3-5c^2def^2-5cd^2e^2f+8d^3e^3\right)\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{105c^{7/2}d^{5/2}\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} - \frac{x\left(e+fx^2\right)^{3/2}(bc-ad)}{7cd(c+dx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(9/2),x]
```

```
[Out] ((d*(b*c + 6*a*d)*e - c*(4*b*c + 3*a*d)*f)*x*Sqrt[e + f*x^2])/(35
*c^2*d^2*(c + d*x^2)^(5/2)) + ((b*c*(4*d^2*e^2 + c*d*e*f - 8*c^2*
f^2) + 3*a*d*(8*d^2*e^2 - 5*c*d*e*f - 2*c^2*f^2))*x*Sqrt[e + f*x^
2])/(105*c^3*d^2*(d*e - c*f)*(c + d*x^2)^(3/2)) - ((b*c - a*d)*x*
(e + f*x^2)^(3/2))/(7*c*d*(c + d*x^2)^(7/2)) + ((6*a*d*(8*d^3*e^3
 - 12*c*d^2*e^2*f + 2*c^2*d*e*f^2 + c^3*f^3) + b*c*(8*d^3*e^3 - 5
*c*d^2*e^2*f - 5*c^2*d*e*f^2 + 8*c^3*f^3))*Sqrt[e + f*x^2]*Ellipt
icE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(105*c^(7/2)*d
^(5/2)*(d*e - c*f)^2*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c +
 d*x^2))]) - (e^(3/2)*Sqrt[f]*(3*a*d*(8*d^2*e^2 - 11*c*d*e*f + c^
2*f^2) + 2*b*c*(2*d^2*e^2 - c*d*e*f + 2*c^2*f^2))*Sqrt[c + d*x^2]
*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(105*c^
4*d^2*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
+ f*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(f*x**2+e)**(3/2)/(d*x**2+c)**(9/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 3.27776, size = 545, normalized size = 1.03

$$\frac{\sqrt{\frac{d}{c}}\left(-x\sqrt{\frac{d}{c}}\left(e+fx^2\right)\left(15c^3(bc-ad)(de-cf)^3-c\left(c+dx^2\right)^2(de-cf)\left(3ad\left(-2c^2f^2-5cdef+8d^2e^2\right)+bc\left(-8c^2f^2+c\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(9/2),x]
```

```
[Out] (Sqrt[d/c]*(-(Sqrt[d/c]*x*(e + f*x^2)*(15*c^3*(b*c - a*d)*(d*e -
c*f)^3 - 3*c^2*(d*e - c*f)^2*(b*c*(d*e - 9*c*f) + 2*a*d*(3*d*e +
c*f))*(c + d*x^2) - c*(d*e - c*f)*(b*c*(4*d^2*e^2 + c*d*e*f - 8*c
^2*f^2) + 3*a*d*(8*d^2*e^2 - 5*c*d*e*f - 2*c^2*f^2))*(c + d*x^2)^
2 - (6*a*d*(8*d^3*e^3 - 12*c*d^2*e^2*f + 2*c^2*d*e*f^2 + c^3*f^3)
 + b*c*(8*d^3*e^3 - 5*c*d^2*e^2*f - 5*c^2*d*e*f^2 + 8*c^3*f^3))*(
c + d*x^2)^3)) + I*e*(c + d*x^2)^3*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (
f*x^2)/e]*((6*a*d*(8*d^3*e^3 - 12*c*d^2*e^2*f + 2*c^2*d*e*f^2 + c
^3*f^3) + b*c*(8*d^3*e^3 - 5*c*d^2*e^2*f - 5*c^2*d*e*f^2 + 8*c^3*
f^3))*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (-(d*e) +
c*f)*(3*a*d*(-16*d^2*e^2 + 16*c*d*e*f + c^2*f^2) + b*c*(-8*d^2*e^
2 + c*d*e*f + 4*c^2*f^2))*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)
/(d*e)])))/(105*c^3*d^3*(d*e - c*f)^2*(c + d*x^2)^(7/2)*Sqrt[e +
f*x^2])
```

Maple [B] time = 0.1, size = 5113, normalized size = 9.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)\left(fx^2+e\right)^{\frac{3}{2}}}{\left(dx^2+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bfx^4+(be+af)x^2+ae\right)\sqrt{fx^2+e}}{(d^4x^8+4\,cd^3x^6+6\,c^2d^2x^4+4\,c^3dx^2+c^4)\sqrt{dx^2+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b*f*x^4 + (b*e + a*f)*x^2 + a*e)*sqrt(f*x^2 + e)/((d^4*
x^8 + 4*c*d^3*x^6 + 6*c^2*d^2*x^4 + 4*c^3*d*x^2 + c^4)*sqrt(d*x^2
 + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(f*x**2+e)**(3/2)/(d*x**2+c)**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)\left(fx^2+e\right)^{\frac{3}{2}}}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(9/2), x)
```

3.35 $\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=551

$$\frac{\sqrt{e}\sqrt{c+dx^2}\left(7af\left(15c^2f^2-11cdef+4d^2e^2\right)-be\left(45c^2f^2-61cdef+24d^2e^2\right)\right)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{105f^{7/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}\left(28adf(de-2cf)-b\left(15c^2f^2-43cdef+24d^2e^2\right)\right)}{105f^3}$$
$$+\frac{x\sqrt{c+dx^2}\left(7adf\left(23c^2f^2-23cdef+8d^2e^2\right)-b\left(-15c^3f^3+103c^2def^2-128cd^2e^2f+48d^3e^3\right)\right)}{105df^3\sqrt{e+fx^2}}$$
$$-\frac{\sqrt{e}\sqrt{c+dx^2}\left(7adf\left(23c^2f^2-23cdef+8d^2e^2\right)-b\left(-15c^3f^3+103c^2def^2-128cd^2e^2f+48d^3e^3\right)\right)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\right.}{105df^{7/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{x\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}(-7adf-5bcf+6bde)}{35f^2}+\frac{bx\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}{7f}$$

```
[Out] ((7*a*d*f*(8*d^2*e^2 - 23*c*d*e*f + 23*c^2*f^2) - b*(48*d^3*e^3 -
 128*c*d^2*e^2*f + 103*c^2*d*e*f^2 - 15*c^3*f^3))*x*Sqrt[c + d*x^
2])/(105*d*f^3*Sqrt[e + f*x^2]) - ((28*a*d*f*(d*e - 2*c*f) - b*(2
4*d^2*e^2 - 43*c*d*e*f + 15*c^2*f^2))*x*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2])/(105*f^3) - ((6*b*d*e - 5*b*c*f - 7*a*d*f)*x*(c + d*x^2)^
(3/2)*Sqrt[e + f*x^2])/(35*f^2) + (b*x*(c + d*x^2)^(5/2)*Sqrt[e +
 f*x^2])/(7*f) - (Sqrt[e]*(7*a*d*f*(8*d^2*e^2 - 23*c*d*e*f + 23*c
^2*f^2) - b*(48*d^3*e^3 - 128*c*d^2*e^2*f + 103*c^2*d*e*f^2 - 15*
c^3*f^3))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]],
1 - (d*e)/(c*f)])/(105*d*f^(7/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x
^2))]*Sqrt[e + f*x^2]) + (Sqrt[e]*(7*a*f*(4*d^2*e^2 - 11*c*d*e*f
+ 15*c^2*f^2) - b*e*(24*d^2*e^2 - 61*c*d*e*f + 45*c^2*f^2))*Sqrt[
c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)
])/(105*f^(7/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*
x^2])
```

Rubi [A] time = 1.79634, antiderivative size = 551, normalized size of antiderivative = 1., number of

steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{\sqrt{e}\sqrt{c+dx^2}\left(7af\left(15c^2f^2-11cdef+4d^2e^2\right)-be\left(45c^2f^2-61cdef+24d^2e^2\right)\right)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{105f^{7/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}\left(28adf(de-2cf)-b\left(15c^2f^2-43cdef+24d^2e^2\right)\right)}{105f^3}$$
$$+\frac{x\sqrt{c+dx^2}\left(7adf\left(23c^2f^2-23cdef+8d^2e^2\right)-b\left(-15c^3f^3+103c^2def^2-128cd^2e^2f+48d^3e^3\right)\right)}{105df^3\sqrt{e+fx^2}}$$
$$-\frac{\sqrt{e}\sqrt{c+dx^2}\left(7adf\left(23c^2f^2-23cdef+8d^2e^2\right)-b\left(-15c^3f^3+103c^2def^2-128cd^2e^2f+48d^3e^3\right)\right)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\right.}{105df^{7/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{x\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}(-7adf-5bcf+6bde)}{35f^2}+\frac{bx\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}{7f}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*(c + d*x^2)^(5/2))/Sqrt[e + f*x^2],x]
```

```
[Out] ((7*a*d*f*(8*d^2*e^2 - 23*c*d*e*f + 23*c^2*f^2) - b*(48*d^3*e^3 -
 128*c*d^2*e^2*f + 103*c^2*d*e*f^2 - 15*c^3*f^3))*x*Sqrt[c + d*x^
2])/(105*d*f^3*Sqrt[e + f*x^2]) - ((28*a*d*f*(d*e - 2*c*f) - b*(2
4*d^2*e^2 - 43*c*d*e*f + 15*c^2*f^2))*x*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2])/(105*f^3) - ((6*b*d*e - 5*b*c*f - 7*a*d*f)*x*(c + d*x^2)^
(3/2)*Sqrt[e + f*x^2])/(35*f^2) + (b*x*(c + d*x^2)^(5/2)*Sqrt[e +
 f*x^2])/(7*f) - (Sqrt[e]*(7*a*d*f*(8*d^2*e^2 - 23*c*d*e*f + 23*c
^2*f^2) - b*(48*d^3*e^3 - 128*c*d^2*e^2*f + 103*c^2*d*e*f^2 - 15*
c^3*f^3))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]],
1 - (d*e)/(c*f)])/(105*d*f^(7/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x
^2))]*Sqrt[e + f*x^2]) + (Sqrt[e]*(7*a*f*(4*d^2*e^2 - 11*c*d*e*f
+ 15*c^2*f^2) - b*e*(24*d^2*e^2 - 61*c*d*e*f + 45*c^2*f^2))*Sqrt[
c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)
])/(105*f^(7/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*
x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)**(5/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.19283, size = 386, normalized size = 0.7

$$\frac{i\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(cf-de)\left(4be\left(15c^2f^2-26cdef+12d^2e^2\right)-7af\left(15c^2f^2-19cdef+8d^2e^2\right)\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*(c + d*x^2)^(5/2))/Sqrt[e + f*x^2],x]
```

```
[Out] (Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2)*(7*a*d*f*(-4*d*e + 11*c*f
+ 3*d*f*x^2) + b*(45*c^2*f^2 + c*d*f*(-61*e + 45*f*x^2) + 3*d^2*(
8*e^2 - 6*e*f*x^2 + 5*f^2*x^4))) - I*e*(7*a*d*f*(8*d^2*e^2 - 23*c
*d*e*f + 23*c^2*f^2) + b*(-48*d^3*e^3 + 128*c*d^2*e^2*f - 103*c^2
*d*e*f^2 + 15*c^3*f^3))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*E
llipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*(-(d*e) + c*f)*
(4*b*e*(12*d^2*e^2 - 26*c*d*e*f + 15*c^2*f^2) - 7*a*f*(8*d^2*e^2
- 19*c*d*e*f + 15*c^2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/
e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(105*Sqrt[d/c]
*f^4*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [B] time = 0.037, size = 1386, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] 1/105*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*(128*((d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d
^2*e^3*f+48*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(
-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d^3*e^4-48*((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d^3*
e^4+60*(-d/c)^(1/2)*x^7*b*c*d^2*f^4-3*(-d/c)^(1/2)*x^7*b*d^3*e*f^
3+98*(-d/c)^(1/2)*x^5*a*c*d^2*f^4-7*(-d/c)^(1/2)*x^5*a*d^3*e*f^3+
90*(-d/c)^(1/2)*x^5*b*c^2*d*f^4+6*(-d/c)^(1/2)*x^5*b*d^3*e^2*f^2+
77*(-d/c)^(1/2)*x^3*a*c^2*d*f^4-28*(-d/c)^(1/2)*x^3*a*d^3*e^2*f^2
+24*(-d/c)^(1/2)*x^3*b*d^3*e^3*f+45*(-d/c)^(1/2)*x*b*c^3*e*f^3+45
*(-d/c)^(1/2)*x^3*b*c^3*f^4+15*(-d/c)^(1/2)*x^9*b*d^3*f^4+105*((d
*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*
f/d/e)^(1/2))*a*c^3*f^4+77*(-d/c)^(1/2)*x*a*c^2*d*e*f^3-28*(-d/c)
^(1/2)*x*a*c*d^2*e^2*f^2-56*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/
2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^3*e^3*f-60*((d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/
d/e)^(1/2))*b*c^3*e*f^3-61*(-d/c)^(1/2)*x*b*c^2*d*e^2*f^2+24*(-d/
c)^(1/2)*x*b*c*d^2*e^3*f+56*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/
```

```
2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^3*e^3*f+15*((d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/
d/e)^(1/2))*b*c^3*e*f^3+21*(-d/c)^(1/2)*x^7*a*d^3*f^4-238*((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/
e)^(1/2))*a*c^2*d*e*f^3-103*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/
2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*d*e^2*f^2+161*
((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),
(c*f/d/e)^(1/2))*a*c^2*d*e*f^3-161*((d*x^2+c)/c)^(1/2)*((f*x^2+e)
/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d^2*e^2*f
^2+189*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)
^(1/2),(c*f/d/e)^(1/2))*a*c*d^2*e^2*f^2+164*((d*x^2+c)/c)^(1/2)*(
(f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^
2*d*e^2*f^2-152*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d^2*e^3*f-19*(-d/c)^(1/2)*x^
5*b*c*d^2*e*f^3+70*(-d/c)^(1/2)*x^3*a*c*d^2*e*f^3+29*(-d/c)^(1/2)
*x^3*b*c^2*d*e*f^3-55*(-d/c)^(1/2)*x^3*b*c*d^2*e^2*f^2)/f^4/(d*f*
x^4+c*f*x^2+d*e*x^2+c*e)/(-d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)(dx^2+c)^{\frac{5}{2}}}{\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^(5/2)/sqrt(f*x^2 + e),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(5/2)/sqrt(f*x^2 + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bd^2x^6+\left(2\,bcd+ad^2\right)x^4+ac^2+\left(bc^2+2\,acd\right)x^2\right)\sqrt{dx^2+c}}{\sqrt{fx^2+e}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^(5/2)/sqrt(f*x^2 + e),x, algorithm="fricas")
```

```
[Out] integral((b*d^2*x^6 + (2*b*c*d + a*d^2)*x^4 + a*c^2 + (b*c^2 + 2*
a*c*d)*x^2)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^2)\,(c + dx^2)^{\frac{5}{2}}}{\sqrt{e + fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x**2+c)**(5/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)*(c + d*x**2)**(5/2)/sqrt(e + f*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)\,(dx^2 + c)^{\frac{5}{2}}}{\sqrt{fx^2 + e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)^(5/2)/sqrt(f*x^2 + e),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(5/2)/sqrt(f*x^2 + e), x)
```

3.36 $\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=396

$$-\frac{x\sqrt{c+dx^2}\left(10adf(de-2cf)-b\left(3c^2f^2-13cdef+8d^2e^2\right)\right)}{15df^2\sqrt{e+fx^2}}+\frac{\sqrt{e}\sqrt{c+dx^2}\left(10adf(de-2cf)-b\left(3c^2f^2-13cdef+8d^2e^2\right)\right)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15df^{5/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{e}\sqrt{c+dx^2}\left(5af(de-3cf)-b\left(4de^2-6cef\right)\right)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15f^{5/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(-5adf-3bcf+4bde)}{15f^2}+\frac{bx\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{5f}$$

```
[Out] -((10*a*d*f*(d*e - 2*c*f) - b*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2
))*x*Sqrt[c + d*x^2])/(15*d*f^2*Sqrt[e + f*x^2]) - ((4*b*d*e - 3*
b*c*f - 5*a*d*f)*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(15*f^2) + (b
*x*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(5*f) + (Sqrt[e]*(10*a*d*f*
(d*e - 2*c*f) - b*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2))*Sqrt[c +
d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(
15*d*f^(5/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2
]) - (Sqrt[e]*(5*a*f*(d*e - 3*c*f) - b*(4*d*e^2 - 6*c*e*f))*Sqrt[
c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)
])/(15*f^(5/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x
^2])
```

Rubi [A] time = 1.28699, antiderivative size = 396, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{x\sqrt{c+dx^2}\left(10adf(de-2cf)-b\left(3c^2f^2-13cdef+8d^2e^2\right)\right)}{15df^2\sqrt{e+fx^2}}+\frac{\sqrt{e}\sqrt{c+dx^2}\left(10adf(de-2cf)-b\left(3c^2f^2-13cdef+8d^2e^2\right)\right)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15df^{5/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{e}\sqrt{c+dx^2}\left(5af(de-3cf)-b\left(4de^2-6cef\right)\right)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15f^{5/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(-5adf-3bcf+4bde)}{15f^2}+\frac{bx\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{5f}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*(c + d*x^2)^(3/2))/Sqrt[e + f*x^2],x]
```

```
[Out] -((10*a*d*f*(d*e - 2*c*f) - b*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2
))*x*Sqrt[c + d*x^2])/(15*d*f^2*Sqrt[e + f*x^2]) - ((4*b*d*e - 3*
b*c*f - 5*a*d*f)*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(15*f^2) + (b
*x*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(5*f) + (Sqrt[e]*(10*a*d*f*
(d*e - 2*c*f) - b*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2))*Sqrt[c +
d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(
15*d*f^(5/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2
]) - (Sqrt[e]*(5*a*f*(d*e - 3*c*f) - b*(4*d*e^2 - 6*c*e*f))*Sqrt[
c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)
])/(15*f^(5/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x
^2])
```

Rubi in Sympy [A] time = 114.943, size = 389, normalized size = 0.98

$$\frac{bx\left(c+dx^2\right)^{\frac{3}{2}}\sqrt{e+fx^2}}{5f} - \frac{\sqrt{c}\sqrt{e+fx^2}\left(20acdf^2-10ad^2ef+3bc^2f^2-13bcdef+8bd^2e^2\right)E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{15\sqrt{d}f^3\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}} + \frac{\sqrt{e}\sqrt{c+dx^2}\left(15acf^2-5adef-6bcef+4bde^2\right)F\left(\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{15f^{\frac{5}{2}}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}\left(5adf+3bcf-4bde\right)}{15f^2} + \frac{x\sqrt{e+fx^2}\left(20acdf^2-10ad^2ef+3bc^2f^2-13bcdef+8bd^2e^2\right)}{15f^3\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)**(3/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] b*x*(c + d*x**2)**(3/2)*sqrt(e + f*x**2)/(5*f) - sqrt(c)*sqrt(e +
 f*x**2)*(20*a*c*d*f**2 - 10*a*d**2*e*f + 3*b*c**2*f**2 - 13*b*c*
d*e*f + 8*b*d**2*e**2)*elliptic_e(atan(sqrt(d)*x/sqrt(c)), -c*f/(
d*e) + 1)/(15*sqrt(d)*f**3*sqrt(c*(e + f*x**2)/(e*(c + d*x**2)))*
sqrt(c + d*x**2)) + sqrt(e)*sqrt(c + d*x**2)*(15*a*c*f**2 - 5*a*d
*e*f - 6*b*c*e*f + 4*b*d*e**2)*elliptic_f(atan(sqrt(f)*x/sqrt(e))
, 1 - d*e/(c*f))/(15*f**(5/2)*sqrt(e*(c + d*x**2)/(c*(e + f*x**2)
))*sqrt(e + f*x**2)) + x*sqrt(c + d*x**2)*sqrt(e + f*x**2)*(5*a*d
*f + 3*b*c*f - 4*b*d*e)/(15*f**2) + x*sqrt(e + f*x**2)*(20*a*c*d*
f**2 - 10*a*d**2*e*f + 3*b*c**2*f**2 - 13*b*c*d*e*f + 8*b*d**2*e*
*2)/(15*f**3*sqrt(c + d*x**2))
```

Mathematica [C] time = 1.48409, size = 279, normalized size = 0.7

$$\frac{-ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(b\left(3c^2f^2-13cdef+8d^2e^2\right)-10adf(de-2cf)\right)E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+fx\sqrt{\frac{d}{c}}\left(c+dx^2\right)\left(e+\right.}{15f^3\sqrt{\frac{d}{c}}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*(c + d*x^2)^(3/2))/Sqrt[e + f*x^2],x]
```

```
[Out] (Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2)*(5*a*d*f + b*(-4*d*e + 6*c
*f + 3*d*f*x^2)) - I*e*(-10*a*d*f*(d*e - 2*c*f) + b*(8*d^2*e^2 -
13*c*d*e*f + 3*c^2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*
EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*(-(d*e) + c*f)
*(5*a*f*(2*d*e - 3*c*f) + b*e*(-8*d*e + 9*c*f))*Sqrt[1 + (d*x^2)/
c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d
*e)])/(15*Sqrt[d/c]*f^3*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [B] time = 0.028, size = 924, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] 1/15*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*(3*(-d/c)^(1/2)*x^7*b*d^2*f^
3+5*(-d/c)^(1/2)*x^5*a*d^2*f^3+9*(-d/c)^(1/2)*x^5*b*c*d*f^3-(-d/c
)^(1/2)*x^5*b*d^2*e*f^2+5*(-d/c)^(1/2)*x^3*a*c*d*f^3+5*(-d/c)^(1/
2)*x^3*a*d^2*e*f^2+6*(-d/c)^(1/2)*x^3*b*c^2*f^3+5*(-d/c)^(1/2)*x^
3*b*c*d*e*f^2-4*(-d/c)^(1/2)*x^3*b*d^2*e^2*f+15*((d*x^2+c)/c)^(1/
2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*
a*c^2*f^3-25*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*
(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d*e*f^2+10*((d*x^2+c)/c)^(1/2)*
((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d
^2*e^2*f-9*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-
d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*e*f^2+17*((d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d
*e^2*f-8*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/
c)^(1/2),(c*f/d/e)^(1/2))*b*d^2*e^3+20*((d*x^2+c)/c)^(1/2)*((f*x^
2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d*e*f
^2-10*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^
(1/2),(c*f/d/e)^(1/2))*a*d^2*e^2*f+3*((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*e*f^2
-13*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1
/2),(c*f/d/e)^(1/2))*b*c*d*e^2*f+8*((d*x^2+c)/c)^(1/2)*((f*x^2+e)
/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d^2*e^3+5*(
```

```
-d/c)^(1/2)*x*a*c*d*e*f^2+6*(-d/c)^(1/2)*x*b*c^2*e*f^2-4*(-d/c)^(
1/2)*x*b*c*d*e^2*f)/f^3/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)/(-d/c)^(1/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)(dx^2+c)^{\frac{3}{2}}}{\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)/sqrt(f*x^2 + e),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)/sqrt(f*x^2 + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bdx^4+(bc+ad)x^2+ac\right)\sqrt{dx^2+c}}{\sqrt{fx^2+e}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)/sqrt(f*x^2 + e),x, algorithm="fricas")
```

```
[Out] integral((b*d*x^4 + (b*c + a*d)*x^2 + a*c)*sqrt(d*x^2 + c)/sqrt(f
*x^2 + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx^2)(c+dx^2)^{\frac{3}{2}}}{\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x**2+c)**(3/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)*(c + d*x**2)**(3/2)/sqrt(e + f*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)\left(dx^2 + c\right)^{\frac{3}{2}}}{\sqrt{fx^2 + e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)/sqrt(f*x^2 + e),x, algorithm="giac")

[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)/sqrt(f*x^2 + e), x)
```

3.37 $\int \frac{(a+bx^2)\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=282

$$-\frac{\sqrt{e}\sqrt{c+dx^2}(be-3af)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3f^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{\sqrt{e}\sqrt{c+dx^2}(-3adf-bcf+2bde)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3df^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x\sqrt{c+dx^2}(-3adf-bcf+2bde)}{3df\sqrt{e+fx^2}}+\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3f}$$

```
[Out] -((2*b*d*e - b*c*f - 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*d*f*Sqrt[e +
f*x^2]) + (b*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*f) + (Sqrt[e]*
(2*b*d*e - b*c*f - 3*a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqr
t[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*d*f^(3/2)*Sqrt[(e*(c + d*x
^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (Sqrt[e]*(b*e - 3*a*f)*S
qrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(
c*f)])/(3*f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e +
f*x^2])
```

Rubi [A] time = 0.601423, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{\sqrt{e}\sqrt{c+dx^2}(be-3af)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3f^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{\sqrt{e}\sqrt{c+dx^2}(-3adf-bcf+2bde)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3df^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x\sqrt{c+dx^2}(-3adf-bcf+2bde)}{3df\sqrt{e+fx^2}}+\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3f}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

```
[Out] -((2*b*d*e - b*c*f - 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*d*f*Sqrt[e +
f*x^2]) + (b*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*f) + (Sqrt[e]*
(2*b*d*e - b*c*f - 3*a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqr
```

```
t[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*d*f^(3/2)*Sqrt[(e*(c + d*x
^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (Sqrt[e]*(b*e - 3*a*f)*S
qrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(
c*f)])/(3*f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e +
f*x^2])
```

Rubi in Sympy [A] time = 68.7722, size = 250, normalized size = 0.89

$$\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3f}+\frac{\sqrt{e}\sqrt{c+dx^2}\,(3af-be)\,F\left(\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{3f^{\frac{3}{2}}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}\,(3adf+bcf-2bde)\,E\left(\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{3df^{\frac{3}{2}}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}+\frac{x\sqrt{c+dx^2}\,(3adf+bcf-2bde)}{3df\sqrt{e+fx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] b*x*sqrt(c + d*x**2)*sqrt(e + f*x**2)/(3*f) + sqrt(e)*sqrt(c + d*
x**2)*(3*a*f - b*e)*elliptic_f(atan(sqrt(f)*x/sqrt(e)), 1 - d*e/(
c*f))/(3*f**(3/2)*sqrt(e*(c + d*x**2)/(c*(e + f*x**2)))*sqrt(e +
f*x**2)) - sqrt(e)*sqrt(c + d*x**2)*(3*a*d*f + b*c*f - 2*b*d*e)*e
lliptic_e(atan(sqrt(f)*x/sqrt(e)), 1 - d*e/(c*f))/(3*d*f**(3/2)*s
qrt(e*(c + d*x**2)/(c*(e + f*x**2)))*sqrt(e + f*x**2)) + x*sqrt(c
 + d*x**2)*(3*a*d*f + b*c*f - 2*b*d*e)/(3*d*f*sqrt(e + f*x**2))
```

Mathematica [C] time = 0.700015, size = 215, normalized size = 0.76

$$\frac{i\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(2be-3af)(cf-de)F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(3adf+bcf-2bde)E\left(i\sinh^{-1}\left(\right.\right.}{3f^2\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

```
[Out] (b*Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2) - I*e*(-2*b*d*e + b*c*f
+ 3*a*d*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*Ar
cSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*(2*b*e - 3*a*f)*(-(d*e) + c*
f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sq
rt[d/c]*x], (c*f)/(d*e)])/(3*Sqrt[d/c]*f^2*Sqrt[c + d*x^2]*Sqrt[e
 + f*x^2])
```

Maple [A] time = 0.024, size = 501, normalized size = 1.8

$$\frac{1}{\left(3\,dfx^4+3\,cfx^2+3\,dex^2+3\,ce\right)f^2}\sqrt{dx^2+c}\sqrt{fx^2+e}\left(\sqrt{-\frac{d}{c}}x^5bdf^2+\sqrt{-\frac{d}{c}}x^3bcf^2+\sqrt{-\frac{d}{c}}x^3bdef+3\,EllipticF\left(x\sqrt{-\frac{d}{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] 1/3*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*((-d/c)^(1/2)*x^5*b*d*f^2+(-d
/c)^(1/2)*x^3*b*c*f^2+(-d/c)^(1/2)*x^3*b*d*e*f+3*EllipticF(x*(-d/
c)^(1/2),(c*f/d/e)^(1/2))*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)
*a*c*f^2-3*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*((d*x^2+c)/c
)^(1/2)*((f*x^2+e)/e)^(1/2)*a*d*e*f-2*((d*x^2+c)/c)^(1/2)*((f*x^2
+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*e*f+2*
((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),
(c*f/d/e)^(1/2))*b*d*e^2+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2
)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e*f+((d*x^2+c)/c)
^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/
2))*b*c*e*f-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x
*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d*e^2+(-d/c)^(1/2)*x*b*c*e*f)/(d
*f*x^4+c*f*x^2+d*e*x^2+c*e)/f^2/(-d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^2+a\right)\sqrt{dx^2+c}}{\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)\sqrt{dx^2+c}}{\sqrt{fx^2+e}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^2)\sqrt{c + dx^2}}{\sqrt{e + fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(c + d*x**2)/sqrt(e + f*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)\sqrt{dx^2 + c}}{\sqrt{fx^2 + e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

3.38 $\int \frac{a+bx^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=206

$$\frac{a\sqrt{e}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{bx\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{b\sqrt{e}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

```
[Out] (b*x*Sqrt[c + d*x^2])/(d*Sqrt[e + f*x^2]) - (b*Sqrt[e]*Sqrt[c + d
*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(d
*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) +
 (a*Sqrt[e]*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]]
, 1 - (d*e)/(c*f)])/(c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2
))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.370599, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{a\sqrt{e}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{bx\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}-\frac{b\sqrt{e}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2])/(d*Sqrt[e + f*x^2]) - (b*Sqrt[e]*Sqrt[c + d
*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(d
*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) +
 (a*Sqrt[e]*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]]
, 1 - (d*e)/(c*f)])/(c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2
))]*Sqrt[e + f*x^2])
```

Rubi in Sympy [A] time = 44.5179, size = 177, normalized size = 0.86

$$\frac{a\sqrt{e}\sqrt{c+dx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}-\frac{b\sqrt{c}\sqrt{e+fx^2}E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{\sqrt{d}f\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}}+\frac{bx\sqrt{e+fx^2}}{f\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] a*sqrt(e)*sqrt(c + d*x**2)*elliptic_f(atan(sqrt(f)*x/sqrt(e)), 1
- d*e/(c*f))/(c*sqrt(f)*sqrt(e*(c + d*x**2)/(c*(e + f*x**2)))*sqr
t(e + f*x**2)) - b*sqrt(c)*sqrt(e + f*x**2)*elliptic_e(atan(sqrt(
d)*x/sqrt(c)), -c*f/(d*e) + 1)/(sqrt(d)*f*sqrt(c*(e + f*x**2)/(e*
(c + d*x**2)))*sqrt(c + d*x**2)) + b*x*sqrt(e + f*x**2)/(f*sqrt(c
 + d*x**2))
```

Mathematica [C] time = 0.192948, size = 131, normalized size = 0.64

$$-\frac{i\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left((af-be)F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+beE\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)\right)}{f\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(b*e*EllipticE[I*Ar
cSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (-(b*e) + a*f)*EllipticF[I*Arc
Sinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(Sqrt[d/c]*f*Sqrt[c + d*x^2]*Sq
rt[e + f*x^2])
```

Maple [A] time = 0.03, size = 158, normalized size = 0.8

$$\frac{1}{f\left(dfx^4+cfx^2+dex^2+ce\right)}\left(\mathit{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)af-\mathit{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)be+\mathit{EllipticE}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)be\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] (EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*f-EllipticF(x*(-d/c)
^(1/2),(c*f/d/e)^(1/2))*b*e+EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1
/2))*b*e)*((f*x^2+e)/e)^(1/2)*((d*x^2+c)/c)^(1/2)*(d*x^2+c)^(1/2)
*(f*x^2+e)^(1/2)/f/(-d/c)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{bx^2+a}{\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="maxima")

[Out] integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2+a}{\sqrt{dx^2+c}\sqrt{fx^2+e}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="fricas")

[Out] integral((b*x^2 + a)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+bx^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)

[Out] Integral((a + b*x**2)/(sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{bx^2+a}{\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="giac")

[Out] integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.39 $\int \frac{a+bx^2}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=209

$$\frac{\sqrt{e}\sqrt{c+dx^2}(be-af)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{e+fx^2}(bc-ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

```
[Out] -(((b*c - a*d)*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[
c]], 1 - (c*f)/(d*e)])/(Sqrt[c]*Sqrt[d]*(d*e - c*f)*Sqrt[c + d*x^
2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])) + (Sqrt[e]*(b*e - a*f)
*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)
/(c*f)])/(c*Sqrt[f]*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^
2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.31425, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{\sqrt{e}\sqrt{c+dx^2}(be-af)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{e+fx^2}(bc-ad)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/((c + d*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] -(((b*c - a*d)*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[
c]], 1 - (c*f)/(d*e)])/(Sqrt[c]*Sqrt[d]*(d*e - c*f)*Sqrt[c + d*x^
2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])) + (Sqrt[e]*(b*e - a*f)
*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)
/(c*f)])/(c*Sqrt[f]*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^
2))]*Sqrt[e + f*x^2])
```

Rubi in Sympy [A] time = 35.3221, size = 173, normalized size = 0.83

$$\frac{\sqrt{e}\sqrt{c+dx^2}\,(af-be)\,F\left(\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}\,(cf-de)}-\frac{\sqrt{e+fx^2}\,(ad-bc)\,E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}\,(cf-de)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/(d*x**2+c)**(3/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] sqrt(e)*sqrt(c + d*x**2)*(a*f - b*e)*elliptic_f(atan(sqrt(f)*x/sq
rt(e)), 1 - d*e/(c*f))/(c*sqrt(f)*sqrt(e*(c + d*x**2)/(c*(e + f*x
**2)))*sqrt(e + f*x**2)*(c*f - d*e)) - sqrt(e + f*x**2)*(a*d - b*
c)*elliptic_e(atan(sqrt(d)*x/sqrt(c)), -c*f/(d*e) + 1)/(sqrt(c)*s
qrt(d)*sqrt(c*(e + f*x**2)/(e*(c + d*x**2)))*sqrt(c + d*x**2)*(c*
f - d*e))
```

Mathematica [C] time = 1.04871, size = 206, normalized size = 0.99

$$\frac{\sqrt{\frac{d}{c}}\left(x\sqrt{\frac{d}{c}}\left(e+fx^2\right)(bc-ad)+ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(bc-ad)E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-ia\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(cf-de)F(}{d\sqrt{c+dx^2}\sqrt{e+fx^2}(cf-de)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/((c + d*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (Sqrt[d/c]*(Sqrt[d/c]*(b*c - a*d)*x*(e + f*x^2) + I*(b*c - a*d)*e
*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt
[d/c]*x], (c*f)/(d*e)] - I*a*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*S
qrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]
))/(d*(-(d*e) + c*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.04, size = 334, normalized size = 1.6

$$\frac{1}{c(cf-de)(dfx^4+cfx^2+dex^2+ce)}\left(-x^3adf\sqrt{-\frac{d}{c}}+x^3bcf\sqrt{-\frac{d}{c}}+EllipticF\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)acf\sqrt{\frac{fx^2+e}{e}}\sqrt{\frac{dx^2+c}{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] (-x^3*a*d*f*(-d/c)^(1/2)+x^3*b*c*f*(-d/c)^(1/2)+EllipticF(x*(-d/c
)^(1/2),(c*f/d/e)^(1/2))*a*c*f*((f*x^2+e)/e)^(1/2)*((d*x^2+c)/c)^
(1/2)-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e*((d*x^2+c)/
c)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(
1/2))*a*d*e*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticE(x*(
-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*e*((d*x^2+c)/c)^(1/2)*((f*x^2+e)
/e)^(1/2)-x*a*d*e*(-d/c)^(1/2)+x*b*c*e*(-d/c)^(1/2))*(f*x^2+e)^(1
/2)*(d*x^2+c)^(1/2)/(-d/c)^(1/2)/c/(c*f-d*e)/(d*f*x^4+c*f*x^2+d*e
*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + a}{(dx^2 + c)^{\frac{3}{2}} \sqrt{fx^2 + e}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{bx^2 + a}{(dx^2 + c)^{\frac{3}{2}} \sqrt{fx^2 + e}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)/((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + bx^2}{(c + dx^2)^{\frac{3}{2}} \sqrt{e + fx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/(d*x**2+c)**(3/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)/((c + d*x**2)**(3/2)*sqrt(e + f*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + a}{(dx^2 + c)^{\frac{3}{2}} \sqrt{fx^2 + e}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)),x, algorithm="giac")

[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

3.40 $\int \frac{a+bx^2}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=284

$$\frac{\sqrt{e+fx^2}(2ad(de-2cf)+bc(cf+de))E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}\sqrt{d}\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(-3acf+ade+2bce)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2\sqrt{e+fx^2}(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x\sqrt{e+fx^2}(bc-ad)}{3c\left(c+dx^2\right)^{3/2}(de-cf)}$$

```
[Out] -((b*c - a*d)*x*Sqrt[e + f*x^2])/(3*c*(d*e - c*f)*(c + d*x^2)^(3/
2)) + ((2*a*d*(d*e - 2*c*f) + b*c*(d*e + c*f))*Sqrt[e + f*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(3*c^(3/2)
*Sqrt[d]*(d*e - c*f)^2*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c
 + d*x^2))]) - (Sqrt[e]*Sqrt[f]*(2*b*c*e + a*d*e - 3*a*c*f)*Sqrt[
c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)
])/(3*c^2*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqr
t[e + f*x^2])
```

Rubi [A] time = 0.63039, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{\sqrt{e+fx^2}(2ad(de-2cf)+bc(cf+de))E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}\sqrt{d}\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(-3acf+ade+2bce)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2\sqrt{e+fx^2}(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x\sqrt{e+fx^2}(bc-ad)}{3c\left(c+dx^2\right)^{3/2}(de-cf)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/((c + d*x^2)^(5/2)*Sqrt[e + f*x^2]),x]
```


```
[Out] -((b*c - a*d)*x*Sqrt[e + f*x^2])/(3*c*(d*e - c*f)*(c + d*x^2)^(3/
2)) + ((2*a*d*(d*e - 2*c*f) + b*c*(d*e + c*f))*Sqrt[e + f*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(3*c^(3/2)
*Sqrt[d]*(d*e - c*f)^2*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c
 + d*x^2))]) - (Sqrt[e]*Sqrt[f]*(2*b*c*e + a*d*e - 3*a*c*f)*Sqrt[
c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)
])/(3*c^2*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqr
t[e + f*x^2])
```

Rubi in Sympy [A] time = 90.132, size = 257, normalized size = 0.9

$$-\frac{x\sqrt{e+fx^2}\,(ad-bc)}{3c\,(c+dx^2)^{\frac{3}{2}}\,(cf-de)}+\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}\,(3acf-ade-2bce)\,F\left(\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{3c^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}\,(cf-de)^2}+\frac{\sqrt{e+fx^2}\,\left(-4acdf+2ad^2e+bc^2f+bcde\right)\,E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{3c^{\frac{3}{2}}\sqrt{d}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}\,(cf-de)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/(d*x**2+c)**(5/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] -x*sqrt(e + f*x**2)*(a*d - b*c)/(3*c*(c + d*x**2)**(3/2)*(c*f - d
*e)) + sqrt(e)*sqrt(f)*sqrt(c + d*x**2)*(3*a*c*f - a*d*e - 2*b*c*
e)*elliptic_f(atan(sqrt(f)*x/sqrt(e)), 1 - d*e/(c*f))/(3*c**2*sqr
t(e*(c + d*x**2)/(c*(e + f*x**2)))*sqrt(e + f*x**2)*(c*f - d*e)**
2) + sqrt(e + f*x**2)*(-4*a*c*d*f + 2*a*d**2*e + b*c**2*f + b*c*d
*e)*elliptic_e(atan(sqrt(d)*x/sqrt(c)), -c*f/(d*e) + 1)/(3*c**(3/
2)*sqrt(d)*sqrt(c*(e + f*x**2)/(e*(c + d*x**2)))*sqrt(c + d*x**2)
*(c*f - d*e)**2)
```

Mathematica [C] time = 2.13819, size = 302, normalized size = 1.06

$$\frac{x\sqrt{\frac{d}{c}}\left(e+fx^2\right)\left(ad\left(-5c^2f+cd\left(3e-4fx^2\right)+2d^2ex^2\right)+bc\left(2c^2f+cdfx^2+d^2ex^2\right)\right)+i\left(c+dx^2\right)\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(cf}{3c^2\sqrt{\frac{d}{c}}(c+}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/((c + d*x^2)^(5/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (Sqrt[d/c]*x*(e + f*x^2)*(b*c*(2*c^2*f + d^2*e*x^2 + c*d*f*x^2) +
 a*d*(-5*c^2*f + 2*d^2*e*x^2 + c*d*(3*e - 4*f*x^2))) + I*e*(2*a*d
*(d*e - 2*c*f) + b*c*(d*e + c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]
*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e
)] + I*(-(d*e) + c*f)*(b*c*e + 2*a*d*e - 3*a*c*f)*(c + d*x^2)*Sqr
t[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c
]*x], (c*f)/(d*e)])/(3*c^2*Sqrt[d/c]*(d*e - c*f)^2*(c + d*x^2)^(3
/2)*Sqrt[e + f*x^2])
```

Maple [B] time = 0.049, size = 1352, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] 1/3*(-4*x^5*a*c*d^2*f^2*(-d/c)^(1/2)+2*x^3*b*c^3*f^2*(-d/c)^(1/2)
-EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c*d^2*e^2*((d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-5*EllipticF(x*(-d/c)^(1/2),(c*f
/d/e)^(1/2))*a*c^2*d*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+
4*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d*e*f*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+x^5*b*c*d^2*e*f*(-d/c)^(1/2)-x^3*a
*c*d^2*e*f*(-d/c)^(1/2)+x^3*b*c^2*d*e*f*(-d/c)^(1/2)-5*x*a*c^2*d*
e*f*(-d/c)^(1/2)+2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*
a*d^3*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*EllipticE(x*(
-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*d^3*e^2*((d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)+2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c
*d^2*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticF(x*(-d/
c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e
)/e)^(1/2)+EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*d*e^2*
((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*EllipticE(x*(-d/c)^(1/2
),(c*f/d/e)^(1/2))*a*c*d^2*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^
(1/2)-EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*e*f*((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticE(x*(-d/c)^(1/2),(c*f/d/
e)^(1/2))*b*c^2*d*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+Ell
ipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c*d^2*e^2*((d*x^2+c)
/c)^(1/2)*((f*x^2+e)/e)^(1/2)+3*EllipticF(x*(-d/c)^(1/2),(c*f/d/e
)^(1/2))*a*c^3*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*x^3*
a*d^3*e^2*(-d/c)^(1/2)+3*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2)
)*x^2*a*c^2*d*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*x^5*a
*d^3*e*f*(-d/c)^(1/2)+x^5*b*c^2*d*f^2*(-d/c)^(1/2)-5*x^3*a*c^2*d*
f^2*(-d/c)^(1/2)+x^3*b*c*d^2*e^2*(-d/c)^(1/2)+3*x*a*c*d^2*e^2*(-d
/c)^(1/2)+2*x*b*c^3*e*f*(-d/c)^(1/2)-5*EllipticF(x*(-d/c)^(1/2),(
c*f/d/e)^(1/2))*x^2*a*c*d^2*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^2*d*e*f*
((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+4*EllipticE(x*(-d/c)^(1/2
),(c*f/d/e)^(1/2))*x^2*a*c*d^2*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)
/e)^(1/2)-EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^2*d*e
*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2))/(f*x^2+e)^(1/2)/(c*f-
d*e)^2/c^2/(-d/c)^(1/2)/(d*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + a}{(dx^2 + c)^{\frac{5}{2}} \sqrt{fx^2 + e}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2+a}{(d^2x^4+2\,cdx^2+c^2)\sqrt{dx^2+c}\sqrt{fx^2+e}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)/((d^2*x^4 + 2*c*d*x^2 + c^2)*sqrt(d*x^2 + c)
*sqrt(f*x^2 + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x**2+c)**(5/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2+a}{(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)), x)
```

3.41 $\int \frac{a+bx^2}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=401

$$\frac{x\sqrt{e+fx^2}(4ad(de-2cf)+bc(3cf+de))}{15c^2(c+dx^2)^{3/2}(de-cf)^2}+\frac{\sqrt{e+fx^2}\left(ad\left(23c^2f^2-23cdef+8d^2e^2\right)+bc\left(-3c^2f^2-7cdef+2d^2e^2\right)\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}\sqrt{d}\sqrt{c+dx^2}(de-cf)^3\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}\left(a\left(15c^2f^2-11cdef+4d^2e^2\right)+bce(de-9cf)\right)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15c^3\sqrt{e+fx^2}(de-cf)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x\sqrt{e+fx^2}(bc-ad)}{5c(c+dx^2)^{5/2}(de-cf)}$$

```
[Out] -((b*c - a*d)*x*Sqrt[e + f*x^2])/(5*c*(d*e - c*f)*(c + d*x^2)^(5/
2)) + ((4*a*d*(d*e - 2*c*f) + b*c*(d*e + 3*c*f))*x*Sqrt[e + f*x^2
])/(15*c^2*(d*e - c*f)^2*(c + d*x^2)^(3/2)) + ((b*c*(2*d^2*e^2 -
7*c*d*e*f - 3*c^2*f^2) + a*d*(8*d^2*e^2 - 23*c*d*e*f + 23*c^2*f^2
))*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*
f)/(d*e)])/(15*c^(5/2)*Sqrt[d]*(d*e - c*f)^3*Sqrt[c + d*x^2]*Sqrt
[(c*(e + f*x^2))/(e*(c + d*x^2))]) - (Sqrt[e]*Sqrt[f]*(b*c*e*(d*e
 - 9*c*f) + a*(4*d^2*e^2 - 11*c*d*e*f + 15*c^2*f^2))*Sqrt[c + d*x
^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*
c^3*(d*e - c*f)^3*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e +
f*x^2])
```

Rubi [A] time = 1.18249, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{x\sqrt{e+fx^2}(4ad(de-2cf)+bc(3cf+de))}{15c^2(c+dx^2)^{3/2}(de-cf)^2}+\frac{\sqrt{e+fx^2}\left(ad\left(23c^2f^2-23cdef+8d^2e^2\right)+bc\left(-3c^2f^2-7cdef+2d^2e^2\right)\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}\sqrt{d}\sqrt{c+dx^2}(de-cf)^3\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}\left(a\left(15c^2f^2-11cdef+4d^2e^2\right)+bce(de-9cf)\right)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15c^3\sqrt{e+fx^2}(de-cf)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x\sqrt{e+fx^2}(bc-ad)}{5c(c+dx^2)^{5/2}(de-cf)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/((c + d*x^2)^(7/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] -((b*c - a*d)*x*Sqrt[e + f*x^2])/(5*c*(d*e - c*f)*(c + d*x^2)^(5/
2)) + ((4*a*d*(d*e - 2*c*f) + b*c*(d*e + 3*c*f))*x*Sqrt[e + f*x^2
])/(15*c^2*(d*e - c*f)^2*(c + d*x^2)^(3/2)) + ((b*c*(2*d^2*e^2 -
7*c*d*e*f - 3*c^2*f^2) + a*d*(8*d^2*e^2 - 23*c*d*e*f + 23*c^2*f^2
))*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*
f)/(d*e)])/(15*c^(5/2)*Sqrt[d]*(d*e - c*f)^3*Sqrt[c + d*x^2]*Sqrt
[(c*(e + f*x^2))/(e*(c + d*x^2))]) - (Sqrt[e]*Sqrt[f]*(b*c*e*(d*e
 - 9*c*f) + a*(4*d^2*e^2 - 11*c*d*e*f + 15*c^2*f^2))*Sqrt[c + d*x
^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*
c^3*(d*e - c*f)^3*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e +
f*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/(d*x**2+c)**(7/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.28568, size = 393, normalized size = 0.98

$$\frac{-x\sqrt{\frac{d}{c}}\left(e+fx^2\right)\left(\left(c+dx^2\right)^2\left(ad\left(-23c^2f^2+23cdef-8d^2e^2\right)+bc\left(3c^2f^2+7cdef-2d^2e^2\right)\right)+3c^2(bc-ad)(de-cf)^2+c\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/((c + d*x^2)^(7/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (-(Sqrt[d/c]*x*(e + f*x^2)*(3*c^2*(b*c - a*d)*(d*e - c*f)^2 + c*(
-(d*e) + c*f)*(4*a*d*(d*e - 2*c*f) + b*c*(d*e + 3*c*f))*(c + d*x^
2) + (a*d*(-8*d^2*e^2 + 23*c*d*e*f - 23*c^2*f^2) + b*c*(-2*d^2*e^
2 + 7*c*d*e*f + 3*c^2*f^2))*(c + d*x^2)^2)) - I*(c + d*x^2)^2*Sqr
t[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(e*(a*d*(-8*d^2*e^2 + 23*c*d
*e*f - 23*c^2*f^2) + b*c*(-2*d^2*e^2 + 7*c*d*e*f + 3*c^2*f^2))*El
lipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (d*e - c*f)*(2*b*c
*e*(d*e - 3*c*f) + a*(8*d^2*e^2 - 19*c*d*e*f + 15*c^2*f^2))*Ellip
ticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(15*c^3*Sqrt[d/c]*(d*
e - c*f)^3*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2])
```

Maple [B] time = 0.059, size = 3039, normalized size = 7.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] 1/15*(9*x^3*b*c^5*f^3*(-d/c)^(1/2)-2*x^5*b*c*d^4*e^3*(-d/c)^(1/2)
-34*x^3*a*c^4*d*f^3*(-d/c)^(1/2)-20*x^3*a*c*d^4*e^3*(-d/c)^(1/2)-
5*x^3*b*c^2*d^3*e^3*(-d/c)^(1/2)-15*x*a*c^2*d^3*e^3*(-d/c)^(1/2)+
9*x*b*c^5*e*f^2*(-d/c)^(1/2)+15*EllipticF(x*(-d/c)^(1/2),(c*f/d/e
)^(1/2))*a*c^5*f^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-23*x^7
*a*c^2*d^3*f^3*(-d/c)^(1/2)+23*x^7*a*c*d^4*e*f^2*(-d/c)^(1/2)+7*x
^7*b*c^2*d^3*e*f^2*(-d/c)^(1/2)-2*x^7*b*c*d^4*e^2*f*(-d/c)^(1/2)+
35*x^5*a*c^2*d^3*e*f^2*(-d/c)^(1/2)+3*x^5*a*c*d^4*e^2*f*(-d/c)^(1
/2)+15*x^5*b*c^3*d^2*e*f^2*(-d/c)^(1/2)+2*x^5*b*c^2*d^3*e^2*f*(-d
/c)^(1/2)-13*x^3*a*c^3*d^2*e*f^2*(-d/c)^(1/2)+43*x^3*a*c^2*d^3*e^
2*f*(-d/c)^(1/2)+8*x^3*b*c^4*d*e*f^2*(-d/c)^(1/2)-2*EllipticF(x*(
-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*b*c*d^4*e^3*((d*x^2+c)/c)^(1/2)*
((f*x^2+e)/e)^(1/2)+12*x^3*b*c^3*d^2*e^2*f*(-d/c)^(1/2)-34*x*a*c^
4*d*e*f^2*(-d/c)^(1/2)+41*x*a*c^3*d^2*e^2*f*(-d/c)^(1/2)-x*b*c^4*
d*e^2*f*(-d/c)^(1/2)-8*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*
x^4*a*d^5*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+8*EllipticE
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*a*d^5*e^3*((d*x^2+c)/c)^(1/2
)*((f*x^2+e)/e)^(1/2)-8*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))
*a*c^2*d^3*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-6*Elliptic
F(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^5*e*f^2*((d*x^2+c)/c)^(1/2)
*((f*x^2+e)/e)^(1/2)-2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*
b*c^3*d^2*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+8*EllipticE
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d^3*e^3*((d*x^2+c)/c)^(1/2
)*((f*x^2+e)/e)^(1/2)-3*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))
*b*c^5*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticE(
x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*d^2*e^3*((d*x^2+c)/c)^(1/2)
*((f*x^2+e)/e)^(1/2)-8*x^7*a*d^5*e^2*f*(-d/c)^(1/2)+3*x^7*b*c^3*d
^2*f^3*(-d/c)^(1/2)-54*x^5*a*c^3*d^2*f^3*(-d/c)^(1/2)+9*x^5*b*c^4
*d*f^3*(-d/c)^(1/2)+15*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*
x^4*a*c^3*d^2*f^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+30*Elli
pticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c^4*d*f^3*((d*x^2+c)/
c)^(1/2)*((f*x^2+e)/e)^(1/2)+23*EllipticE(x*(-d/c)^(1/2),(c*f/d/e
)^(1/2))*x^4*a*c^2*d^3*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1
/2)-23*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*a*c*d^4*e^2*
f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-3*EllipticE(x*(-d/c)^(1
/2),(c*f/d/e)^(1/2))*x^4*b*c^3*d^2*e*f^2*((d*x^2+c)/c)^(1/2)*((f*
x^2+e)/e)^(1/2)-7*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*b
*c^2*d^3*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-68*Ellipti
cF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c^3*d^2*e*f^2*((d*x^2+c)
/c)^(1/2)*((f*x^2+e)/e)^(1/2)+46*EllipticE(x*(-d/c)^(1/2),(c*f/d/
e)^(1/2))*x^2*a*c^3*d^2*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)-46*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c^2*d^3*e
^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-6*EllipticE(x*(-d/c)
^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^4*d*e*f^2*((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)-14*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2
```

```
*b*c^3*d^2*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+54*Ellip
ticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c^2*d^3*e^2*f*((d*x^2+
c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-12*EllipticF(x*(-d/c)^(1/2),(c*f/
d/e)^(1/2))*x^2*b*c^4*d*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)+2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*b*c*d^4*e^3*
((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-16*EllipticF(x*(-d/c)^(1/
2),(c*f/d/e)^(1/2))*x^2*a*c*d^4*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e
)/e)^(1/2)-4*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^2*
d^3*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+16*EllipticE(x*(-
d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c*d^4*e^3*((d*x^2+c)/c)^(1/2)*(
(f*x^2+e)/e)^(1/2)+4*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^
2*b*c^2*d^3*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-34*Ellipt
icF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^4*d*e*f^2*((d*x^2+c)/c)^(
1/2)*((f*x^2+e)/e)^(1/2)+27*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1
/2))*a*c^3*d^2*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+8*El
lipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^4*d*e^2*f*((d*x^2+c)/
c)^(1/2)*((f*x^2+e)/e)^(1/2)+23*EllipticE(x*(-d/c)^(1/2),(c*f/d/e
)^(1/2))*a*c^4*d*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-23
*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^3*d^2*e^2*f*((d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-7*EllipticE(x*(-d/c)^(1/2),(c*f
/d/e)^(1/2))*b*c^4*d*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2
)+16*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^3*d^2*e^2*
f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-34*EllipticF(x*(-d/c)^(
1/2),(c*f/d/e)^(1/2))*x^4*a*c^2*d^3*e*f^2*((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)+27*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4
*a*c*d^4*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-6*Elliptic
F(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^4*b*c^3*d^2*e*f^2*((d*x^2+c)/
c)^(1/2)*((f*x^2+e)/e)^(1/2)+8*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)
^(1/2))*x^4*b*c^2*d^3*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/
2)-8*x^5*a*d^5*e^3*(-d/c)^(1/2))/(f*x^2+e)^(1/2)/(c*f-d*e)^3/c^3/
(-d/c)^(1/2)/(d*x^2+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + a}{(dx^2 + c)^{\frac{7}{2}} \sqrt{fx^2 + e}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/((d*x^2 + c)^(7/2)*sqrt(f*x^2 + e)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(7/2)*sqrt(f*x^2 + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2 + a}{(d^3x^6 + 3\,cd^2x^4 + 3\,c^2dx^2 + c^3)\sqrt{dx^2 + c}\sqrt{fx^2 + e}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/((d*x^2 + c)^(7/2)*sqrt(f*x^2 + e)),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)/((d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3)
*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/(d*x**2+c)**(7/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + a}{(dx^2 + c)^{\frac{7}{2}} \sqrt{fx^2 + e}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/((d*x^2 + c)^(7/2)*sqrt(f*x^2 + e)),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(7/2)*sqrt(f*x^2 + e)), x)
```

3.42 $\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=501

$$-\frac{\sqrt{e}\sqrt{c+dx^2}\left(10adf(2de-3cf)-b\left(15c^2f^2-41cdef+24d^2e^2\right)\right)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15f^{7/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{\sqrt{c+dx^2}\left(5af\left(3c^2f^2-13cdef+8d^2e^2\right)-2be\left(19c^2f^2-44cdef+24d^2e^2\right)\right)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15\sqrt{e}f^{7/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{x\sqrt{c+dx^2}\left(5af\left(3c^2f^2-13cdef+8d^2e^2\right)-2be\left(19c^2f^2-44cdef+24d^2e^2\right)\right)}{15ef^3\sqrt{e+fx^2}}$$
$$-\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}(be(24de-23cf)-5af(4de-3cf))}{15ef^3}$$
$$+\frac{dx\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}(6be-5af)}{5ef^2}-\frac{x\left(c+dx^2\right)^{5/2}(be-af)}{ef\sqrt{e+fx^2}}$$

```
[Out] -((5*a*f*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2) - 2*b*e*(24*d^2*e^2
 - 44*c*d*e*f + 19*c^2*f^2))*x*Sqrt[c + d*x^2])/(15*e*f^3*Sqrt[e
+ f*x^2]) - ((b*e - a*f)*x*(c + d*x^2)^(5/2))/(e*f*Sqrt[e + f*x^2
]) - (d*(b*e*(24*d*e - 23*c*f) - 5*a*f*(4*d*e - 3*c*f))*x*Sqrt[c
+ d*x^2]*Sqrt[e + f*x^2])/(15*e*f^3) + (d*(6*b*e - 5*a*f)*x*(c +
d*x^2)^(3/2)*Sqrt[e + f*x^2])/(5*e*f^2) + ((5*a*f*(8*d^2*e^2 - 13
*c*d*e*f + 3*c^2*f^2) - 2*b*e*(24*d^2*e^2 - 44*c*d*e*f + 19*c^2*f
^2))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (
d*e)/(c*f)])/(15*Sqrt[e]*f^(7/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x
^2))]*Sqrt[e + f*x^2]) - (Sqrt[e]*(10*a*d*f*(2*d*e - 3*c*f) - b*(
24*d^2*e^2 - 41*c*d*e*f + 15*c^2*f^2))*Sqrt[c + d*x^2]*EllipticF[
ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*f^(7/2)*Sqrt[(
e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 1.69633, antiderivative size = 501, normalized size of antiderivative = 1., number of

steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{\sqrt{e}\sqrt{c+dx^2}\left(10adf(2de-3cf)-b\left(15c^2f^2-41cdef+24d^2e^2\right)\right)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15f^{7/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{\sqrt{c+dx^2}\left(5af\left(3c^2f^2-13cdef+8d^2e^2\right)-2be\left(19c^2f^2-44cdef+24d^2e^2\right)\right)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15\sqrt{e}f^{7/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{x\sqrt{c+dx^2}\left(5af\left(3c^2f^2-13cdef+8d^2e^2\right)-2be\left(19c^2f^2-44cdef+24d^2e^2\right)\right)}{15ef^3\sqrt{e+fx^2}}$$
$$-\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}(be(24de-23cf)-5af(4de-3cf))}{15ef^3}$$
$$+\frac{dx\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}(6be-5af)}{5ef^2}-\frac{x\left(c+dx^2\right)^{5/2}(be-af)}{ef\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*(c + d*x^2)^(5/2))/(e + f*x^2)^(3/2),x]
```

```
[Out] -((5*a*f*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2) - 2*b*e*(24*d^2*e^2
 - 44*c*d*e*f + 19*c^2*f^2))*x*Sqrt[c + d*x^2])/(15*e*f^3*Sqrt[e
+ f*x^2]) - ((b*e - a*f)*x*(c + d*x^2)^(5/2))/(e*f*Sqrt[e + f*x^2
]) - (d*(b*e*(24*d*e - 23*c*f) - 5*a*f*(4*d*e - 3*c*f))*x*Sqrt[c
+ d*x^2]*Sqrt[e + f*x^2])/(15*e*f^3) + (d*(6*b*e - 5*a*f)*x*(c +
d*x^2)^(3/2)*Sqrt[e + f*x^2])/(5*e*f^2) + ((5*a*f*(8*d^2*e^2 - 13
*c*d*e*f + 3*c^2*f^2) - 2*b*e*(24*d^2*e^2 - 44*c*d*e*f + 19*c^2*f
^2))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (
d*e)/(c*f)])/(15*Sqrt[e]*f^(7/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x
^2))]*Sqrt[e + f*x^2]) - (Sqrt[e]*(10*a*d*f*(2*d*e - 3*c*f) - b*(
24*d^2*e^2 - 41*c*d*e*f + 15*c^2*f^2))*Sqrt[c + d*x^2]*EllipticF[
ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*f^(7/2)*Sqrt[(
e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)**(5/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.05784, size = 369, normalized size = 0.74

$$\frac{-ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(cf-de)\left(5adf(9cf-8de)+b\left(15c^2f^2-64cdef+48d^2e^2\right)\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-ide\sqrt{\frac{dx^2}{c}+\cdots}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*(c + d*x^2)^(5/2))/(e + f*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d/c]*f*x*(c + d*x^2)*(5*a*f*(-6*c*d*e*f + 3*c^2*f^2 + d^2*e
*(4*e + f*x^2)) + b*e*(-15*c^2*f^2 + c*d*f*(41*e + 11*f*x^2) - 3*
d^2*(8*e^2 + 2*e*f*x^2 - f^2*x^4))) - I*d*e*(-5*a*f*(8*d^2*e^2 -
13*c*d*e*f + 3*c^2*f^2) + 2*b*e*(24*d^2*e^2 - 44*c*d*e*f + 19*c^2
*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSin
h[Sqrt[d/c]*x], (c*f)/(d*e)] - I*e*(-(d*e) + c*f)*(5*a*d*f*(-8*d*
e + 9*c*f) + b*(48*d^2*e^2 - 64*c*d*e*f + 15*c^2*f^2))*Sqrt[1 + (
d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (
c*f)/(d*e)])/(15*Sqrt[d/c]*e*f^4*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [B] time = 0.069, size = 1169, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x)
```

```
[Out] 1/15*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*(-88*((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d^
2*e^3*f-48*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-
d/c)^(1/2),(c*f/d/e)^(1/2))*b*d^3*e^4+48*((d*x^2+c)/c)^(1/2)*((f*
x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d^3*e
^4+3*(-d/c)^(1/2)*x^7*b*d^3*e*f^3+5*(-d/c)^(1/2)*x^5*a*d^3*e*f^3-
6*(-d/c)^(1/2)*x^5*b*d^3*e^2*f^2+15*(-d/c)^(1/2)*x^3*a*c^2*d*f^4+
20*(-d/c)^(1/2)*x^3*a*d^3*e^2*f^2-24*(-d/c)^(1/2)*x^3*b*d^3*e^3*f
-15*(-d/c)^(1/2)*x*b*c^3*e*f^3-30*(-d/c)^(1/2)*x*a*c^2*d*e*f^3+20
*(-d/c)^(1/2)*x*a*c*d^2*e^2*f^2+40*((d*x^2+c)/c)^(1/2)*((f*x^2+e)
/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^3*e^3*f+1
5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2
),(c*f/d/e)^(1/2))*b*c^3*e*f^3+41*(-d/c)^(1/2)*x*b*c^2*d*e^2*f^2-
24*(-d/c)^(1/2)*x*b*c*d^2*e^3*f-40*((d*x^2+c)/c)^(1/2)*((f*x^2+e)
/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^3*e^3*f+4
5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2
),(c*f/d/e)^(1/2))*a*c^2*d*e*f^3+38*((d*x^2+c)/c)^(1/2)*((f*x^2+e
)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*d*e^2*
f^2-15*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)
^(1/2),(c*f/d/e)^(1/2))*a*c^2*d*e*f^3+65*((d*x^2+c)/c)^(1/2)*((f*
```

```
x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d^2
*e^2*f^2-85*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(
-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d^2*e^2*f^2-79*((d*x^2+c)/c)^(1/
2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*
b*c^2*d*e^2*f^2+112*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellip
ticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d^2*e^3*f+14*(-d/c)^(1/2
)*x^5*b*c*d^2*e*f^3-25*(-d/c)^(1/2)*x^3*a*c*d^2*e*f^3-4*(-d/c)^(1
/2)*x^3*b*c^2*d*e*f^3+35*(-d/c)^(1/2)*x^3*b*c*d^2*e^2*f^2+15*x*a*
c^3*f^4*(-d/c)^(1/2))/f^4/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)/(-d/c)^(1
/2)/e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)(dx^2+c)^{\frac{5}{2}}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^(5/2)/(f*x^2 + e)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(5/2)/(f*x^2 + e)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bd^2x^6+\left(2\,bcd+ad^2\right)x^4+ac^2+\left(bc^2+2\,acd\right)x^2\right)\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^(5/2)/(f*x^2 + e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*d^2*x^6 + (2*b*c*d + a*d^2)*x^4 + a*c^2 + (b*c^2 + 2*
a*c*d)*x^2)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(5/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)\left(dx^2 + c\right)^{\frac{5}{2}}}{\left(fx^2 + e\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*(d*x^2 + c)^(5/2)/(f*x^2 + e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(5/2)/(f*x^2 + e)^(3/2), x)
```

3.43 $\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=358

$$-\frac{\sqrt{e}\sqrt{c+dx^2}(-3adf-3bcf+4bde)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3f^{5/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{\sqrt{c+dx^2}(be(8de-7cf)-3af(2de-cf))E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3\sqrt{e}f^{5/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}(4be-3af)}{3ef^2}-\frac{x\sqrt{c+dx^2}(be(8de-7cf)-3af(2de-cf))}{3ef^2\sqrt{e+fx^2}}-\frac{x\left(c+dx^2\right)^{3/2}(be-af)}{ef\sqrt{e+fx^2}}$$

```
[Out] -((b*e*(8*d*e - 7*c*f) - 3*a*f*(2*d*e - c*f))*x*Sqrt[c + d*x^2])/
(3*e*f^2*Sqrt[e + f*x^2]) - ((b*e - a*f)*x*(c + d*x^2)^(3/2))/(e*
f*Sqrt[e + f*x^2]) + (d*(4*b*e - 3*a*f)*x*Sqrt[c + d*x^2]*Sqrt[e
+ f*x^2])/(3*e*f^2) + ((b*e*(8*d*e - 7*c*f) - 3*a*f*(2*d*e - c*f)
)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e
)/(c*f)])/(3*Sqrt[e]*f^(5/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))
]*Sqrt[e + f*x^2]) - (Sqrt[e]*(4*b*d*e - 3*b*c*f - 3*a*d*f)*Sqrt[
c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)
])/(3*f^(5/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^
2])
```

Rubi [A] time = 1.09576, antiderivative size = 358, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{\sqrt{e}\sqrt{c+dx^2}(-3adf-3bcf+4bde)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3f^{5/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{\sqrt{c+dx^2}(be(8de-7cf)-3af(2de-cf))E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3\sqrt{e}f^{5/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}(4be-3af)}{3ef^2}-\frac{x\sqrt{c+dx^2}(be(8de-7cf)-3af(2de-cf))}{3ef^2\sqrt{e+fx^2}}-\frac{x\left(c+dx^2\right)^{3/2}(be-af)}{ef\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(3/2),x]
```

```
[Out] -((b*e*(8*d*e - 7*c*f) - 3*a*f*(2*d*e - c*f))*x*Sqrt[c + d*x^2])/
(3*e*f^2*Sqrt[e + f*x^2]) - ((b*e - a*f)*x*(c + d*x^2)^(3/2))/(e*
f*Sqrt[e + f*x^2]) + (d*(4*b*e - 3*a*f)*x*Sqrt[c + d*x^2]*Sqrt[e
+ f*x^2])/(3*e*f^2) + ((b*e*(8*d*e - 7*c*f) - 3*a*f*(2*d*e - c*f)
)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e
)/(c*f)])/(3*Sqrt[e]*f^(5/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))
]*Sqrt[e + f*x^2]) - (Sqrt[e]*(4*b*d*e - 3*b*c*f - 3*a*d*f)*Sqrt[
c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)
])/(3*f^(5/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^
2])
```

Rubi in Sympy [A] time = 113.906, size = 345, normalized size = 0.96

$$\frac{\sqrt{c}\sqrt{d}\sqrt{e+fx^2}\left(3acf^2-6adef-7bcef+8bde^2\right)E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{3ef^3\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}} - \frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}\,(3af-4be)}{3ef^2} - \frac{dx\sqrt{e+fx^2}\left(3acf^2-6adef-7bcef+8bde^2\right)}{3ef^3\sqrt{c+dx^2}} + \frac{\sqrt{e}\sqrt{c+dx^2}\,(3adf+3bcf-4bde)\,F\left(\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{3f^{\frac{5}{2}}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{x\left(c+dx^2\right)^{\frac{3}{2}}(af-be)}{ef\sqrt{e+fx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)**(3/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] sqrt(c)*sqrt(d)*sqrt(e + f*x**2)*(3*a*c*f**2 - 6*a*d*e*f - 7*b*c*
e*f + 8*b*d*e**2)*elliptic_e(atan(sqrt(d)*x/sqrt(c)), -c*f/(d*e)
+ 1)/(3*e*f**3*sqrt(c*(e + f*x**2)/(e*(c + d*x**2)))*sqrt(c + d*x
**2)) - d*x*sqrt(c + d*x**2)*sqrt(e + f*x**2)*(3*a*f - 4*b*e)/(3*
e*f**2) - d*x*sqrt(e + f*x**2)*(3*a*c*f**2 - 6*a*d*e*f - 7*b*c*e*
f + 8*b*d*e**2)/(3*e*f**3*sqrt(c + d*x**2)) + sqrt(e)*sqrt(c + d*
x**2)*(3*a*d*f + 3*b*c*f - 4*b*d*e)*elliptic_f(atan(sqrt(f)*x/sqr
t(e)), 1 - d*e/(c*f))/(3*f**(5/2)*sqrt(e*(c + d*x**2)/(c*(e + f*x
**2)))*sqrt(e + f*x**2)) + x*(c + d*x**2)**(3/2)*(a*f - b*e)/(e*f
*sqrt(e + f*x**2))
```

Mathematica [C] time = 1.30167, size = 260, normalized size = 0.73

$$\frac{fx\sqrt{\frac{d}{c}}\left(c+dx^2\right)\left(3af(cf-de)+be\left(-3cf+4de+dfx^2\right)\right)-ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(cf-de)(6adf+3bcf-8bde)F\left(i\sinh\right.}{3ef^3\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d/c]*f*x*(c + d*x^2)*(3*a*f*(-(d*e) + c*f) + b*e*(4*d*e - 3
*c*f + d*f*x^2)) - I*d*e*(-3*a*f*(-2*d*e + c*f) + b*e*(-8*d*e + 7
*c*f))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSin
h[Sqrt[d/c]*x], (c*f)/(d*e)] - I*e*(-(d*e) + c*f)*(-8*b*d*e + 3*b
*c*f + 6*a*d*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF
[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(3*Sqrt[d/c]*e*f^3*Sqrt[c
+ d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.038, size = 750, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

```
[Out] 1/3*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*((-d/c)^(1/2)*x^5*b*d^2*e*f^2
+3*(-d/c)^(1/2)*x^3*a*c*d*f^3-3*(-d/c)^(1/2)*x^3*a*d^2*e*f^2-2*(-
d/c)^(1/2)*x^3*b*c*d*e*f^2+4*(-d/c)^(1/2)*x^3*b*d^2*e^2*f+6*((d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/
d/e)^(1/2))*a*c*d*e*f^2-6*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)
*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^2*e^2*f+3*((d*x^2+
c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e
)^(1/2))*b*c^2*e*f^2-11*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*E
llipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d*e^2*f+8*((d*x^2+c)
/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^
(1/2))*b*d^2*e^3-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellipt
icE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d*e*f^2+6*((d*x^2+c)/c)^(
1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2)
)*a*d^2*e^2*f+7*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d*e^2*f-8*((d*x^2+c)/c)^(1/2
)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b
*d^2*e^3+3*x*a*c^2*f^3*(-d/c)^(1/2)-3*(-d/c)^(1/2)*x*a*c*d*e*f^2-
3*(-d/c)^(1/2)*x*b*c^2*e*f^2+4*(-d/c)^(1/2)*x*b*c*d*e^2*f)/(d*f*x
^4+c*f*x^2+d*e*x^2+c*e)/f^3/e/(-d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)\left(dx^2 + c\right)^{\frac{3}{2}}}{\left(fx^2 + e\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bdx^4 + (bc + ad)x^2 + ac\right)\sqrt{dx^2 + c}}{(fx^2 + e)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*d*x^4 + (b*c + a*d)*x^2 + a*c)*sqrt(d*x^2 + c)/(f*x^2
 + e)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2\right)\left(c + dx^2\right)^{\frac{3}{2}}}{(e + fx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(3/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)*(c + d*x**2)**(3/2)/(e + f*x**2)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)\left(dx^2 + c\right)^{\frac{3}{2}}}{(fx^2 + e)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^(3/2), x)
```

3.44 $\int \frac{(a+bx^2)\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=258

$$-\frac{\sqrt{c+dx^2}(2be-af)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}f^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x\sqrt{c+dx^2}(be-af)}{ef\sqrt{e+fx^2}}+\frac{x\sqrt{c+dx^2}(2be-af)}{ef\sqrt{e+fx^2}}+\frac{b\sqrt{e}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{f^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

```
[Out] -(((b*e - a*f)*x*Sqrt[c + d*x^2])/(e*f*Sqrt[e + f*x^2])) + ((2*b*
e - a*f)*x*Sqrt[c + d*x^2])/(e*f*Sqrt[e + f*x^2]) - ((2*b*e - a*f
)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e
)/(c*f)])/(Sqrt[e]*f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*
Sqrt[e + f*x^2]) + (b*Sqrt[e]*Sqrt[c + d*x^2]*EllipticF[ArcTan[(S
qrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(f^(3/2)*Sqrt[(e*(c + d*x^2
))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.55995, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{\sqrt{c+dx^2}(2be-af)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}f^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x\sqrt{c+dx^2}(be-af)}{ef\sqrt{e+fx^2}}+\frac{x\sqrt{c+dx^2}(2be-af)}{ef\sqrt{e+fx^2}}+\frac{b\sqrt{e}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{f^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```


```
[Out] -(((b*e - a*f)*x*Sqrt[c + d*x^2])/(e*f*Sqrt[e + f*x^2])) + ((2*b*
e - a*f)*x*Sqrt[c + d*x^2])/(e*f*Sqrt[e + f*x^2]) - ((2*b*e - a*f
)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e
)/(c*f)])/(Sqrt[e]*f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*
Sqrt[e + f*x^2]) + (b*Sqrt[e]*Sqrt[c + d*x^2]*EllipticF[ArcTan[(S
qrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(f^(3/2)*Sqrt[(e*(c + d*x^2
))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi in Sympy [A] time = 73.7741, size = 230, normalized size = 0.89

$$\frac{b\sqrt{e}\sqrt{c+dx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{f^{\frac{3}{2}}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}+\frac{\sqrt{c}\sqrt{d}\sqrt{e+fx^2}\,(af-2be)\,E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{ef^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}}-\frac{dx\sqrt{e+fx^2}\,(af-2be)}{ef^2\sqrt{c+dx^2}}+\frac{x\sqrt{c+dx^2}\,(af-be)}{ef\sqrt{e+fx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] b*sqrt(e)*sqrt(c + d*x**2)*elliptic_f(atan(sqrt(f)*x/sqrt(e)), 1
- d*e/(c*f))/(f**(3/2)*sqrt(e*(c + d*x**2)/(c*(e + f*x**2)))*sqrt
(e + f*x**2)) + sqrt(c)*sqrt(d)*sqrt(e + f*x**2)*(a*f - 2*b*e)*el
liptic_e(atan(sqrt(d)*x/sqrt(c)), -c*f/(d*e) + 1)/(e*f**2*sqrt(c*
(e + f*x**2)/(e*(c + d*x**2)))*sqrt(c + d*x**2)) - d*x*sqrt(e + f
*x**2)*(a*f - 2*b*e)/(e*f**2*sqrt(c + d*x**2)) + x*sqrt(c + d*x**
2)*(a*f - b*e)/(e*f*sqrt(e + f*x**2))
```

Mathematica [C] time = 0.593644, size = 208, normalized size = 0.81

$$\frac{fx\sqrt{\frac{d}{c}}\left(c+dx^2\right)(af-be)-ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(adf+bcf-2bde)F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-ide\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(2b}{ef^2\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d/c]*f*(-(b*e) + a*f)*x*(c + d*x^2) - I*d*e*(2*b*e - a*f)*S
qrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d
/c]*x], (c*f)/(d*e)] - I*e*(-2*b*d*e + b*c*f + a*d*f)*Sqrt[1 + (d
*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c
*f)/(d*e)])/(Sqrt[d/c]*e*f^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.035, size = 393, normalized size = 1.5

$$\frac{1}{(dfx^4+cfx^2+dex^2+ce)f^2e}\sqrt{dx^2+c}\sqrt{fx^2+e}\left(x^3adf^2\sqrt{-\frac{d}{c}}-\sqrt{-\frac{d}{c}}x^3bdef+EllipticF\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)\sqrt{\frac{dx^2+c}{c}}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

```
[Out] (d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*(x^3*a*d*f^2*(-d/c)^(1/2)-(-d/c)^
(1/2)*x^3*b*d*e*f+EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*((d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*a*d*e*f+((d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*e
*f-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(
1/2),(c*f/d/e)^(1/2))*b*d*e^2-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e*f+2*((d*x^2+
c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e
)^(1/2))*b*d*e^2+x*a*c*f^2*(-d/c)^(1/2)-(-d/c)^(1/2)*x*b*c*e*f)/(
d*f*x^4+c*f*x^2+d*e*x^2+c*e)/f^2/e/(-d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)\sqrt{dx^2+c}}{\left(fx^2+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)\sqrt{dx^2+c}}{\left(fx^2+e\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)\sqrt{c+dx^2}}{\left(e+fx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)

[Out] Integral((a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)\sqrt{dx^2 + c}}{(fx^2 + e)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2),x, algorithm="giac")

[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

3.45 $\int \frac{a+bx^2}{\sqrt{c+dx^2}(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=209

$$\frac{\sqrt{c+dx^2}(be-af)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{e}\sqrt{c+dx^2}(bc-ad)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

```
[Out] ((b*e - a*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]
], 1 - (d*e)/(c*f)])/(Sqrt[e]*Sqrt[f]*(d*e - c*f)*Sqrt[(e*(c + d*
x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - ((b*c - a*d)*Sqrt[e]*Sq
rt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c
*f)])/(c*Sqrt[f]*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))
]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.312203, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{\sqrt{c+dx^2}(be-af)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{e}\sqrt{c+dx^2}(bc-ad)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

```
[Out] ((b*e - a*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]
], 1 - (d*e)/(c*f)])/(Sqrt[e]*Sqrt[f]*(d*e - c*f)*Sqrt[(e*(c + d*
x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - ((b*c - a*d)*Sqrt[e]*Sq
rt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c
*f)])/(c*Sqrt[f]*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))
]*Sqrt[e + f*x^2])
```

Rubi in Sympy [A] time = 37.0874, size = 173, normalized size = 0.83

$$-\frac{\sqrt{c}\sqrt{e+fx^2}\,(ad-bc)\,F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{\sqrt{d}e\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}\,(cf-de)}+\frac{\sqrt{c+dx^2}\,(af-be)\,E\left(\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}\,(cf-de)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] -sqrt(c)*sqrt(e + f*x**2)*(a*d - b*c)*elliptic_f(atan(sqrt(d)*x/s
qrt(c)), -c*f/(d*e) + 1)/(sqrt(d)*e*sqrt(c*(e + f*x**2)/(e*(c + d
*x**2)))*sqrt(c + d*x**2)*(c*f - d*e)) + sqrt(c + d*x**2)*(a*f -
b*e)*elliptic_e(atan(sqrt(f)*x/sqrt(e)), 1 - d*e/(c*f))/(sqrt(e)*
sqrt(f)*sqrt(e*(c + d*x**2)/(c*(e + f*x**2)))*sqrt(e + f*x**2)*(c
*f - d*e))
```

Mathematica [C] time = 0.608327, size = 212, normalized size = 1.01

$$\frac{fx\sqrt{\frac{d}{c}}\left(c+dx^2\right)(af-be)-ide\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(be-af)E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-ibe\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(cf-de)F\left(\right.}{ef\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}(cf-de)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d/c]*f*(-(b*e) + a*f)*x*(c + d*x^2) - I*d*e*(b*e - a*f)*Sqr
t[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c
]*x], (c*f)/(d*e)] - I*b*e*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqr
t[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/
(Sqrt[d/c]*e*f*(-(d*e) + c*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.044, size = 349, normalized size = 1.7

$$\frac{1}{ef(cf-de)(dfx^4+cfx^2+dex^2+ce)}\left(x^3adf^2\sqrt{-\frac{d}{c}}-\sqrt{-\frac{d}{c}}x^3bdef+\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{fx^2+e}{e}}EllipticF\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)bc\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

```
[Out] (x^3*a*d*f^2*(-d/c)^(1/2)-(-d/c)^(1/2)*x^3*b*d*e*f+((d*x^2+c)/c)^
(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2
))*b*c*e*f-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-
d/c)^(1/2),(c*f/d/e)^(1/2))*b*d*e^2-((d*x^2+c)/c)^(1/2)*((f*x^2+e
)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e*f+((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f
/d/e)^(1/2))*b*d*e^2+x*a*c*f^2*(-d/c)^(1/2)-(-d/c)^(1/2)*x*b*c*e*
f)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/(-d/c)^(1/2)/e/f/(c*f-d*e)/(d*
f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + a}{\sqrt{dx^2 + c}(fx^2 + e)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2 + a}{\sqrt{dx^2 + c}(fx^2 + e)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + bx^2}{\sqrt{c + dx^2}\,(e + fx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)/(sqrt(c + d*x**2)*(e + f*x**2)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + a}{\sqrt{dx^2 + c}(fx^2 + e)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)),x, algorithm="giac")

[Out] integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

3.46 $\int \frac{a+bx^2}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=272

$$-\frac{x(bc-ad)}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)}+\frac{\sqrt{e}\sqrt{c+dx^2}(-2adf+bcf+bde)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{f}\sqrt{c+dx^2}(-acf-ade+2bce)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{e}\sqrt{e+fx^2}(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

```
[Out] -(((b*c - a*d)*x)/(c*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
) - (Sqrt[f]*(2*b*c*e - a*d*e - a*c*f)*Sqrt[c + d*x^2]*EllipticE[
ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[e]*(d*e -
c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (
Sqrt[e]*(b*d*e + b*c*f - 2*a*d*f)*Sqrt[c + d*x^2]*EllipticF[ArcTa
n[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[f]*(d*e - c*f)^
2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.641017, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$-\frac{x(bc-ad)}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)}+\frac{\sqrt{e}\sqrt{c+dx^2}(-2adf+bcf+bde)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{f}\sqrt{c+dx^2}(-acf-ade+2bce)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{e}\sqrt{e+fx^2}(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```



```
[Out] -(((b*c - a*d)*x)/(c*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
) - (Sqrt[f]*(2*b*c*e - a*d*e - a*c*f)*Sqrt[c + d*x^2]*EllipticE[
ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[e]*(d*e -
c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (
Sqrt[e]*(b*d*e + b*c*f - 2*a*d*f)*Sqrt[c + d*x^2]*EllipticF[ArcTa
n[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[f]*(d*e - c*f)^
2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi in Sympy [A] time = 71.8301, size = 236, normalized size = 0.87

$$-\frac{\sqrt{e}\sqrt{c+dx^2}\,(2adf-bcf-bde)\,F\left(\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}\,(cf-de)^2}-\frac{x\,(ad-bc)}{c\sqrt{c+dx^2}\sqrt{e+fx^2}\,(cf-de)}+\frac{\sqrt{f}\sqrt{c+dx^2}\,(acf+ade-2bce)\,E\left(\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{c\sqrt{e}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}\,(cf-de)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/(d*x**2+c)**(3/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] -sqrt(e)*sqrt(c + d*x**2)*(2*a*d*f - b*c*f - b*d*e)*elliptic_f(at
an(sqrt(f)*x/sqrt(e)), 1 - d*e/(c*f))/(c*sqrt(f)*sqrt(e*(c + d*x*
*2)/(c*(e + f*x**2)))*sqrt(e + f*x**2)*(c*f - d*e)**2) - x*(a*d -
 b*c)/(c*sqrt(c + d*x**2)*sqrt(e + f*x**2)*(c*f - d*e)) + sqrt(f)
*sqrt(c + d*x**2)*(a*c*f + a*d*e - 2*b*c*e)*elliptic_e(atan(sqrt(
f)*x/sqrt(e)), 1 - d*e/(c*f))/(c*sqrt(e)*sqrt(e*(c + d*x**2)/(c*(
e + f*x**2)))*sqrt(e + f*x**2)*(c*f - d*e)**2)
```

Mathematica [C] time = 1.25059, size = 262, normalized size = 0.96

$$\frac{\sqrt{\frac{d}{c}}\left(x\sqrt{\frac{d}{c}}\left(a\left(c^2f^2+cdf^2x^2+d^2e\left(e+fx^2\right)\right)-bce\left(cf+d\left(e+2fx^2\right)\right)\right)-ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(bc-ad)(cf-de)F\left(i\,\mathrm{si}\right.\right.}{de\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d/c]*(Sqrt[d/c]*x*(a*(c^2*f^2 + c*d*f^2*x^2 + d^2*e*(e + f*
x^2)) - b*c*e*(c*f + d*(e + 2*f*x^2))) - I*d*e*(2*b*c*e - a*(d*e
+ c*f))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSi
nh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*(b*c - a*d)*e*(-(d*e) + c*f)*Sq
rt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/
c]*x], (c*f)/(d*e)]))/(d*e*(d*e - c*f)^2*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2])
```

Maple [A] time = 0.043, size = 581, normalized size = 2.1

$$\frac{1}{ce\,(cf-de)^2\,(dfx^4+cfx^2+dex^2+ce)}\left(x^3acdf^2\sqrt{-\frac{d}{c}}+x^3ad^2ef\sqrt{-\frac{d}{c}}-2\,x^3bcdef\sqrt{-\frac{d}{c}}-EllipticF\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)ac\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

```
[Out] (x^3*a*c*d*f^2*(-d/c)^(1/2)+x^3*a*d^2*e*f*(-d/c)^(1/2)-2*x^3*b*c*
d*e*f*(-d/c)^(1/2)-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*
d*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticF(x*(-d/c)^
(1/2),(c*f/d/e)^(1/2))*a*d^2*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e
)^(1/2)+EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^2*e*f*((d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticF(x*(-d/c)^(1/2),(c*f/
d/e)^(1/2))*b*c*d*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-Ell
ipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d*e*f*((d*x^2+c)/c)^(1
/2)*((f*x^2+e)/e)^(1/2)-EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))
*a*d^2*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticE(x*
(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*d*e^2*((d*x^2+c)/c)^(1/2)*((f*x
^2+e)/e)^(1/2)+x*a*c^2*f^2*(-d/c)^(1/2)+x*a*d^2*e^2*(-d/c)^(1/2)-
x*b*c^2*e*f*(-d/c)^(1/2)-x*b*c*d*e^2*(-d/c)^(1/2))*(f*x^2+e)^(1/2
)*(d*x^2+c)^(1/2)/e/c/(-d/c)^(1/2)/(c*f-d*e)^2/(d*f*x^4+c*f*x^2+d
*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + a}{(dx^2 + c)^{\frac{3}{2}}(fx^2 + e)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)),x, algorithm="maxima
```

```
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2 + a}{(dfx^4 + (de + cf)x^2 + ce)\sqrt{dx^2 + c}\sqrt{fx^2 + e}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)),x, algorithm="fricas
```

```
[Out] integral((b*x^2 + a)/((d*f*x^4 + (d*e + c*f)*x^2 + c*e)*sqrt(d*x^
2 + c)*sqrt(f*x^2 + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/(d*x**2+c)**(3/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + a}{(dx^2 + c)^{\frac{3}{2}}(fx^2 + e)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

3.47 $\int \frac{a+bx^2}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=375

$$\frac{\sqrt{f}\sqrt{c+dx^2}\left(a\left(-3c^2f^2-7cdef+2d^2e^2\right)+bce(7cf+de)\right)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2\sqrt{e}\sqrt{e+fx^2}(de-cf)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x(2ad(de-3cf)+bc(3cf+de))}{3c^2\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)^2}-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(ad(de-9cf)+bc(3cf+5de))F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2\sqrt{e+fx^2}(de-cf)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x(bc-ad)}{3c\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}(de-cf)}$$

```
[Out] -((b*c - a*d)*x)/(3*c*(d*e - c*f)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^
2]) + ((2*a*d*(d*e - 3*c*f) + b*c*(d*e + 3*c*f))*x)/(3*c^2*(d*e -
 c*f)^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]) + (Sqrt[f]*(b*c*e*(d*e +
 7*c*f) + a*(2*d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2))*Sqrt[c + d*x^2]*
EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*S
qrt[e]*(d*e - c*f)^3*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
 + f*x^2]) - (Sqrt[e]*Sqrt[f]*(a*d*(d*e - 9*c*f) + b*c*(5*d*e + 3
*c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 -
 (d*e)/(c*f)])/(3*c^2*(d*e - c*f)^3*Sqrt[(e*(c + d*x^2))/(c*(e +
f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 1.12029, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{\sqrt{f}\sqrt{c+dx^2}\left(a\left(-3c^2f^2-7cdef+2d^2e^2\right)+bce(7cf+de)\right)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2\sqrt{e}\sqrt{e+fx^2}(de-cf)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x(2ad(de-3cf)+bc(3cf+de))}{3c^2\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)^2}-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(ad(de-9cf)+bc(3cf+5de))F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2\sqrt{e+fx^2}(de-cf)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x(bc-ad)}{3c\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}(de-cf)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] -((b*c - a*d)*x)/(3*c*(d*e - c*f)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^
2]) + ((2*a*d*(d*e - 3*c*f) + b*c*(d*e + 3*c*f))*x)/(3*c^2*(d*e -
 c*f)^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]) + (Sqrt[f]*(b*c*e*(d*e +
 7*c*f) + a*(2*d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2))*Sqrt[c + d*x^2]*
EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*S
qrt[e]*(d*e - c*f)^3*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
 + f*x^2]) - (Sqrt[e]*Sqrt[f]*(a*d*(d*e - 9*c*f) + b*c*(5*d*e + 3
*c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 -
 (d*e)/(c*f)])/(3*c^2*(d*e - c*f)^3*Sqrt[(e*(c + d*x^2))/(c*(e +
f*x^2))]*Sqrt[e + f*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/(d*x**2+c)**(5/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 3.82665, size = 428, normalized size = 1.14

$$\frac{-ide\left(c+dx^2\right)\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(a\left(-3c^2f^2-7cdef+2d^2e^2\right)+bce(7cf+de)\right)E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+x\sqrt{\frac{d}{c}}\left(a\left(3c^4f\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d/c]*x*(-(b*c*e*(3*c^3*f^2 + d^3*e*x^2*(e + f*x^2) + c*d^2*
f*x^2*(4*e + 7*f*x^2) + c^2*d*f*(5*e + 11*f*x^2))) + a*(3*c^4*f^3
 + 6*c^3*d*f^3*x^2 - 2*d^4*e^2*x^2*(e + f*x^2) + c^2*d^2*f*(8*e^2
 + 8*e*f*x^2 + 3*f^2*x^4) + c*d^3*e*(-3*e^2 + 4*e*f*x^2 + 7*f^2*x
^4))) - I*d*e*(b*c*e*(d*e + 7*c*f) + a*(2*d^2*e^2 - 7*c*d*e*f - 3
*c^2*f^2))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*El
lipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*e*(-(d*e) + c*f)
*(2*a*d*(d*e - 3*c*f) + b*c*(d*e + 3*c*f))*(c + d*x^2)*Sqrt[1 + (
d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (
c*f)/(d*e)])/(3*c^2*Sqrt[d/c]*e*(-(d*e) + c*f)^3*(c + d*x^2)^(3/2
)*Sqrt[e + f*x^2])
```

Maple [B] time = 0.056, size = 1742, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x)
```

```
[Out] -1/3*(-2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*d^4*e^3*
((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-7*x^5*a*c*d^3*e*f^2*(-d/c
)^(1/2)+7*x^5*b*c^2*d^2*e*f^2*(-d/c)^(1/2)+x^5*b*c*d^3*e^2*f*(-d/
c)^(1/2)-3*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^3*d*
e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticF(x*(-d/c
)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c^2*d^2*e^2*f*((d*x^2+c)/c)^(1/2)*
((f*x^2+e)/e)^(1/2)+3*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x
^2*a*c^2*d^2*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+7*Elli
pticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c*d^3*e^2*f*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-7*EllipticE(x*(-d/c)^(1/2),(c*f/d/
e)^(1/2))*x^2*b*c^2*d^2*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)+6*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*c^2*d^2*e*
f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-8*EllipticF(x*(-d/c)^
(1/2),(c*f/d/e)^(1/2))*x^2*a*c*d^3*e^2*f*((d*x^2+c)/c)^(1/2)*((f*
x^2+e)/e)^(1/2)+EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*b*c
*d^3*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-3*x^5*a*c^2*d^2*
f^3*(-d/c)^(1/2)+2*x^5*a*d^4*e^2*f*(-d/c)^(1/2)-6*x^3*a*c^3*d*f^3
*(-d/c)^(1/2)+x^3*b*c*d^3*e^3*(-d/c)^(1/2)+3*x*a*c*d^3*e^3*(-d/c)
^(1/2)+3*x*b*c^4*e*f^2*(-d/c)^(1/2)-3*x*a*c^4*f^3*(-d/c)^(1/2)-8*
x^3*a*c^2*d^2*e*f^2*(-d/c)^(1/2)-4*x^3*a*c*d^3*e^2*f*(-d/c)^(1/2)
+11*x^3*b*c^3*d*e*f^2*(-d/c)^(1/2)+4*x^3*b*c^2*d^2*e^2*f*(-d/c)^(
1/2)-8*x*a*c^2*d^2*e^2*f*(-d/c)^(1/2)+5*x*b*c^3*d*e^2*f*(-d/c)^(1
/2)+2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*d^4*e^3*((d
*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticF(x*(-d/c)^(1/2),(
c*f/d/e)^(1/2))*a*c*d^3*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/
2)-3*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^4*e*f^2*((d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticF(x*(-d/c)^(1/2),(c*f/d
/e)^(1/2))*b*c^2*d^2*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-
2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d^3*e^3*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticE(x*(-d/c)^(1/2),(c*f/d/e)
^(1/2))*b*c^2*d^2*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*x
^3*a*d^4*e^3*(-d/c)^(1/2)-EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2
))*x^2*b*c*d^3*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+6*Elli
pticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^3*d*e*f^2*((d*x^2+c)/c)
^(1/2)*((f*x^2+e)/e)^(1/2)-8*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(
1/2))*a*c^2*d^2*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*E
llipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c^3*d*e^2*f*((d*x^2+c)
/c)^(1/2)*((f*x^2+e)/e)^(1/2)+3*EllipticE(x*(-d/c)^(1/2),(c*f/d/e
)^(1/2))*a*c^3*d*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+7*
EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c^2*d^2*e^2*f*((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-7*EllipticE(x*(-d/c)^(1/2),(c*f/
d/e)^(1/2))*b*c^3*d*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)
)/(f*x^2+e)^(1/2)/(c*f-d*e)^3/c^2/(-d/c)^(1/2)/e/(d*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + a}{(dx^2 + c)^{\frac{5}{2}}(fx^2 + e)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)),x, algorithm="maxima
```

```
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2 + a}{(d^2fx^6 + (d^2e + 2\,cdf)x^4 + c^2e + (2\,cde + c^2f)x^2)\sqrt{dx^2 + c}\sqrt{fx^2 + e}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)),x, algorithm="fricas
```

```
[Out] integral((b*x^2 + a)/((d^2*f*x^6 + (d^2*e + 2*c*d*f)*x^4 + c^2*e
+ (2*c*d*e + c^2*f)*x^2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/(d*x**2+c)**(5/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + a}{(dx^2 + c)^{\frac{5}{2}}(fx^2 + e)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)),x, algorithm="giac")

[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)), x)
```

3.48 $\int \frac{e+fx^2}{\sqrt{a+bx^2}(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=209

$$\frac{\sqrt{c}\sqrt{a+bx^2}(be-af)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}(de-cf)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] -(((d*e - c*f)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[
c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(b*e - a*f)
*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)
/(a*d)])/(a*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^
2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 0.306223, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{\sqrt{c}\sqrt{a+bx^2}(be-af)F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{a+bx^2}(de-cf)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x^2)/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(((d*e - c*f)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[
c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(b*e - a*f)
*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)
/(a*d)])/(a*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^
2))]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 35.6353, size = 173, normalized size = 0.83

$$\frac{\sqrt{a}\sqrt{c+dx^2}\,(af-be)\,F\left(\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}+1\right)}{\sqrt{b}c\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{a+bx^2}\,(ad-bc)}-\frac{\sqrt{a+bx^2}\,(cf-de)\,E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**2+e)/(d*x**2+c)**(3/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(c + d*x**2)*(a*f - b*e)*elliptic_f(atan(sqrt(b)*x/sq
rt(a)), -a*d/(b*c) + 1)/(sqrt(b)*c*sqrt(a*(c + d*x**2)/(c*(a + b*
x**2)))*sqrt(a + b*x**2)*(a*d - b*c)) - sqrt(a + b*x**2)*(c*f - d
*e)*elliptic_e(atan(sqrt(d)*x/sqrt(c)), 1 - b*c/(a*d))/(sqrt(c)*s
qrt(d)*sqrt(c*(a + b*x**2)/(a*(c + d*x**2)))*sqrt(c + d*x**2)*(a*
d - b*c))
```

Mathematica [C] time = 0.64809, size = 212, normalized size = 1.01

$$\frac{dx\sqrt{\frac{b}{a}}\left(a+bx^2\right)(de-cf)-ibc\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(cf-de)E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-icf\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{dx^2}{c}+1}(ad-bc)F\left(i\right.}{cd\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}(ad-bc)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(e + f*x^2)/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[b/a]*d*(d*e - c*f)*x*(a + b*x^2) - I*b*c*(-(d*e) + c*f)*Sqr
t[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a
]*x], (a*d)/(b*c)] - I*c*(-(b*c) + a*d)*f*Sqrt[1 + (b*x^2)/a]*Sqr
t[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/
(Sqrt[b/a]*c*d*(-(b*c) + a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] time = 0.042, size = 349, normalized size = 1.7

$$\frac{1}{cd\left(ad-bc\right)\left(bdx^4+adx^2+cx^2b+ac\right)}\left(-x^3bcdf\sqrt{-\frac{b}{a}}+x^3bd^2e\sqrt{-\frac{b}{a}}+EllipticF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)acdf\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+}{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^2+e)/(d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] (-x^3*b*c*d*f*(-b/a)^(1/2)+x^3*b*d^2*e*(-b/a)^(1/2)+EllipticF(x*(
-b/a)^(1/2),(a*d/b/c)^(1/2))*a*c*d*f*((b*x^2+a)/a)^(1/2)*((d*x^2+
c)/c)^(1/2)-EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^2*f*((b
*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+EllipticE(x*(-b/a)^(1/2),(a*
d/b/c)^(1/2))*b*c^2*f*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-Ell
ipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*d*e*((b*x^2+a)/a)^(1/2
)*((d*x^2+c)/c)^(1/2)-x*a*c*d*f*(-b/a)^(1/2)+x*a*d^2*e*(-b/a)^(1/
2))*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/(-b/a)^(1/2)/c/d/(a*d-b*c)/(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^2 + e}{\sqrt{bx^2 + a}(dx^2 + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{fx^2 + e}{\sqrt{bx^2 + a}(dx^2 + c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="fricas")
```

```
[Out] integral((f*x^2 + e)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e + fx^2}{\sqrt{a + bx^2}\,(c + dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e)/(d*x**2+c)**(3/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral((e + f*x**2)/(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^2 + e}{\sqrt{bx^2 + a}(dx^2 + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="giac")

[Out] integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

3.49 $\int \frac{e+fx^2}{\sqrt{a-bx^2}(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=247

$$\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}(ad+bc)}+\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(de-cf)E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{cd\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}(ad+bc)}+\frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}F\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

```
[Out] ((d*e - c*f)*x*Sqrt[a - b*x^2])/(c*(b*c + a*d)*Sqrt[c + d*x^2]) +
 (Sqrt[a]*Sqrt[b]*(d*e - c*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]
*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(c*d*(b*
c + a*d)*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c]) + (Sqrt[a]*f*Sqrt[1
 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sq
rt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[c + d*x^
2])
```

Rubi [A] time = 0.746629, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226

$$\frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}(ad+bc)}+\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(de-cf)E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{cd\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}(ad+bc)}+\frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}F\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x^2)/(Sqrt[a - b*x^2]*(c + d*x^2)^(3/2)),x]
```

```
[Out] ((d*e - c*f)*x*Sqrt[a - b*x^2])/(c*(b*c + a*d)*Sqrt[c + d*x^2]) +
 (Sqrt[a]*Sqrt[b]*(d*e - c*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]
*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(c*d*(b*
c + a*d)*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c]) + (Sqrt[a]*f*Sqrt[1
 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*x)/Sq
rt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt[c + d*x^
2])
```

Rubi in Sympy [A] time = 113.146, size = 209, normalized size = 0.85

$$-\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,(cf-de)\,E\left(\operatorname{asin}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}\right)}{cd\sqrt{1+\frac{dx^2}{c}}\sqrt{a-bx^2}\,(ad+bc)}+\frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}F\left(\operatorname{asin}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}-\frac{x\sqrt{a-bx^2}\,(cf-de)}{c\sqrt{c+dx^2}\,(ad+bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**2+e)/(d*x**2+c)**(3/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(a)*sqrt(b)*sqrt(1 - b*x**2/a)*sqrt(c + d*x**2)*(c*f - d*e)*
elliptic_e(asin(sqrt(b)*x/sqrt(a)), -a*d/(b*c))/(c*d*sqrt(1 + d*x
**2/c)*sqrt(a - b*x**2)*(a*d + b*c)) + sqrt(a)*f*sqrt(1 - b*x**2/
a)*sqrt(1 + d*x**2/c)*elliptic_f(asin(sqrt(b)*x/sqrt(a)), -a*d/(b
*c))/(sqrt(b)*d*sqrt(a - b*x**2)*sqrt(c + d*x**2)) - x*sqrt(a - b
*x**2)*(c*f - d*e)/(c*sqrt(c + d*x**2)*(a*d + b*c))
```

Mathematica [C] time = 1.23432, size = 220, normalized size = 0.89

$$\frac{dx\sqrt{-\frac{b}{a}}\left(a-bx^2\right)(de-cf)+ibc\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(cf-de)E\left(i\sinh^{-1}\left(\sqrt{-\frac{b}{a}}x\right)\middle|-\frac{ad}{bc}\right)-icf\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc}{cd\sqrt{-\frac{b}{a}}\sqrt{a-bx^2}\sqrt{c+dx^2}(ad+bc)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(e + f*x^2)/(Sqrt[a - b*x^2]*(c + d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[-(b/a)]*d*(d*e - c*f)*x*(a - b*x^2) + I*b*c*(-(d*e) + c*f)*
Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[
-(b/a)]*x], -((a*d)/(b*c))] - I*c*(b*c + a*d)*f*Sqrt[1 - (b*x^2)/
a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*
d)/(b*c))])/(Sqrt[-(b/a)]*c*d*(b*c + a*d)*Sqrt[a - b*x^2]*Sqrt[c
+ d*x^2])
```

Maple [A] time = 0.073, size = 359, normalized size = 1.5

$$\frac{1}{cd\,(ad+bc)\,(bdx^4-adx^2+cx^2b-ac)}\left(-x^3bcdf\sqrt{\frac{b}{a}}+x^3bd^2e\sqrt{\frac{b}{a}}-EllipticF\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)acdf\sqrt{-\frac{bx^2-a}{a}}\sqrt{\frac{dx^2+}{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^2+e)/(d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x)
```

```
[Out] (-x^3*b*c*d*f*(b/a)^(1/2)+x^3*b*d^2*e*(b/a)^(1/2)-EllipticF(x*(b/
a)^(1/2),(-a*d/b/c)^(1/2))*a*c*d*f*(-(b*x^2-a)/a)^(1/2)*((d*x^2+c
)/c)^(1/2)-EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b*c^2*f*(-(b
*x^2-a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)+EllipticE(x*(b/a)^(1/2),(-a*
d/b/c)^(1/2))*b*c^2*f*(-(b*x^2-a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)-El
lipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b*c*d*e*(-(b*x^2-a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)+x*a*c*d*f*(b/a)^(1/2)-x*a*d^2*e*(b/a)^(1/
2))*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(b/a)^(1/2)/c/d/(a*d+b*c)/(b
*d*x^4-a*d*x^2+b*c*x^2-a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^2 + e}{\sqrt{-bx^2 + a}(dx^2 + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="maxima"
```

```
[Out] integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{fx^2 + e}{\sqrt{-bx^2 + a}(dx^2 + c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="fricas"
```

```
[Out] integral((f*x^2 + e)/(sqrt(-b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e + fx^2}{\sqrt{a - bx^2}\,(c + dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**2+e)/(d*x**2+c)**(3/2)/(-b*x**2+a)**(1/2),x)

[Out] Integral((e + f*x**2)/(sqrt(a - b*x**2)*(c + d*x**2)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^2 + e}{\sqrt{-bx^2 + a}(dx^2 + c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="giac")

[Out] integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

3.50 $\int \frac{e+fx^2}{\sqrt{a+bx^2}(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=237

$$\frac{x\sqrt{a+bx^2}(cf+de)}{c\sqrt{c-dx^2}(ad+bc)}-\frac{\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}(cf+de)E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}(ad+bc)}+\frac{e\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

```
[Out] ((d*e + c*f)*x*Sqrt[a + b*x^2])/(c*(b*c + a*d)*Sqrt[c - d*x^2]) -
 ((d*e + c*f)*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSi
n[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[c]*Sqrt[d]*(b*c +
a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[c - d*x^2]) + (e*Sqrt[1 + (b*x^2)/a
]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b
*c)/(a*d))])/(Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.711783, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226

$$\frac{x\sqrt{a+bx^2}(cf+de)}{c\sqrt{c-dx^2}(ad+bc)}-\frac{\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}(cf+de)E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}(ad+bc)}+\frac{e\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x^2)/(Sqrt[a + b*x^2]*(c - d*x^2)^(3/2)),x]
```

```
[Out] ((d*e + c*f)*x*Sqrt[a + b*x^2])/(c*(b*c + a*d)*Sqrt[c - d*x^2]) -
 ((d*e + c*f)*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSi
n[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[c]*Sqrt[d]*(b*c +
a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[c - d*x^2]) + (e*Sqrt[1 + (b*x^2)/a
]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b
*c)/(a*d))])/(Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[c - d*x^2])
```

Rubi in Sympy [A] time = 112.439, size = 204, normalized size = 0.86

$$\frac{x\sqrt{a+bx^2}\,(cf+de)}{c\sqrt{c-dx^2}\,(ad+bc)}+\frac{e\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}F\left(\operatorname{asin}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}-\frac{\sqrt{1-\frac{dx^2}{c}}\sqrt{a+bx^2}\,(cf+de)\,E\left(\operatorname{asin}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}\,(ad+bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**2+e)/(-d*x**2+c)**(3/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] x*sqrt(a + b*x**2)*(c*f + d*e)/(c*sqrt(c - d*x**2)*(a*d + b*c)) +
 e*sqrt(1 + b*x**2/a)*sqrt(1 - d*x**2/c)*elliptic_f(asin(sqrt(d)*
x/sqrt(c)), -b*c/(a*d))/(sqrt(c)*sqrt(d)*sqrt(a + b*x**2)*sqrt(c
- d*x**2)) - sqrt(1 - d*x**2/c)*sqrt(a + b*x**2)*(c*f + d*e)*elli
ptic_e(asin(sqrt(d)*x/sqrt(c)), -b*c/(a*d))/(sqrt(c)*sqrt(d)*sqrt
(1 + b*x**2/a)*sqrt(c - d*x**2)*(a*d + b*c))
```

Mathematica [C] time = 0.687131, size = 213, normalized size = 0.9

$$\frac{dx\sqrt{\frac{b}{a}}\left(a+bx^2\right)(cf+de)-ibc\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(cf+de)E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)\middle|-\frac{ad}{bc}\right)+icf\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)F\left(\right.}{cd\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c-dx^2}(ad+bc)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(e + f*x^2)/(Sqrt[a + b*x^2]*(c - d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[b/a]*d*(d*e + c*f)*x*(a + b*x^2) - I*b*c*(d*e + c*f)*Sqrt[1
 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x
], -((a*d)/(b*c))] + I*c*(b*c + a*d)*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1
 - (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], -((a*d)/(b*c))])/
(Sqrt[b/a]*c*d*(b*c + a*d)*Sqrt[a + b*x^2]*Sqrt[c - d*x^2])
```

Maple [A] time = 0.06, size = 345, normalized size = 1.5

$$\frac{1}{c\,(ad+bc)\,(bdx^4+adx^2-cx^2b-ac)}\left(-x^3bcf\sqrt{\frac{d}{c}}-x^3bde\sqrt{\frac{d}{c}}-EllipticF\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)ade\sqrt{\frac{bx^2+a}{a}}\sqrt{-\frac{dx^2-c}{c}}-E\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^2+e)/(-d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] (-x^3*b*c*f*(d/c)^(1/2)-x^3*b*d*e*(d/c)^(1/2)-EllipticF(x*(d/c)^(
1/2),(-b*c/a/d)^(1/2))*a*d*e*((b*x^2+a)/a)^(1/2)*(-(d*x^2-c)/c)^(
1/2)-EllipticF(x*(d/c)^(1/2),(-b*c/a/d)^(1/2))*b*c*e*((b*x^2+a)/a
)^(1/2)*(-(d*x^2-c)/c)^(1/2)+EllipticE(x*(d/c)^(1/2),(-b*c/a/d)^(
1/2))*a*c*f*((b*x^2+a)/a)^(1/2)*(-(d*x^2-c)/c)^(1/2)+EllipticE(x*
(d/c)^(1/2),(-b*c/a/d)^(1/2))*a*d*e*((b*x^2+a)/a)^(1/2)*(-(d*x^2-
c)/c)^(1/2)-x*a*c*f*(d/c)^(1/2)-x*a*d*e*(d/c)^(1/2))*(b*x^2+a)^(1
/2)*(-d*x^2+c)^(1/2)/(d/c)^(1/2)/c/(a*d+b*c)/(b*d*x^4+a*d*x^2-b*c
*x^2-a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^2 + e}{\sqrt{bx^2 + a}(-dx^2 + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(-d*x^2 + c)^(3/2)),x, algorithm="maxima"
```

```
[Out] integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(-d*x^2 + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{fx^2 + e}{\sqrt{bx^2 + a}(dx^2 - c)\sqrt{-dx^2 + c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(-d*x^2 + c)^(3/2)),x, algorithm="fricas"
```

```
[Out] integral(-(f*x^2 + e)/(sqrt(b*x^2 + a)*(d*x^2 - c)*sqrt(-d*x^2 +
c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e + fx^2}{\sqrt{a + bx^2}\,(c - dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**2+e)/(-d*x**2+c)**(3/2)/(b*x**2+a)**(1/2),x)

[Out] Integral((e + f*x**2)/(sqrt(a + b*x**2)*(c - d*x**2)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^2+e}{\sqrt{bx^2+a}(-dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(-d*x^2 + c)^(3/2)),x, algorithm="giac")

[Out] integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(-d*x^2 + c)^(3/2)), x)
```

3.51 $\int \frac{e+fx^2}{\sqrt{a-bx^2}(c-dx^2)^{3/2}}\,dx$

Optimal. Leaf size=242

$$-\frac{x\sqrt{a-bx^2}(cf+de)}{c\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}(cf+de)E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}(bc-ad)}+\frac{e\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}$$

```
[Out] -(((d*e + c*f)*x*Sqrt[a - b*x^2])/(c*(b*c - a*d)*Sqrt[c - d*x^2])
) + ((d*e + c*f)*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[Ar
cSin[(Sqrt[d]*x)/Sqrt[c]], (b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*(b*c -
a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[c - d*x^2]) + (e*Sqrt[1 - (b*x^2)/a
]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], (b*c
)/(a*d)])/(Sqrt[c]*Sqrt[d]*Sqrt[a - b*x^2]*Sqrt[c - d*x^2])
```

Rubi [A] time = 0.755732, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219

$$-\frac{x\sqrt{a-bx^2}(cf+de)}{c\sqrt{c-dx^2}(bc-ad)}+\frac{\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}(cf+de)E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}(bc-ad)}+\frac{e\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x^2)/(Sqrt[a - b*x^2]*(c - d*x^2)^(3/2)),x]
```

```
[Out] -(((d*e + c*f)*x*Sqrt[a - b*x^2])/(c*(b*c - a*d)*Sqrt[c - d*x^2])
) + ((d*e + c*f)*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[Ar
cSin[(Sqrt[d]*x)/Sqrt[c]], (b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*(b*c -
a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[c - d*x^2]) + (e*Sqrt[1 - (b*x^2)/a
]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], (b*c
)/(a*d)])/(Sqrt[c]*Sqrt[d]*Sqrt[a - b*x^2]*Sqrt[c - d*x^2])
```

Rubi in Sympy [A] time = 147.743, size = 202, normalized size = 0.83

$$\frac{\sqrt{a}e\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}F\left(\operatorname{asin}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|\frac{ad}{bc}\right)}{\sqrt{b}c\sqrt{a-bx^2}\sqrt{c-dx^2}}+\frac{x\sqrt{a-bx^2}\,(cf+de)}{c\sqrt{c-dx^2}\,(ad-bc)}-\frac{\sqrt{1-\frac{dx^2}{c}}\sqrt{a-bx^2}\,(cf+de)\,E\left(\operatorname{asin}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**2+e)/(-d*x**2+c)**(3/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*e*sqrt(1 - b*x**2/a)*sqrt(1 - d*x**2/c)*elliptic_f(asin(s
qrt(b)*x/sqrt(a)), a*d/(b*c))/(sqrt(b)*c*sqrt(a - b*x**2)*sqrt(c
- d*x**2)) + x*sqrt(a - b*x**2)*(c*f + d*e)/(c*sqrt(c - d*x**2)*(
a*d - b*c)) - sqrt(1 - d*x**2/c)*sqrt(a - b*x**2)*(c*f + d*e)*ell
iptic_e(asin(sqrt(d)*x/sqrt(c)), b*c/(a*d))/(sqrt(c)*sqrt(d)*sqrt
(1 - b*x**2/a)*sqrt(c - d*x**2)*(a*d - b*c))
```

Mathematica [C] time = 0.73136, size = 221, normalized size = 0.91

$$\frac{dx\sqrt{-\frac{b}{a}}\left(a-bx^2\right)(cf+de)+ibc\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}(cf+de)E\left(i\sinh^{-1}\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+icf\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}(ad-bc)}{cd\sqrt{-\frac{b}{a}}\sqrt{a-bx^2}\sqrt{c-dx^2}(ad-bc)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(e + f*x^2)/(Sqrt[a - b*x^2]*(c - d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[-(b/a)]*d*(d*e + c*f)*x*(a - b*x^2) + I*b*c*(d*e + c*f)*Sqr
t[1 - (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b
/a)]*x], (a*d)/(b*c)] + I*c*(-(b*c) + a*d)*f*Sqrt[1 - (b*x^2)/a]*
Sqrt[1 - (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[-(b/a)]*x], (a*d)/(b
*c)])/(Sqrt[-(b/a)]*c*d*(-(b*c) + a*d)*Sqrt[a - b*x^2]*Sqrt[c - d
*x^2])
```

Maple [A] time = 0.063, size = 354, normalized size = 1.5

$$\frac{1}{c\,(ad-bc)\,(bdx^4-adx^2-cx^2b+ac)}\left(-x^3bcf\sqrt{\frac{d}{c}}-x^3bde\sqrt{\frac{d}{c}}+EllipticF\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)ade\sqrt{-\frac{dx^2-c}{c}}\sqrt{-\frac{bx^2-a}{a}}-I\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^2+e)/(-d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x)
```

```
[Out] (-x^3*b*c*f*(d/c)^(1/2)-x^3*b*d*e*(d/c)^(1/2)+EllipticF(x*(d/c)^(
1/2),(b*c/a/d)^(1/2))*a*d*e*(-(d*x^2-c)/c)^(1/2)*(-(b*x^2-a)/a)^(
1/2)-EllipticF(x*(d/c)^(1/2),(b*c/a/d)^(1/2))*b*c*e*(-(d*x^2-c)/c
)^(1/2)*(-(b*x^2-a)/a)^(1/2)-EllipticE(x*(d/c)^(1/2),(b*c/a/d)^(1
/2))*a*c*f*(-(d*x^2-c)/c)^(1/2)*(-(b*x^2-a)/a)^(1/2)-EllipticE(x*
(d/c)^(1/2),(b*c/a/d)^(1/2))*a*d*e*(-(d*x^2-c)/c)^(1/2)*(-(b*x^2-
a)/a)^(1/2)+x*a*c*f*(d/c)^(1/2)+x*a*d*e*(d/c)^(1/2))*(-b*x^2+a)^(
1/2)*(-d*x^2+c)^(1/2)/(d/c)^(1/2)/c/(a*d-b*c)/(b*d*x^4-a*d*x^2-b*
c*x^2+a*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^2 + e}{\sqrt{-bx^2 + a}(-dx^2 + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*(-d*x^2 + c)^(3/2)),x, algorithm="maxima
```

```
[Out] integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*(-d*x^2 + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{fx^2 + e}{\sqrt{-bx^2 + a}(dx^2 - c)\sqrt{-dx^2 + c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*(-d*x^2 + c)^(3/2)),x, algorithm="fricas
```

```
[Out] integral(-(f*x^2 + e)/(sqrt(-b*x^2 + a)*(d*x^2 - c)*sqrt(-d*x^2 +
 c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e + fx^2}{\sqrt{a - bx^2}\,(c - dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**2+e)/(-d*x**2+c)**(3/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral((e + f*x**2)/(sqrt(a - b*x**2)*(c - d*x**2)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{fx^2 + e}{\sqrt{-bx^2 + a}(-dx^2 + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*(-d*x^2 + c)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*(-d*x^2 + c)^(3/2)), x)
```

3.52 $\int \frac{a+bx^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx$

Optimal. Leaf size=191

$$\frac{a\sqrt{dx^2+2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{\sqrt{2}\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}+\frac{bx\sqrt{dx^2+2}}{d\sqrt{fx^2+3}}-\frac{\sqrt{2}b\sqrt{dx^2+2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{d\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}$$

```
[Out] (b*x*Sqrt[2 + d*x^2])/(d*Sqrt[3 + f*x^2]) - (Sqrt[2]*b*Sqrt[2 + d
*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(d
*Sqrt[f]*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2]) + (a*Sqrt
[2 + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f
)])/(Sqrt[2]*Sqrt[f]*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2
])
```

Rubi [A] time = 0.343267, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{a\sqrt{dx^2+2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{\sqrt{2}\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}+\frac{bx\sqrt{dx^2+2}}{d\sqrt{fx^2+3}}-\frac{\sqrt{2}b\sqrt{dx^2+2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{d\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/(Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2]),x]
```


```
[Out] (b*x*Sqrt[2 + d*x^2])/(d*Sqrt[3 + f*x^2]) - (Sqrt[2]*b*Sqrt[2 + d
*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(d
*Sqrt[f]*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2]) + (a*Sqrt
[2 + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f
)])/(Sqrt[2]*Sqrt[f]*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2
])
```

Rubi in Sympy [A] time = 37.8688, size = 180, normalized size = 0.94

$$\frac{\sqrt{3}a\sqrt{dx^2+2}F\left(\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{f}x}{3}\right)\middle|-\frac{3d}{2f}+1\right)}{2\sqrt{f}\sqrt{\frac{3dx^2+6}{2fx^2+6}}\sqrt{fx^2+3}}+\frac{bx\sqrt{fx^2+3}}{f\sqrt{dx^2+2}}-\frac{\sqrt{2}b\sqrt{fx^2+3}E\left(\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{d}x}{2}\right)\middle|1-\frac{2f}{3d}\right)}{\sqrt{d}f\sqrt{\frac{2fx^2+6}{3dx^2+6}}\sqrt{dx^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/(d*x**2+2)**(1/2)/(f*x**2+3)**(1/2),x)
```

```
[Out] sqrt(3)*a*sqrt(d*x**2 + 2)*elliptic_f(atan(sqrt(3)*sqrt(f)*x/3),
-3*d/(2*f) + 1)/(2*sqrt(f)*sqrt((3*d*x**2 + 6)/(2*f*x**2 + 6))*sq
rt(f*x**2 + 3)) + b*x*sqrt(f*x**2 + 3)/(f*sqrt(d*x**2 + 2)) - sqr
t(2)*b*sqrt(f*x**2 + 3)*elliptic_e(atan(sqrt(2)*sqrt(d)*x/2), 1 -
 2*f/(3*d))/(sqrt(d)*f*sqrt((2*f*x**2 + 6)/(3*d*x**2 + 6))*sqrt(d
*x**2 + 2))
```

Mathematica [C] time = 0.138845, size = 81, normalized size = 0.42

$$-\frac{i\left((af-3b)F\left(i\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)+3bE\left(i\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)\right)}{\sqrt{3}\sqrt{d}f}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/(Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2]),x]
```

```
[Out] ((-I)*(3*b*EllipticE[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)]
 + (-3*b + a*f)*EllipticF[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(
3*d)]))/(Sqrt[3]*Sqrt[d]*f)
```

Maple [A] time = 0.051, size = 105, normalized size = 0.6

$$\frac{\sqrt{2}}{2\,d}\left(EllipticF\left(\frac{x\sqrt{3}}{3}\sqrt{-f},\frac{\sqrt{3}\sqrt{2}}{2}\sqrt{\frac{d}{f}}\right)ad-2\,EllipticF\left(1/3\,x\sqrt{3}\sqrt{-f},1/2\,\sqrt{3}\sqrt{2}\sqrt{\frac{d}{f}}\right)b+2\,EllipticE\left(1/3\,x\sqrt{3}\sqrt{-f},1/2\,\sqrt{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x)
```

```
[Out] 1/2*2^(1/2)*(EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*3^(1/2)*2^(1/
2)*(1/f*d)^(1/2))*a*d-2*EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*3^
(1/2)*2^(1/2)*(1/f*d)^(1/2))*b+2*EllipticE(1/3*x*3^(1/2)*(-f)^(1/
2),1/2*3^(1/2)*2^(1/2)*(1/f*d)^(1/2))*b)/(-f)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{bx^2+a}{\sqrt{dx^2+2}\sqrt{fx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)),x, algorithm="maxima")

[Out] integrate((b*x^2 + a)/(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2 + a}{\sqrt{dx^2 + 2}\sqrt{fx^2 + 3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)),x, algorithm="fricas")

[Out] integral((b*x^2 + a)/(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + bx^2}{\sqrt{dx^2 + 2}\sqrt{fx^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/(d*x**2+2)**(1/2)/(f*x**2+3)**(1/2),x)

[Out] Integral((a + b*x**2)/(sqrt(d*x**2 + 2)*sqrt(f*x**2 + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + a}{\sqrt{dx^2 + 2}\sqrt{fx^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)),x, algorithm="giac")

[Out] integrate((b*x^2 + a)/(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)), x)
```

3.53 $\int \frac{(a+bx^2)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx$

Optimal. Leaf size=262

$$-\frac{\sqrt{2}\sqrt{dx^2+2}(b-af)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{f^{3/2}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}+\frac{\sqrt{2}\sqrt{dx^2+2}(-3adf+6bd-2bf)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{3df^{3/2}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}-\frac{x\sqrt{dx^2+2}(-3adf+6bd-2bf)}{3df\sqrt{fx^2+3}}+\frac{bx\sqrt{dx^2+2}\sqrt{fx^2+3}}{3f}$$

```
[Out] -((6*b*d - 2*b*f - 3*a*d*f)*x*Sqrt[2 + d*x^2])/(3*d*f*Sqrt[3 + f*
x^2]) + (b*x*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2])/(3*f) + (Sqrt[2]*(6
*b*d - 2*b*f - 3*a*d*f)*Sqrt[2 + d*x^2]*EllipticE[ArcTan[(Sqrt[f]
*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(3*d*f^(3/2)*Sqrt[(2 + d*x^2)/(3
+ f*x^2)]*Sqrt[3 + f*x^2]) - (Sqrt[2]*(b - a*f)*Sqrt[2 + d*x^2]*E
llipticF[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(f^(3/2)*
Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2])
```

Rubi [A] time = 0.547819, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{\sqrt{2}\sqrt{dx^2+2}(b-af)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{f^{3/2}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}+\frac{\sqrt{2}\sqrt{dx^2+2}(-3adf+6bd-2bf)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{3df^{3/2}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}-\frac{x\sqrt{dx^2+2}(-3adf+6bd-2bf)}{3df\sqrt{fx^2+3}}+\frac{bx\sqrt{dx^2+2}\sqrt{fx^2+3}}{3f}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*Sqrt[2 + d*x^2])/Sqrt[3 + f*x^2],x]
```

```
[Out] -((6*b*d - 2*b*f - 3*a*d*f)*x*Sqrt[2 + d*x^2])/(3*d*f*Sqrt[3 + f*
x^2]) + (b*x*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2])/(3*f) + (Sqrt[2]*(6
*b*d - 2*b*f - 3*a*d*f)*Sqrt[2 + d*x^2]*EllipticE[ArcTan[(Sqrt[f]
*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(3*d*f^(3/2)*Sqrt[(2 + d*x^2)/(3
```

```
+ f*x^2)]*Sqrt[3 + f*x^2]) - (Sqrt[2]*(b - a*f)*Sqrt[2 + d*x^2]*E
llipticF[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(f^(3/2)*
Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2])
```

Rubi in Sympy [A] time = 59.8932, size = 245, normalized size = 0.94

$$\frac{bx\sqrt{dx^2+2}\sqrt{fx^2+3}}{3f} - \frac{\sqrt{3}\left(-af+b\right)\sqrt{dx^2+2}F\left(\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{f}x}{3}\right)\middle|-\frac{3d}{2f}+1\right)}{f^{\frac{3}{2}}\sqrt{\frac{3dx^2+6}{2fx^2+6}}\sqrt{fx^2+3}} - \frac{x\sqrt{dx^2+2}\left(-3adf+6bd-2bf\right)}{3df\sqrt{fx^2+3}} + \frac{\sqrt{3}\sqrt{dx^2+2}\left(-3adf+6bd-2bf\right)E\left(\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{f}x}{3}\right)\middle|-\frac{3d}{2f}+1\right)}{3df^{\frac{3}{2}}\sqrt{\frac{3dx^2+6}{2fx^2+6}}\sqrt{fx^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+2)**(1/2)/(f*x**2+3)**(1/2),x)
```

```
[Out] b*x*sqrt(d*x**2 + 2)*sqrt(f*x**2 + 3)/(3*f) - sqrt(3)*(-a*f + b)*
sqrt(d*x**2 + 2)*elliptic_f(atan(sqrt(3)*sqrt(f)*x/3), -3*d/(2*f)
 + 1)/(f**(3/2)*sqrt((3*d*x**2 + 6)/(2*f*x**2 + 6))*sqrt(f*x**2 +
 3)) - x*sqrt(d*x**2 + 2)*(-3*a*d*f + 6*b*d - 2*b*f)/(3*d*f*sqrt(
f*x**2 + 3)) + sqrt(3)*sqrt(d*x**2 + 2)*(-3*a*d*f + 6*b*d - 2*b*f
)*elliptic_e(atan(sqrt(3)*sqrt(f)*x/3), -3*d/(2*f) + 1)/(3*d*f**(
3/2)*sqrt((3*d*x**2 + 6)/(2*f*x**2 + 6))*sqrt(f*x**2 + 3))
```

Mathematica [C] time = 0.282959, size = 142, normalized size = 0.54

$$\frac{i\sqrt{3}(3d-2f)(af-2b)F\left(i\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)+i\sqrt{3}(-3adf+6bd-2bf)E\left(i\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)+b\sqrt{d}fx\sqrt{dx^2+2}\sqrt{fx^2+3}}{3\sqrt{d}f^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*Sqrt[2 + d*x^2])/Sqrt[3 + f*x^2],x]
```

```
[Out] (b*Sqrt[d]*f*x*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2] + I*Sqrt[3]*(6*b*d
 - 2*b*f - 3*a*d*f)*EllipticE[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*
f)/(3*d)] + I*Sqrt[3]*(3*d - 2*f)*(-2*b + a*f)*EllipticF[I*ArcSin
h[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)])/(3*Sqrt[d]*f^2)
```

Maple [A] time = 0.028, size = 367, normalized size = 1.4

$$\frac{1}{\left(3\,dfx^{4}+9\,dx^{2}+6\,fx^{2}+18\right)fd}\sqrt{dx^{2}+2}\sqrt{fx^{2}+3}\left(x^{5}bd^{2}f\sqrt{-f}+3\,\sqrt{2}EllipticE\left(1/3\,x\sqrt{3}\sqrt{-f},1/2\,\sqrt{3}\sqrt{2}\sqrt{\frac{d}{f}}\right)adf\sqrt{dx^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x)
```

```
[Out] 1/3*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)*(x^5*b*d^2*f*(-f)^(1/2)+3*2^(
1/2)*EllipticE(1/3*x*3^(1/2)*(-f)^(1/2),1/2*3^(1/2)*2^(1/2)*(1/f*
d)^(1/2))*a*d*f*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)+3*x^3*b*d^2*(-f)^
(1/2)+2*x^3*b*d*f*(-f)^(1/2)-6*2^(1/2)*EllipticE(1/3*x*3^(1/2)*(-
f)^(1/2),1/2*3^(1/2)*2^(1/2)*(1/f*d)^(1/2))*b*d*(d*x^2+2)^(1/2)*(
f*x^2+3)^(1/2)+2*2^(1/2)*EllipticE(1/3*x*3^(1/2)*(-f)^(1/2),1/2*3
^(1/2)*2^(1/2)*(1/f*d)^(1/2))*b*f*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)
+3*2^(1/2)*EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*3^(1/2)*2^(1/2)
*(1/f*d)^(1/2))*b*d*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)-2*2^(1/2)*Ell
ipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*3^(1/2)*2^(1/2)*(1/f*d)^(1/2)
)*b*f*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)+6*x*b*d*(-f)^(1/2))/(d*f*x^
4+3*d*x^2+2*f*x^2+6)/f/(-f)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^{2}+a\right)\sqrt{dx^{2}+2}}{\sqrt{fx^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*sqrt(d*x^2 + 2)/sqrt(f*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + 2)/sqrt(f*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^{2}+a\right)\sqrt{dx^{2}+2}}{\sqrt{fx^{2}+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*sqrt(d*x^2 + 2)/sqrt(f*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)*sqrt(d*x^2 + 2)/sqrt(f*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^2)\sqrt{dx^2 + 2}}{\sqrt{fx^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)*(d*x**2+2)**(1/2)/(f*x**2+3)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(d*x**2 + 2)/sqrt(f*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)\sqrt{dx^2 + 2}}{\sqrt{fx^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*sqrt(d*x^2 + 2)/sqrt(f*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + 2)/sqrt(f*x^2 + 3), x)
```

3.54 $\int \left(a+bx^2\right) \sqrt{2+dx^2}\sqrt{3+fx^2}\,dx$

Optimal. Leaf size=356

$$\frac{x\sqrt{dx^2+2}\left(5adf(3d+2f)-2b\left(9d^2-6df+4f^2\right)\right)}{15d^2f\sqrt{fx^2+3}} - \frac{\sqrt{2}\sqrt{dx^2+2}\left(5adf(3d+2f)-2b\left(9d^2-6df+4f^2\right)\right)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{15d^2f^{3/2}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}} - \frac{\sqrt{2}\sqrt{dx^2+2}(-10adf+3bd+2bf)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{5df^{3/2}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}} + \frac{x\sqrt{dx^2+2}\sqrt{fx^2+3}(5adf+3bd-4bf)}{15df} + \frac{bx\left(dx^2+2\right)^{3/2}\sqrt{fx^2+3}}{5d}$$

```
[Out] ((5*a*d*f*(3*d + 2*f) - 2*b*(9*d^2 - 6*d*f + 4*f^2))*x*Sqrt[2 + d
*x^2])/(15*d^2*f*Sqrt[3 + f*x^2]) + ((3*b*d - 4*b*f + 5*a*d*f)*x*
Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2])/(15*d*f) + (b*x*(2 + d*x^2)^(3/2
)*Sqrt[3 + f*x^2])/(5*d) - (Sqrt[2]*(5*a*d*f*(3*d + 2*f) - 2*b*(9
*d^2 - 6*d*f + 4*f^2))*Sqrt[2 + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*
x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(15*d^2*f^(3/2)*Sqrt[(2 + d*x^2)/(
3 + f*x^2)]*Sqrt[3 + f*x^2]) - (Sqrt[2]*(3*b*d + 2*b*f - 10*a*d*f
)*Sqrt[2 + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d
)/(2*f)])/(5*d*f^(3/2)*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x
^2])
```

Rubi [A] time = 0.925818, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{x\sqrt{dx^2+2}\left(5adf(3d+2f)-2b\left(9d^2-6df+4f^2\right)\right)}{15d^2f\sqrt{fx^2+3}} - \frac{\sqrt{2}\sqrt{dx^2+2}\left(5adf(3d+2f)-2b\left(9d^2-6df+4f^2\right)\right)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{15d^2f^{3/2}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}} - \frac{\sqrt{2}\sqrt{dx^2+2}(-10adf+3bd+2bf)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{5df^{3/2}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}} + \frac{x\sqrt{dx^2+2}\sqrt{fx^2+3}(5adf+3bd-4bf)}{15df} + \frac{bx\left(dx^2+2\right)^{3/2}\sqrt{fx^2+3}}{5d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2],x]
```

```
[Out] ((5*a*d*f*(3*d + 2*f) - 2*b*(9*d^2 - 6*d*f + 4*f^2))*x*Sqrt[2 + d
*x^2])/(15*d^2*f*Sqrt[3 + f*x^2]) + ((3*b*d - 4*b*f + 5*a*d*f)*x*
Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2])/(15*d*f) + (b*x*(2 + d*x^2)^(3/2
)*Sqrt[3 + f*x^2])/(5*d) - (Sqrt[2]*(5*a*d*f*(3*d + 2*f) - 2*b*(9
*d^2 - 6*d*f + 4*f^2))*Sqrt[2 + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*
x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(15*d^2*f^(3/2)*Sqrt[(2 + d*x^2)/(
3 + f*x^2)]*Sqrt[3 + f*x^2]) - (Sqrt[2]*(3*b*d + 2*b*f - 10*a*d*f
)*Sqrt[2 + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d
)/(2*f)])/(5*d*f^(3/2)*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x
^2])
```

Rubi in Sympy [A] time = 101.835, size = 350, normalized size = 0.98

$$\frac{bx\left(dx^2+2\right)^{\frac{3}{2}}\sqrt{fx^2+3}}{5d}+\frac{x\sqrt{dx^2+2}\sqrt{fx^2+3}\,(5adf+3bd-4bf)}{15df}-\frac{x\sqrt{dx^2+2}\left(-15ad^2f-10adf^2+18bd^2-12bdf+8bf^2\right)}{15d^2f\sqrt{fx^2+3}}+\frac{\sqrt{3}\sqrt{dx^2+2}\left(-15ad^2f-10adf^2+18bd^2-12bdf+8bf^2\right)E\left(\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{f}x}{3}\right)\middle|-\frac{3d}{2f}+1\right)}{15d^2f^{\frac{3}{2}}\sqrt{\frac{3dx^2+6}{2fx^2+6}}\sqrt{fx^2+3}}-\frac{2\sqrt{2}\sqrt{fx^2+3}\,(-10adf+3bd+2bf)\,F\left(\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{d}x}{2}\right)\middle|1-\frac{2f}{3d}\right)}{15d^{\frac{3}{2}}f\sqrt{\frac{2fx^2+6}{3dx^2+6}}\sqrt{dx^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+2)**(1/2)*(f*x**2+3)**(1/2),x)
```

```
[Out] b*x*(d*x**2 + 2)**(3/2)*sqrt(f*x**2 + 3)/(5*d) + x*sqrt(d*x**2 +
2)*sqrt(f*x**2 + 3)*(5*a*d*f + 3*b*d - 4*b*f)/(15*d*f) - x*sqrt(d
*x**2 + 2)*(-15*a*d**2*f - 10*a*d*f**2 + 18*b*d**2 - 12*b*d*f + 8
*b*f**2)/(15*d**2*f*sqrt(f*x**2 + 3)) + sqrt(3)*sqrt(d*x**2 + 2)*
(-15*a*d**2*f - 10*a*d*f**2 + 18*b*d**2 - 12*b*d*f + 8*b*f**2)*el
liptic_e(atan(sqrt(3)*sqrt(f)*x/3), -3*d/(2*f) + 1)/(15*d**2*f**(
3/2)*sqrt((3*d*x**2 + 6)/(2*f*x**2 + 6))*sqrt(f*x**2 + 3)) - 2*sq
rt(2)*sqrt(f*x**2 + 3)*(-10*a*d*f + 3*b*d + 2*b*f)*elliptic_f(ata
n(sqrt(2)*sqrt(d)*x/2), 1 - 2*f/(3*d))/(15*d**(3/2)*f*sqrt((2*f*x
**2 + 6)/(3*d*x**2 + 6))*sqrt(d*x**2 + 2))
```

Mathematica [C] time = 0.533412, size = 186, normalized size = 0.52

$$\frac{i\sqrt{3}\left(2b\left(9d^2-6df+4f^2\right)-5adf(3d+2f)\right)E\left(i\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)+\sqrt{d}fx\sqrt{dx^2+2}\sqrt{fx^2+3}\left(5adf+3bd\left(fx^2+1\right)+\ldots\right.}{15d^{3/2}f^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2],x]
```

```
[Out] (Sqrt[d]*f*x*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2]*(2*b*f + 5*a*d*f + 3
*b*d*(1 + f*x^2)) + I*Sqrt[3]*(-5*a*d*f*(3*d + 2*f) + 2*b*(9*d^2
- 6*d*f + 4*f^2))*EllipticE[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)
/(3*d)] + I*Sqrt[3]*(3*d - 2*f)*(-6*b*d + 2*b*f + 5*a*d*f)*Ellipt
icF[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)])/(15*d^(3/2)*f^2
)
```

Maple [B] time = 0.033, size = 775, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2),x)
```

```
[Out] 1/15*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)*(3*x^7*b*d^3*f^2*(-f)^(1/2)+
5*x^5*a*d^3*f^2*(-f)^(1/2)+12*x^5*b*d^3*f*(-f)^(1/2)+8*x^5*b*d^2*
f^2*(-f)^(1/2)+15*x^3*a*d^3*f*(-f)^(1/2)+10*x^3*a*d^2*f^2*(-f)^(1
/2)+15*2^(1/2)*EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*3^(1/2)*2^(
1/2)*(1/f*d)^(1/2))*a*d^2*f*(f*x^2+3)^(1/2)*(d*x^2+2)^(1/2)-10*2^
(1/2)*EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*3^(1/2)*2^(1/2)*(1/f
*d)^(1/2))*a*d*f^2*(f*x^2+3)^(1/2)*(d*x^2+2)^(1/2)+15*2^(1/2)*Ell
ipticE(1/3*x*3^(1/2)*(-f)^(1/2),1/2*3^(1/2)*2^(1/2)*(1/f*d)^(1/2)
)*a*d^2*f*(f*x^2+3)^(1/2)*(d*x^2+2)^(1/2)+10*2^(1/2)*EllipticE(1/
3*x*3^(1/2)*(-f)^(1/2),1/2*3^(1/2)*2^(1/2)*(1/f*d)^(1/2))*a*d*f^2
*(f*x^2+3)^(1/2)*(d*x^2+2)^(1/2)+9*x^3*b*d^3*(-f)^(1/2)+30*x^3*b*
d^2*f*(-f)^(1/2)+4*x^3*b*d*f^2*(-f)^(1/2)+9*2^(1/2)*EllipticF(1/3
*x*3^(1/2)*(-f)^(1/2),1/2*3^(1/2)*2^(1/2)*(1/f*d)^(1/2))*b*d^2*(f
*x^2+3)^(1/2)*(d*x^2+2)^(1/2)-18*2^(1/2)*EllipticF(1/3*x*3^(1/2)*
(-f)^(1/2),1/2*3^(1/2)*2^(1/2)*(1/f*d)^(1/2))*b*d*f*(f*x^2+3)^(1/
2)*(d*x^2+2)^(1/2)+8*2^(1/2)*EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1
/2*3^(1/2)*2^(1/2)*(1/f*d)^(1/2))*b*f^2*(f*x^2+3)^(1/2)*(d*x^2+2)
^(1/2)-18*2^(1/2)*EllipticE(1/3*x*3^(1/2)*(-f)^(1/2),1/2*3^(1/2)*
2^(1/2)*(1/f*d)^(1/2))*b*d^2*(f*x^2+3)^(1/2)*(d*x^2+2)^(1/2)+12*2
^(1/2)*EllipticE(1/3*x*3^(1/2)*(-f)^(1/2),1/2*3^(1/2)*2^(1/2)*(1/
f*d)^(1/2))*b*d*f*(f*x^2+3)^(1/2)*(d*x^2+2)^(1/2)-8*2^(1/2)*Ellip
ticE(1/3*x*3^(1/2)*(-f)^(1/2),1/2*3^(1/2)*2^(1/2)*(1/f*d)^(1/2))*
b*f^2*(f*x^2+3)^(1/2)*(d*x^2+2)^(1/2)+30*x*a*d^2*f*(-f)^(1/2)+18*
x*b*d^2*(-f)^(1/2)+12*x*b*d*f*(-f)^(1/2))/(d*f*x^4+3*d*x^2+2*f*x^
```

```
2+6)/f/d^2/(-f)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)\sqrt{dx^2 + 2}\sqrt{fx^2 + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^2 + a)\sqrt{dx^2 + 2}\sqrt{fx^2 + 3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + bx^2)\sqrt{dx^2 + 2}\sqrt{fx^2 + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+2)**(1/2)*(f*x**2+3)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(d*x**2 + 2)*sqrt(f*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^2 + a)\sqrt{dx^2 + 2}\sqrt{fx^2 + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3), x)
```

3.55 $\int \frac{-b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}}\,dx$

Optimal. Leaf size=113

$$-\frac{\sqrt{b-\sqrt{b^2-4ac}}\left(\sqrt{b^2-4ac}+b\right)E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*(b + Sqrt[b^2 - 4*a*c])*EllipticE[
ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]], (b - Sqr
t[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c]))
```

Rubi [A] time = 0.488136, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 87, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023

$$-\frac{\sqrt{b-\sqrt{b^2-4ac}}\left(\sqrt{b^2-4ac}+b\right)E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(Sqrt[1 + (2*c*x^2)/(-
b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]),x]
```

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*(b + Sqrt[b^2 - 4*a*c])*EllipticE[
ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]], (b - Sqr
t[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c]))
```

Rubi in Sympy [A] time = 47.7069, size = 100, normalized size = 0.88

$$-\frac{\sqrt{2}\sqrt{b-\sqrt{-4ac+b^2}}\left(b+\sqrt{-4ac+b^2}\right)E\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)\middle|\frac{b-\sqrt{-4ac+b^2}}{b+\sqrt{-4ac+b^2}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-b+2*c*x**2-(-4*a*c+b**2)**(1/2))/(1+2*c*x**2/(-b-
(-4*a*c+b**2)**(1/2)))**(1/2)/(1+2*c*x**2/(-b+(-4*a*c+b**2)**(1/2)))**(1/2),x)
```

```
[Out] -sqrt(2)*sqrt(b - sqrt(-4*a*c + b**2))*(b + sqrt(-4*a*c + b**2))*
elliptic_e(asin(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2))),
```

```
(b - sqrt(-4*a*c + b**2))/(b + sqrt(-4*a*c + b**2)))/(2*sqrt(c))
```

Mathematica [C] time = 0.661414, size = 104, normalized size = 0.92

$$-2i\sqrt{2}a\sqrt{\frac{c}{\sqrt{b^2-4ac}-b}}E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{\sqrt{b^2-4ac}-b}}x\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(Sqrt[1 + (2*c*x^2)/(-
b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]),x]
```

```
[Out] (-2*I)*Sqrt[2]*a*Sqrt[c/(-b + Sqrt[b^2 - 4*a*c])]*EllipticE[I*Arc
Sinh[Sqrt[2]*Sqrt[c/(-b + Sqrt[b^2 - 4*a*c])]*x], (b - Sqrt[b^2 -
 4*a*c])/(b + Sqrt[b^2 - 4*a*c])]
```

Maple [F] time = 0.29, size = 0, normalized size = 0.

$$\int 1\left(2\,cx^2-\sqrt{-4\,ac+b^2}-b\right)\frac{1}{\sqrt{1+2\,\frac{cx^2}{-b-\sqrt{-4\,ac+b^2}}}}\frac{1}{\sqrt{1+2\,\frac{cx^2}{-b+\sqrt{-4\,ac+b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^2-(-4*a*c+b^2)^(1/2)-b)/(1+2*c*x^2/(-b-(-4*a*c+b^2)^(1/2)))^(1/2)/(
b+(-4*a*c+b^2)^(1/2)))^(1/2),x)
```

```
[Out] int((2*c*x^2-(-4*a*c+b^2)^(1/2)-b)/(1+2*c*x^2/(-b-(-4*a*c+b^2)^(1
/2)))^(1/2)/(1+2*c*x^2/(-b+(-4*a*c+b^2)^(1/2)))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,cx^2-b-\sqrt{b^2-4\,ac}}{\sqrt{-\frac{2\,cx^2}{b+\sqrt{b^2-4\,ac}}+1}\sqrt{-\frac{2\,cx^2}{b-\sqrt{b^2-4\,ac}}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2 - b - sqrt(b^2 - 4*a*c))/(sqrt(-2*c*x^2/(b + sqrt(b^2 - 4*a*c)
2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)),x, algorithm="maxima")
```

```
[Out] integrate((2*c*x^2 - b - sqrt(b^2 - 4*a*c))/(sqrt(-2*c*x^2/(b + s
qrt(b^2 - 4*a*c)) + 1)*sqrt(-2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{2\,cx^2 - b - \sqrt{b^2 - 4\,ac}}{\sqrt{-\frac{2\,cx^2 - b - \sqrt{b^2 - 4\,ac}}{b + \sqrt{b^2 - 4\,ac}}}\sqrt{-\frac{2\,cx^2 - b + \sqrt{b^2 - 4\,ac}}{b - \sqrt{b^2 - 4\,ac}}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2 - b - sqrt(b^2 - 4*a*c))/(sqrt(-2*c*x^2/(b + sqrt(b^2 - 4*a*c)
2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)),x, algorithm="fricas")
```

```
[Out] integral((2*c*x^2 - b - sqrt(b^2 - 4*a*c))/(sqrt(-(2*c*x^2 - b -
sqrt(b^2 - 4*a*c))/(b + sqrt(b^2 - 4*a*c)))*sqrt(-(2*c*x^2 - b +
sqrt(b^2 - 4*a*c))/(b - sqrt(b^2 - 4*a*c)))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{-b + 2cx^2 - \sqrt{-4ac + b^2}}{\sqrt{\frac{-b+2cx^2-\sqrt{-4ac+b^2}}{-b-\sqrt{-4ac+b^2}}}\sqrt{\frac{-b+2cx^2+\sqrt{-4ac+b^2}}{-b+\sqrt{-4ac+b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b+2*c*x**2-(-4*a*c+b**2)**(1/2))/(1+2*c*x**2/(-b-(-4*a*c+b**2)**(1
b+(-4*a*c+b**2)**(1/2)))**(1/2),x)
```

```
[Out] Integral((-b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(sqrt((-b + 2*c*x*
*2 - sqrt(-4*a*c + b**2))/(-b - sqrt(-4*a*c + b**2)))*sqrt((-b +
2*c*x**2 + sqrt(-4*a*c + b**2))/(-b + sqrt(-4*a*c + b**2)))), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2 - b - sqrt(b^2 - 4*a*c))/(sqrt(-2*c*x^2/(b + sqrt(b^2 - 4*a*c)
2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.56 $\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

Optimal. Leaf size=526

$$\frac{x\left(b-\sqrt{b^2-4ac}\right)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}}{\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{\left(b-\sqrt{b^2-4ac}\right)\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}F\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}-\frac{\left(b-\sqrt{b^2-4ac}\right)\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}E\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] ((b - Sqrt[b^2 - 4*a*c])*x*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a
*c])])/Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])] - ((b - Sqrt[b
^2 - 4*a*c])*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c])]*EllipticE[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
 Sqrt[b^2 - 4*a*c]]], (-2*Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*
c])])/(Sqrt[2]*Sqrt[c]*Sqrt[(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]
))/(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))]*Sqrt[1 + (2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c])]) + ((b - Sqrt[b^2 - 4*a*c])*Sqrt[b + Sqrt[
b^2 - 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Ellipti
cF[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]], (-2*S
qrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt
[(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c]))]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 1.86344, antiderivative size = 526, normalized size of antiderivative = 1., number of

steps used = 5, number of rules used = 5, integrand size = 81, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{x\left(b-\sqrt{b^2-4ac}\right)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}}{\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{\left(b-\sqrt{b^2-4ac}\right)\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}F\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}-\frac{\left(b-\sqrt{b^2-4ac}\right)\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}E\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]
```

```
[Out] ((b - Sqrt[b^2 - 4*a*c])*x*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a
*c])])/Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])] - ((b - Sqrt[b
^2 - 4*a*c])*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c])]*EllipticE[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
 Sqrt[b^2 - 4*a*c]]], (-2*Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*
c])])/(Sqrt[2]*Sqrt[c]*Sqrt[(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]
))/(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))]*Sqrt[1 + (2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c])]) + ((b - Sqrt[b^2 - 4*a*c])*Sqrt[b + Sqrt[
b^2 - 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Ellipti
cF[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]], (-2*S
qrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt
[(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c]))]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*c*x**2-(-4*a*c+b**2)**(1/2)+b)/(1+2*c*x**2/(b-(-
4*a*c+b**2)**(1/2)))**(1/2)/(1+2*c*x**2/(b+(-4*a*c+b**2)**(1/2)))**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.698521, size = 203, normalized size = 0.39

$$-\frac{i\left(\left(\sqrt{b^2-4ac}+b\right)E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b-\sqrt{b^2-4ac}}}x\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)-2\sqrt{b^2-4ac}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b-\sqrt{b^2-4ac}}}x\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)\right)}{\sqrt{2}\sqrt{\frac{c}{b-\sqrt{b^2-4ac}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*
```

```
[Out] ((-I)*((b + Sqrt[b^2 - 4*a*c])*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c
/(b - Sqrt[b^2 - 4*a*c])]*x], (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b
^2 - 4*a*c])] - 2*Sqrt[b^2 - 4*a*c]*EllipticF[I*ArcSinh[Sqrt[2]*S
qrt[c/(b - Sqrt[b^2 - 4*a*c])]*x], (b - Sqrt[b^2 - 4*a*c])/(b + S
qrt[b^2 - 4*a*c])]))/(Sqrt[2]*Sqrt[c/(b - Sqrt[b^2 - 4*a*c])])
```

Maple [F] time = 0.2, size = 0, normalized size = 0.

$$\int 1\left(2\,cx^2-\sqrt{-4\,ac+b^2}+b\right)\frac{1}{\sqrt{1+2\,\frac{cx^2}{b-\sqrt{-4\,ac+b^2}}}}\frac{1}{\sqrt{1+2\,\frac{cx^2}{b+\sqrt{-4\,ac+b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1
4*a*c+b^2)^(1/2)))^(1/2),x)
```

```
[Out] int((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/
2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,cx^2+b-\sqrt{b^2-4\,ac}}{\sqrt{\frac{2\,cx^2}{b+\sqrt{b^2-4\,ac}}+1}\sqrt{\frac{2\,cx^2}{b-\sqrt{b^2-4\,ac}}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/(sqrt(2*c*x^2/(b + sqrt(b^2 - 4*a*c))
```

```
[Out] integrate((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/(sqrt(2*c*x^2/(b + sq
rt(b^2 - 4*a*c)) + 1)*sqrt(2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{2\,cx^2+b-\sqrt{b^2-4\,ac}}{\sqrt{\frac{2\,cx^2+b+\sqrt{b^2-4\,ac}}{b+\sqrt{b^2-4\,ac}}}\sqrt{\frac{2\,cx^2+b-\sqrt{b^2-4\,ac}}{b-\sqrt{b^2-4\,ac}}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/(sqrt(2*c*x^2/(b + sqrt(b^2 - 4*a*c))
```

```
[Out] integral((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/(sqrt((2*c*x^2 + b + s
qrt(b^2 - 4*a*c))/(b + sqrt(b^2 - 4*a*c)))*sqrt((2*c*x^2 + b - sq
rt(b^2 - 4*a*c))/(b - sqrt(b^2 - 4*a*c)))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b+2cx^2-\sqrt{-4ac+b^2}}{\sqrt{\frac{b+2cx^2-\sqrt{-4ac+b^2}}{b-\sqrt{-4ac+b^2}}}\sqrt{\frac{b+2cx^2+\sqrt{-4ac+b^2}}{b+\sqrt{-4ac+b^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**2-(-4*a*c+b**2)**(1/2)+b)/(1+2*c*x**2/(b-(-4*a*c+b**2)**(1/2
4*a*c+b**2)**(1/2)))**(1/2),x)
```

```
[Out] Integral((b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(sqrt((b + 2*c*x**2
 - sqrt(-4*a*c + b**2))/(b - sqrt(-4*a*c + b**2)))*sqrt((b + 2*c*
x**2 + sqrt(-4*a*c + b**2))/(b + sqrt(-4*a*c + b**2)))), x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/(sqrt(2*c*x^2/(b + sqrt(b^2 - 4*a*c))
```

```
[Out] Timed out
```

3.57 $\int \frac{(a+bx^2)\sqrt{c+dx^2}}{e+fx^2}\,dx$

Optimal. Leaf size=128

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)(-2adf-bcf+2bde)}{2\sqrt{d}f^2}+\frac{(be-af)\sqrt{de-cf}\tanh^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{e}\sqrt{c+dx^2}}\right)}{\sqrt{e}f^2}+\frac{bx\sqrt{c+dx^2}}{2f}$$

```
[Out] (b*x*Sqrt[c + d*x^2])/(2*f) - ((2*b*d*e - b*c*f - 2*a*d*f)*ArcTan
h[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*Sqrt[d]*f^2) + ((b*e - a*f)*Sq
rt[d*e - c*f]*ArcTanh[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2
])])/(Sqrt[e]*f^2)
```

Rubi [A] time = 0.406057, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)(-2adf-bcf+2bde)}{2\sqrt{d}f^2}+\frac{(be-af)\sqrt{de-cf}\tanh^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{e}\sqrt{c+dx^2}}\right)}{\sqrt{e}f^2}+\frac{bx\sqrt{c+dx^2}}{2f}$$

Antiderivative was successfully verified.

```
[In]  Int[((a + b*x^2)*Sqrt[c + d*x^2])/(e + f*x^2),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2])/(2*f) - ((2*b*d*e - b*c*f - 2*a*d*f)*ArcTan
h[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*Sqrt[d]*f^2) + ((b*e - a*f)*Sq
rt[d*e - c*f]*ArcTanh[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2
])])/(Sqrt[e]*f^2)
```

Rubi in Sympy [A] time = 47.2829, size = 117, normalized size = 0.91

$$\frac{bx\sqrt{c+dx^2}}{2f}+\frac{(af-be)\sqrt{cf-de}\,\mathrm{atan}\left(\frac{x\sqrt{cf-de}}{\sqrt{e}\sqrt{c+dx^2}}\right)}{\sqrt{e}f^2}+\frac{(2adf+bcf-2bde)\,\mathrm{atanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)*(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

```
[Out] b*x*sqrt(c + d*x**2)/(2*f) + (a*f - b*e)*sqrt(c*f - d*e)*atan(x*s
qrt(c*f - d*e)/(sqrt(e)*sqrt(c + d*x**2)))/(sqrt(e)*f**2) + (2*a*
d*f + b*c*f - 2*b*d*e)*atanh(sqrt(d)*x/sqrt(c + d*x**2))/(2*sqrt(
```

```
d)*f**2)
```

Mathematica [A] time = 0.41897, size = 124, normalized size = 0.97

$$\frac{\frac{\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)(2adf+bcf-2bde)}{\sqrt{d}}-\frac{2(be-af)\sqrt{cf-de}\tan^{-1}\left(\frac{x\sqrt{cf-de}}{\sqrt{e}\sqrt{c+dx^2}}\right)}{\sqrt{e}}+bfx\sqrt{c+dx^2}}{2f^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((a + b*x^2)*Sqrt[c + d*x^2])/(e + f*x^2),x]
```

```
[Out] (b*f*x*Sqrt[c + d*x^2] - (2*(b*e - a*f)*Sqrt[-(d*e) + c*f]*ArcTan
[(Sqrt[-(d*e) + c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])])/Sqrt[e] + ((-
2*b*d*e + b*c*f + 2*a*d*f)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/Sq
rt[d])/(2*f^2)
```

Maple [B] time = 0.056, size = 1942, normalized size = 15.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e),x)
```

```
[Out] 1/2*b*x*(d*x^2+c)^(1/2)/f+1/2*b/f*c/d^(1/2)*ln(x*d^(1/2)+(d*x^2+c
)^(1/2))-1/2/(-e*f)^(1/2)*((x+(-e*f)^(1/2)/f)^2*d-2*d*(-e*f)^(1/2
)/f*(x+(-e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2)*a+1/2/(-e*f)^(1/2)/f*((
x+(-e*f)^(1/2)/f)^2*d-2*d*(-e*f)^(1/2)/f*(x+(-e*f)^(1/2)/f)+(c*f-
d*e)/f)^(1/2)*b*e+1/2/f*d^(1/2)*ln((-d*(-e*f)^(1/2)/f+(x+(-e*f)^(
1/2)/f)*d)/d^(1/2)+((x+(-e*f)^(1/2)/f)^2*d-2*d*(-e*f)^(1/2)/f*(x+
(-e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2))*a-1/2/f^2*d^(1/2)*ln((-d*(-e*
f)^(1/2)/f+(x+(-e*f)^(1/2)/f)*d)/d^(1/2)+((x+(-e*f)^(1/2)/f)^2*d-
2*d*(-e*f)^(1/2)/f*(x+(-e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2))*b*e+1/2
/(-e*f)^(1/2)/((c*f-d*e)/f)^(1/2)*ln((2*(c*f-d*e)/f-2*d*(-e*f)^(1
/2)/f*(x+(-e*f)^(1/2)/f)+2*((c*f-d*e)/f)^(1/2)*((x+(-e*f)^(1/2)/f
)^2*d-2*d*(-e*f)^(1/2)/f*(x+(-e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2))/(
x+(-e*f)^(1/2)/f))*c*a-1/2/(-e*f)^(1/2)/f/((c*f-d*e)/f)^(1/2)*ln(
(2*(c*f-d*e)/f-2*d*(-e*f)^(1/2)/f*(x+(-e*f)^(1/2)/f)+2*((c*f-d*e)
/f)^(1/2)*((x+(-e*f)^(1/2)/f)^2*d-2*d*(-e*f)^(1/2)/f*(x+(-e*f)^(1
/2)/f)+(c*f-d*e)/f)^(1/2))/(x+(-e*f)^(1/2)/f))*c*b*e-1/2/(-e*f)^(
1/2)/f/((c*f-d*e)/f)^(1/2)*ln((2*(c*f-d*e)/f-2*d*(-e*f)^(1/2)/f*(
x+(-e*f)^(1/2)/f)+2*((c*f-d*e)/f)^(1/2)*((x+(-e*f)^(1/2)/f)^2*d-2
*d*(-e*f)^(1/2)/f*(x+(-e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2))/(x+(-e*f
)^(1/2)/f))*d*e*a+1/2/(-e*f)^(1/2)/f^2/((c*f-d*e)/f)^(1/2)*ln((2*
(c*f-d*e)/f-2*d*(-e*f)^(1/2)/f*(x+(-e*f)^(1/2)/f)+2*((c*f-d*e)/f)
```

```
^(1/2)*((x+(-e*f)^(1/2)/f)^2*d-2*d*(-e*f)^(1/2)/f*(x+(-e*f)^(1/2)
/f)+(c*f-d*e)/f)^(1/2))/(x+(-e*f)^(1/2)/f))*d*e^2*b+1/2/(-e*f)^(1
/2)*((x-(-e*f)^(1/2)/f)^2*d+2*d*(-e*f)^(1/2)/f*(x-(-e*f)^(1/2)/f)
+(c*f-d*e)/f)^(1/2)*a-1/2/(-e*f)^(1/2)/f*((x-(-e*f)^(1/2)/f)^2*d+
2*d*(-e*f)^(1/2)/f*(x-(-e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2)*b*e+1/2/
f*d^(1/2)*ln((d*(-e*f)^(1/2)/f+(x-(-e*f)^(1/2)/f)*d)/d^(1/2)+((x-
(-e*f)^(1/2)/f)^2*d+2*d*(-e*f)^(1/2)/f*(x-(-e*f)^(1/2)/f)+(c*f-d*
e)/f)^(1/2))*a-1/2/f^2*d^(1/2)*ln((d*(-e*f)^(1/2)/f+(x-(-e*f)^(1/
2)/f)*d)/d^(1/2)+((x-(-e*f)^(1/2)/f)^2*d+2*d*(-e*f)^(1/2)/f*(x-(-
e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2))*b*e-1/2/(-e*f)^(1/2)/((c*f-d*e)
/f)^(1/2)*ln((2*(c*f-d*e)/f+2*d*(-e*f)^(1/2)/f*(x-(-e*f)^(1/2)/f)
+2*((c*f-d*e)/f)^(1/2)*((x-(-e*f)^(1/2)/f)^2*d+2*d*(-e*f)^(1/2)/f
*(x-(-e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2))/(x-(-e*f)^(1/2)/f))*c*a+1
/2/(-e*f)^(1/2)/f/((c*f-d*e)/f)^(1/2)*ln((2*(c*f-d*e)/f+2*d*(-e*f
)^(1/2)/f*(x-(-e*f)^(1/2)/f)+2*((c*f-d*e)/f)^(1/2)*((x-(-e*f)^(1/
2)/f)^2*d+2*d*(-e*f)^(1/2)/f*(x-(-e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2
))/(x-(-e*f)^(1/2)/f))*c*b*e+1/2/(-e*f)^(1/2)/f/((c*f-d*e)/f)^(1/
2)*ln((2*(c*f-d*e)/f+2*d*(-e*f)^(1/2)/f*(x-(-e*f)^(1/2)/f)+2*((c*
f-d*e)/f)^(1/2)*((x-(-e*f)^(1/2)/f)^2*d+2*d*(-e*f)^(1/2)/f*(x-(-e
*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2))/(x-(-e*f)^(1/2)/f))*d*e*a-1/2/(-
e*f)^(1/2)/f^2/((c*f-d*e)/f)^(1/2)*ln((2*(c*f-d*e)/f+2*d*(-e*f)^(
1/2)/f*(x-(-e*f)^(1/2)/f)+2*((c*f-d*e)/f)^(1/2)*((x-(-e*f)^(1/2)/
f)^2*d+2*d*(-e*f)^(1/2)/f*(x-(-e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2))/
(x-(-e*f)^(1/2)/f))*d*e^2*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32636, size = 1, normalized size = 0.01

$$\left[\frac{2\sqrt{dx^2+c}b\sqrt{d}fx-(be-af)\sqrt{d}\sqrt{\frac{de-cf}{e}}\log\left(\frac{(8d^2e^2-8cdef+c^2f^2)x^4+c^2e^2+2(4cde^2-3c^2ef)x^2-4(ce^2x+(2de^2-cef)x^3)\sqrt{dx^2+c}\sqrt{\frac{de-cf}{e}}}{f^2x^4+2efx^2+e^2}\right.}{4\sqrt{d}f^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(d*x^2 + c)*b*sqrt(d)*f*x - (b*e - a*f)*sqrt(d)*sqrt(
(d*e - c*f)/e)*log(((8*d^2*e^2 - 8*c*d*e*f + c^2*f^2)*x^4 + c^2*e
^2 + 2*(4*c*d*e^2 - 3*c^2*e*f)*x^2 - 4*(c*e^2*x + (2*d*e^2 - c*e*
f)*x^3)*sqrt(d*x^2 + c)*sqrt((d*e - c*f)/e))/(f^2*x^4 + 2*e*f*x^2
 + e^2)) - (2*b*d*e - (b*c + 2*a*d)*f)*log(-2*sqrt(d*x^2 + c)*d*x
 - (2*d*x^2 + c)*sqrt(d)))/(sqrt(d)*f^2), 1/4*(2*sqrt(d*x^2 + c)*
b*sqrt(-d)*f*x - (b*e - a*f)*sqrt(-d)*sqrt((d*e - c*f)/e)*log(((8
*d^2*e^2 - 8*c*d*e*f + c^2*f^2)*x^4 + c^2*e^2 + 2*(4*c*d*e^2 - 3*
c^2*e*f)*x^2 - 4*(c*e^2*x + (2*d*e^2 - c*e*f)*x^3)*sqrt(d*x^2 + c
)*sqrt((d*e - c*f)/e))/(f^2*x^4 + 2*e*f*x^2 + e^2)) - 2*(2*b*d*e
- (b*c + 2*a*d)*f)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)))/(sqrt(-d)*
f^2), 1/4*(2*sqrt(d*x^2 + c)*b*sqrt(d)*f*x - 2*(b*e - a*f)*sqrt(d
)*sqrt(-(d*e - c*f)/e)*arctan(-1/2*((2*d*e - c*f)*x^2 + c*e)/(sqr
t(d*x^2 + c)*e*x*sqrt(-(d*e - c*f)/e))) - (2*b*d*e - (b*c + 2*a*d
)*f)*log(-2*sqrt(d*x^2 + c)*d*x - (2*d*x^2 + c)*sqrt(d)))/(sqrt(d
)*f^2), 1/2*(sqrt(d*x^2 + c)*b*sqrt(-d)*f*x - (b*e - a*f)*sqrt(-d
)*sqrt(-(d*e - c*f)/e)*arctan(-1/2*((2*d*e - c*f)*x^2 + c*e)/(sqr
t(d*x^2 + c)*e*x*sqrt(-(d*e - c*f)/e))) - (2*b*d*e - (b*c + 2*a*d
)*f)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)))/(sqrt(-d)*f^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^2)\sqrt{c + dx^2}}{e + fx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(1/2)/(f*x**2+e),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2), x)
```

GIAC/XCAS [A] time = 0.28336, size = 227, normalized size = 1.77

$$\frac{\sqrt{dx^2+c}\,bx}{2\,f} - \frac{\left(ac\sqrt{d}f^2 - bc\sqrt{d}fe - ad^{\frac{3}{2}}fe + bd^{\frac{3}{2}}e^2\right)\arctan\left(\frac{\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2 f - cf + 2\,de}{2\,\sqrt{cdfe - d^2e^2}}\right)}{\sqrt{cdfe - d^2e^2}\,f^2} - \frac{(bcf + 2\,adf - 2\,bde)\ln\left(\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2\right)}{4\,\sqrt{d}f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*b*x/f - (a*c*sqrt(d)*f^2 - b*c*sqrt(d)*f*e -
a*d^(3/2)*f*e + b*d^(3/2)*e^2)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^
2 + c))^2*f - c*f + 2*d*e)/sqrt(c*d*f*e - d^2*e^2))/(sqrt(c*d*f*e
 - d^2*e^2)*f^2) - 1/4*(b*c*f + 2*a*d*f - 2*b*d*e)*ln((sqrt(d)*x
- sqrt(d*x^2 + c))^2)/(sqrt(d)*f^2)
```

3.58 $\int \frac{(a+bx^2)^3}{(c+dx^2)\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=304

$$\frac{b\left(8a^2f^2-8abef+3b^2e^2\right)\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{8df^{5/2}}-\frac{b^2x\sqrt{e+fx^2}(bc-ad)}{2d^2f}-\frac{3b^2x\sqrt{e+fx^2}(be-2af)}{8df^2}+\frac{b^2x\left(a+bx^2\right)\sqrt{e+fx^2}}{4df}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}d^3\sqrt{de-cf}}+\frac{b(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d^3\sqrt{f}}+\frac{b(bc-ad)(be-2af)\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2d^2f^{3/2}}$$

```
[Out] -(b^2*(b*c - a*d)*x*Sqrt[e + f*x^2])/(2*d^2*f) - (3*b^2*(b*e - 2*
a*f)*x*Sqrt[e + f*x^2])/(8*d*f^2) + (b^2*x*(a + b*x^2)*Sqrt[e + f
*x^2])/(4*d*f) - ((b*c - a*d)^3*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[
c]*Sqrt[e + f*x^2])])/(Sqrt[c]*d^3*Sqrt[d*e - c*f]) + (b*(b*c - a
*d)^2*ArcTanh[(Sqrt[f]*x)/Sqrt[e + f*x^2]])/(d^3*Sqrt[f]) + (b*(b
*c - a*d)*(b*e - 2*a*f)*ArcTanh[(Sqrt[f]*x)/Sqrt[e + f*x^2]])/(2*
d^2*f^(3/2)) + (b*(3*b^2*e^2 - 8*a*b*e*f + 8*a^2*f^2)*ArcTanh[(Sq
rt[f]*x)/Sqrt[e + f*x^2]])/(8*d*f^(5/2))
```

Rubi [A] time = 0.909412, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{b\left(8a^2f^2-8abef+3b^2e^2\right)\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{8df^{5/2}}-\frac{b^2x\sqrt{e+fx^2}(bc-ad)}{2d^2f}-\frac{3b^2x\sqrt{e+fx^2}(be-2af)}{8df^2}+\frac{b^2x\left(a+bx^2\right)\sqrt{e+fx^2}}{4df}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}d^3\sqrt{de-cf}}+\frac{b(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d^3\sqrt{f}}+\frac{b(bc-ad)(be-2af)\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2d^2f^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^3/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] -(b^2*(b*c - a*d)*x*Sqrt[e + f*x^2])/(2*d^2*f) - (3*b^2*(b*e - 2*
a*f)*x*Sqrt[e + f*x^2])/(8*d*f^2) + (b^2*x*(a + b*x^2)*Sqrt[e + f
*x^2])/(4*d*f) - ((b*c - a*d)^3*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[
c]*Sqrt[e + f*x^2])])/(Sqrt[c]*d^3*Sqrt[d*e - c*f]) + (b*(b*c - a
```

```
*d)^2*ArcTanh[(Sqrt[f]*x)/Sqrt[e + f*x^2]])/(d^3*Sqrt[f]) + (b*(b
*c - a*d)*(b*e - 2*a*f)*ArcTanh[(Sqrt[f]*x)/Sqrt[e + f*x^2]])/(2*
d^2*f^(3/2)) + (b*(3*b^2*e^2 - 8*a*b*e*f + 8*a^2*f^2)*ArcTanh[(Sq
rt[f]*x)/Sqrt[e + f*x^2]])/(8*d*f^(5/2))
```

Rubi in Sympy [A] time = 100.828, size = 280, normalized size = 0.92

$$\frac{b^2x\left(a+bx^2\right)\sqrt{e+fx^2}}{4df}+\frac{3b^2x\sqrt{e+fx^2}\,(2af-be)}{8df^2}+\frac{b^2x\sqrt{e+fx^2}\,(ad-bc)}{2d^2f}+\frac{b\left(8a^2f^2-8abef+3b^2e^2\right)\operatorname{atanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{8df^{\frac{5}{2}}}+\frac{b\,(ad-bc)\,(2af-be)\operatorname{atanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2d^2f^{\frac{3}{2}}}+\frac{b\,(ad-bc)^2\operatorname{atanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d^3\sqrt{f}}+\frac{(ad-bc)^3\operatorname{atanh}\left(\frac{x\sqrt{cf-de}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}d^3\sqrt{cf-de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**3/(d*x**2+c)/(f*x**2+e)**(1/2),x)
```

```
[Out] b**2*x*(a + b*x**2)*sqrt(e + f*x**2)/(4*d*f) + 3*b**2*x*sqrt(e +
f*x**2)*(2*a*f - b*e)/(8*d*f**2) + b**2*x*sqrt(e + f*x**2)*(a*d -
 b*c)/(2*d**2*f) + b*(8*a**2*f**2 - 8*a*b*e*f + 3*b**2*e**2)*atan
h(sqrt(f)*x/sqrt(e + f*x**2))/(8*d*f**(5/2)) + b*(a*d - b*c)*(2*a
*f - b*e)*atanh(sqrt(f)*x/sqrt(e + f*x**2))/(2*d**2*f**(3/2)) + b
*(a*d - b*c)**2*atanh(sqrt(f)*x/sqrt(e + f*x**2))/(d**3*sqrt(f))
+ (a*d - b*c)**3*atanh(x*sqrt(c*f - d*e)/(sqrt(c)*sqrt(e + f*x**2
)))/(sqrt(c)*d**3*sqrt(c*f - d*e))
```

Mathematica [A] time = 0.423479, size = 194, normalized size = 0.64

$$\frac{\frac{b\log\left(\sqrt{f}\sqrt{e+fx^2}+fx\right)\left(24a^2d^2f^2-12abdf(2cf+de)+b^2\left(8c^2f^2+4cdef+3d^2e^2\right)\right)}{f^{5/2}}+\frac{b^2dx\sqrt{e+fx^2}\left(12adf+b\left(-4cf-3de+2dfx^2\right)\right)}{f^2}+\frac{8(ad-bc)^3\tan^{-1}\left(\frac{x\sqrt{de-}}{\sqrt{c}\sqrt{e+}}\right.}{\sqrt{c}\sqrt{de-cf}}}{8d^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^3/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] ((b^2*d*x*Sqrt[e + f*x^2]*(12*a*d*f + b*(-3*d*e - 4*c*f + 2*d*f*x
^2)))/f^2 + (8*(-(b*c) + a*d)^3*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[
c]*Sqrt[e + f*x^2])])/(Sqrt[c]*Sqrt[d*e - c*f]) + (b*(24*a^2*d^2*
f^2 - 12*a*b*d*f*(d*e + 2*c*f) + b^2*(3*d^2*e^2 + 4*c*d*e*f + 8*c
```

```
^2*f^2))*Log[f*x + Sqrt[f]*Sqrt[e + f*x^2]])/f^(5/2))/(8*d^3)
```

Maple [B] time = 0.062, size = 1541, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^3/(d*x^2+c)/(f*x^2+e)^(1/2),x)
```

```
[Out] b^3/d^3*c^2*ln(x*f^(1/2)+(f*x^2+e)^(1/2))/f^(1/2)+3/2*b^2/d*x/f*(
f*x^2+e)^(1/2)*a-1/2*b^3/d^2*x/f*(f*x^2+e)^(1/2)*c-3/2*b^2/d*e/f^
(3/2)*ln(x*f^(1/2)+(f*x^2+e)^(1/2))*a+1/2*b^3/d^2*e/f^(3/2)*ln(x*
f^(1/2)+(f*x^2+e)^(1/2))*c+1/4*b^3/d*x^3/f*(f*x^2+e)^(1/2)-3/8*b^
3/d*e/f^2*x*(f*x^2+e)^(1/2)+3/8*b^3/d*e^2/f^(5/2)*ln(x*f^(1/2)+(f
*x^2+e)^(1/2))+3*b/d*a^2*ln(x*f^(1/2)+(f*x^2+e)^(1/2))/f^(1/2)-3*
b^2/d^2*c*a*ln(x*f^(1/2)+(f*x^2+e)^(1/2))/f^(1/2)+1/2/(-c*d)^(1/2
)/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d-2*f*(-c*d)^(1/2)/d*(x+(
-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*f-2*f
*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x+(-c*d)^
(1/2)/d))*a^3-3/2/d/(-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f
-d*e)/d-2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1
/2)*((x+(-c*d)^(1/2)/d)^2*f-2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)
-(c*f-d*e)/d)^(1/2))/(x+(-c*d)^(1/2)/d))*a^2*c*b+3/2/d^2/(-c*d)^(
1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d-2*f*(-c*d)^(1/2)/d*(
x+(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*f-
2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x+(-c*
d)^(1/2)/d))*a*c^2*b^2-1/2/d^3/(-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*
ln((-2*(c*f-d*e)/d-2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*(-(c*f
-d*e)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*f-2*f*(-c*d)^(1/2)/d*(x+(-c*
d)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x+(-c*d)^(1/2)/d))*c^3*b^3-1/2/(
-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d+2*f*(-c*d)^(1
/2)/d*(x-(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x-(-c*d)^(1/2)/
d)^2*f+2*f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/
(x-(-c*d)^(1/2)/d))*a^3+3/2/d/(-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*l
n((-2*(c*f-d*e)/d+2*f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*(-(c*f-
d*e)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*f+2*f*(-c*d)^(1/2)/d*(x-(-c*d
)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x-(-c*d)^(1/2)/d))*a^2*c*b-3/2/d^
2/(-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d+2*f*(-c*d)
^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x-(-c*d)^(1/
2)/d)^2*f+2*f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2
))/(x-(-c*d)^(1/2)/d))*a*c^2*b^2+1/2/d^3/(-c*d)^(1/2)/(-(c*f-d*e)
/d)^(1/2)*ln((-2*(c*f-d*e)/d+2*f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d
)+2*(-(c*f-d*e)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*f+2*f*(-c*d)^(1/2)
/d*(x-(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x-(-c*d)^(1/2)/d))*c^3
*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^3/((d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.9531, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^3/((d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*f^(
5/2)*log((((d^2*e^2 - 8*c*d*e*f + 8*c^2*f^2)*x^4 + c^2*e^2 - 2*(3
*c*d*e^2 - 4*c^2*e*f)*x^2)*sqrt(-c*d*e + c^2*f) + 4*((c*d^2*e^2 -
 3*c^2*d*e*f + 2*c^3*f^2)*x^3 - (c^2*d*e^2 - c^3*e*f)*x)*sqrt(f*x
^2 + e))/(d^2*x^4 + 2*c*d*x^2 + c^2)) - 2*(2*b^3*d^2*f*x^3 - (3*b
^3*d^2*e + 4*(b^3*c*d - 3*a*b^2*d^2)*f)*x)*sqrt(-c*d*e + c^2*f)*s
qrt(f*x^2 + e)*sqrt(f) - (3*b^3*d^2*e^2 + 4*(b^3*c*d - 3*a*b^2*d^
2)*e*f + 8*(b^3*c^2 - 3*a*b^2*c*d + 3*a^2*b*d^2)*f^2)*sqrt(-c*d*e
 + c^2*f)*log(-2*sqrt(f*x^2 + e)*f*x - (2*f*x^2 + e)*sqrt(f)))/(s
qrt(-c*d*e + c^2*f)*d^3*f^(5/2)), -1/16*(8*(b^3*c^3 - 3*a*b^2*c^2
*d + 3*a^2*b*c*d^2 - a^3*d^3)*f^(5/2)*arctan(1/2*((d*e - 2*c*f)*x
^2 - c*e)/(sqrt(c*d*e - c^2*f)*sqrt(f*x^2 + e)*x)) - 2*(2*b^3*d^2
*f*x^3 - (3*b^3*d^2*e + 4*(b^3*c*d - 3*a*b^2*d^2)*f)*x)*sqrt(c*d*
e - c^2*f)*sqrt(f*x^2 + e)*sqrt(f) - (3*b^3*d^2*e^2 + 4*(b^3*c*d
- 3*a*b^2*d^2)*e*f + 8*(b^3*c^2 - 3*a*b^2*c*d + 3*a^2*b*d^2)*f^2)
*sqrt(c*d*e - c^2*f)*log(-2*sqrt(f*x^2 + e)*f*x - (2*f*x^2 + e)*s
qrt(f)))/(sqrt(c*d*e - c^2*f)*d^3*f^(5/2)), -1/8*(2*(b^3*c^3 - 3*
a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-f)*f^2*log((((d^2*e^
2 - 8*c*d*e*f + 8*c^2*f^2)*x^4 + c^2*e^2 - 2*(3*c*d*e^2 - 4*c^2*e
*f)*x^2)*sqrt(-c*d*e + c^2*f) + 4*((c*d^2*e^2 - 3*c^2*d*e*f + 2*c
^3*f^2)*x^3 - (c^2*d*e^2 - c^3*e*f)*x)*sqrt(f*x^2 + e))/(d^2*x^4
+ 2*c*d*x^2 + c^2)) - (2*b^3*d^2*f*x^3 - (3*b^3*d^2*e + 4*(b^3*c*
d - 3*a*b^2*d^2)*f)*x)*sqrt(-c*d*e + c^2*f)*sqrt(f*x^2 + e)*sqrt(
-f) - (3*b^3*d^2*e^2 + 4*(b^3*c*d - 3*a*b^2*d^2)*e*f + 8*(b^3*c^2
 - 3*a*b^2*c*d + 3*a^2*b*d^2)*f^2)*sqrt(-c*d*e + c^2*f)*arctan(sq
rt(-f)*x/sqrt(f*x^2 + e)))/(sqrt(-c*d*e + c^2*f)*d^3*sqrt(-f)*f^2
), -1/8*(4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sq
rt(-f)*f^2*arctan(1/2*((d*e - 2*c*f)*x^2 - c*e)/(sqrt(c*d*e - c^2
*f)*sqrt(f*x^2 + e)*x)) - (2*b^3*d^2*f*x^3 - (3*b^3*d^2*e + 4*(b^
3*c*d - 3*a*b^2*d^2)*f)*x)*sqrt(c*d*e - c^2*f)*sqrt(f*x^2 + e)*sq
rt(-f) - (3*b^3*d^2*e^2 + 4*(b^3*c*d - 3*a*b^2*d^2)*e*f + 8*(b^3*
c^2 - 3*a*b^2*c*d + 3*a^2*b*d^2)*f^2)*sqrt(c*d*e - c^2*f)*arctan(
sqrt(-f)*x/sqrt(f*x^2 + e)))/(sqrt(c*d*e - c^2*f)*d^3*sqrt(-f)*f^
2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^3}{\left(c+dx^2\right)\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**3/(d*x**2+c)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**3/((c + d*x**2)*sqrt(e + f*x**2)), x)
```

GIAC/XCAS [A] time = 0.273489, size = 392, normalized size = 1.29

$$\frac{1}{8}\left(\frac{2\,b^3x^2}{df}-\frac{4\,b^3cd^4f^2-12\,ab^2d^5f^2+3\,b^3d^5fe}{d^6f^3}\right)\sqrt{fx^2+e}x+\frac{\left(b^3c^3\sqrt{f}-3\,ab^2c^2d\sqrt{f}+3\,a^2bcd^2\sqrt{f}-a^3d^3\sqrt{f}\right)\arctan\left(\frac{\left(\sqrt{f}x-\sqrt{fx^2+e}\right)^2d+2\,cf-de}{2\sqrt{-c^2f^2+cdfe}}\right)}{\sqrt{-c^2f^2+cdfe}d^3}-\frac{\left(8\,b^3c^2f^2-24\,ab^2cdf^2+24\,a^2bd^2f^2+4\,b^3cdfe-12\,ab^2d^2fe+3\,b^3d^2e^2\right)\ln\left(\left(\sqrt{f}x-\sqrt{fx^2+e}\right)^2\right)}{16\,d^3f^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^3/((d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="giac")
```

```
[Out] 1/8*(2*b^3*x^2/(d*f) - (4*b^3*c*d^4*f^2 - 12*a*b^2*d^5*f^2 + 3*b^
3*d^5*f*e)/(d^6*f^3))*sqrt(f*x^2 + e)*x + (b^3*c^3*sqrt(f) - 3*a*
b^2*c^2*d*sqrt(f) + 3*a^2*b*c*d^2*sqrt(f) - a^3*d^3*sqrt(f))*arct
an(1/2*((sqrt(f)*x - sqrt(f*x^2 + e))^2*d + 2*c*f - d*e)/sqrt(-c^
2*f^2 + c*d*f*e))/(sqrt(-c^2*f^2 + c*d*f*e)*d^3) - 1/16*(8*b^3*c^
2*f^2 - 24*a*b^2*c*d*f^2 + 24*a^2*b*d^2*f^2 + 4*b^3*c*d*f*e - 12*
a*b^2*d^2*f*e + 3*b^3*d^2*e^2)*ln((sqrt(f)*x - sqrt(f*x^2 + e))^2
)/(d^3*f^(5/2))
```

3.59 $\int \frac{(a+bx^2)^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=166

$$\frac{(bc-ad)^2\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}d^2\sqrt{de-cf}}-\frac{b(bc-ad)\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d^2\sqrt{f}}-\frac{b(be-2af)\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2df^{3/2}}+\frac{b^2x\sqrt{e+fx^2}}{2df}$$

```
[Out] (b^2*x*Sqrt[e + f*x^2])/(2*d*f) + ((b*c - a*d)^2*ArcTan[(Sqrt[d*e
 - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])])/(Sqrt[c]*d^2*Sqrt[d*e - c*
f]) - (b*(b*c - a*d)*ArcTanh[(Sqrt[f]*x)/Sqrt[e + f*x^2]])/(d^2*S
qrt[f]) - (b*(b*e - 2*a*f)*ArcTanh[(Sqrt[f]*x)/Sqrt[e + f*x^2]])/
(2*d*f^(3/2))
```

Rubi [A] time = 0.418915, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$

$$\frac{(bc-ad)^2\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}d^2\sqrt{de-cf}}-\frac{b(bc-ad)\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d^2\sqrt{f}}-\frac{b(be-2af)\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2df^{3/2}}+\frac{b^2x\sqrt{e+fx^2}}{2df}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)^2/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (b^2*x*Sqrt[e + f*x^2])/(2*d*f) + ((b*c - a*d)^2*ArcTan[(Sqrt[d*e
 - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])])/(Sqrt[c]*d^2*Sqrt[d*e - c*
f]) - (b*(b*c - a*d)*ArcTanh[(Sqrt[f]*x)/Sqrt[e + f*x^2]])/(d^2*S
qrt[f]) - (b*(b*e - 2*a*f)*ArcTanh[(Sqrt[f]*x)/Sqrt[e + f*x^2]])/
(2*d*f^(3/2))
```

Rubi in Sympy [A] time = 54.8896, size = 148, normalized size = 0.89

$$\frac{b^2x\sqrt{e+fx^2}}{2df}+\frac{b\,(2af-be)\operatorname{atanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2df^{\frac{3}{2}}}+\frac{b\,(ad-bc)\operatorname{atanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d^2\sqrt{f}}+\frac{(ad-bc)^2\operatorname{atanh}\left(\frac{x\sqrt{cf-de}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}d^2\sqrt{cf-de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**2/(d*x**2+c)/(f*x**2+e)**(1/2),x)
```

```
[Out] b**2*x*sqrt(e + f*x**2)/(2*d*f) + b*(2*a*f - b*e)*atanh(sqrt(f)*x
/sqrt(e + f*x**2))/(2*d*f**(3/2)) + b*(a*d - b*c)*atanh(sqrt(f)*x
```

```
/sqrt(e + f*x**2))/(d**2*sqrt(f)) + (a*d - b*c)**2*atanh(x*sqrt(c
*f - d*e)/(sqrt(c)*sqrt(e + f*x**2)))/(sqrt(c)*d**2*sqrt(c*f - d*
e))
```

Mathematica [A] time = 0.248661, size = 133, normalized size = 0.8

$$\frac{-\frac{b\log\left(\sqrt{f}\sqrt{e+fx^2}+fx\right)(-4adf+2bcf+bde)}{f^{3/2}}+\frac{2(bc-ad)^2\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}\sqrt{de-cf}}+\frac{b^2dx\sqrt{e+fx^2}}{f}}{2d^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)^2/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] ((b^2*d*x*Sqrt[e + f*x^2])/f + (2*(b*c - a*d)^2*ArcTan[(Sqrt[d*e
- c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])])/(Sqrt[c]*Sqrt[d*e - c*f]) -
 (b*(b*d*e + 2*b*c*f - 4*a*d*f)*Log[f*x + Sqrt[f]*Sqrt[e + f*x^2]
])/f^(3/2))/(2*d^2)
```

Maple [B] time = 0.023, size = 1052, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x)
```

```
[Out] 1/2*b^2*x*(f*x^2+e)^(1/2)/f/d-1/2*b^2/d*e/f^(3/2)*ln(x*f^(1/2)+(f
*x^2+e)^(1/2))+2*b/d*a*ln(x*f^(1/2)+(f*x^2+e)^(1/2))/f^(1/2)-b^2/
d^2*c*ln(x*f^(1/2)+(f*x^2+e)^(1/2))/f^(1/2)+1/2/(-c*d)^(1/2)/(-(c
*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d-2*f*(-c*d)^(1/2)/d*(x+(-c*d)^
(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*f-2*f*(-c*d
)^(1/2)/d*(x+(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x+(-c*d)^(1/2)/
d))*a^2-1/d/(-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d-
2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x+
(-c*d)^(1/2)/d)^2*f-2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)-(c*f-d*
e)/d)^(1/2))/(x+(-c*d)^(1/2)/d))*c*a*b+1/2/d^2/(-c*d)^(1/2)/(-(c*
f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d-2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(
1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*f-2*f*(-c*d)
^(1/2)/d*(x+(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x+(-c*d)^(1/2)/d
))*b^2*c^2-1/2/(-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)
/d+2*f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*(
(x-(-c*d)^(1/2)/d)^2*f+2*f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)-(c*f
-d*e)/d)^(1/2))/(x-(-c*d)^(1/2)/d))*a^2+1/d/(-c*d)^(1/2)/(-(c*f-d
*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d+2*f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2
)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*f+2*f*(-c*d)^(1
```

```
/2)/d*(x-(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x-(-c*d)^(1/2)/d))*
c*a*b-1/2/d^2/(-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/
d+2*f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((
x-(-c*d)^(1/2)/d)^2*f+2*f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)-(c*f-
d*e)/d)^(1/2))/(x-(-c*d)^(1/2)/d))*b^2*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.66891, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-c*d*e + c^2*f)*sqrt(f*x^2 + e)*b^2*d*sqrt(f)*x + (b
^2*c^2 - 2*a*b*c*d + a^2*d^2)*f^(3/2)*log((((d^2*e^2 - 8*c*d*e*f
+ 8*c^2*f^2)*x^4 + c^2*e^2 - 2*(3*c*d*e^2 - 4*c^2*e*f)*x^2)*sqrt(
-c*d*e + c^2*f) + 4*((c*d^2*e^2 - 3*c^2*d*e*f + 2*c^3*f^2)*x^3 -
(c^2*d*e^2 - c^3*e*f)*x)*sqrt(f*x^2 + e))/(d^2*x^4 + 2*c*d*x^2 +
c^2)) - (b^2*d*e + 2*(b^2*c - 2*a*b*d)*f)*sqrt(-c*d*e + c^2*f)*lo
g(-2*sqrt(f*x^2 + e)*f*x - (2*f*x^2 + e)*sqrt(f)))/(sqrt(-c*d*e +
 c^2*f)*d^2*f^(3/2)), 1/4*(2*sqrt(c*d*e - c^2*f)*sqrt(f*x^2 + e)*
b^2*d*sqrt(f)*x + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*f^(3/2)*arcta
n(1/2*((d*e - 2*c*f)*x^2 - c*e)/(sqrt(c*d*e - c^2*f)*sqrt(f*x^2 +
 e)*x)) - (b^2*d*e + 2*(b^2*c - 2*a*b*d)*f)*sqrt(c*d*e - c^2*f)*l
og(-2*sqrt(f*x^2 + e)*f*x - (2*f*x^2 + e)*sqrt(f)))/(sqrt(c*d*e -
 c^2*f)*d^2*f^(3/2)), 1/4*(2*sqrt(-c*d*e + c^2*f)*sqrt(f*x^2 + e)
*b^2*d*sqrt(-f)*x + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-f)*f*lo
g((((d^2*e^2 - 8*c*d*e*f + 8*c^2*f^2)*x^4 + c^2*e^2 - 2*(3*c*d*e^
2 - 4*c^2*e*f)*x^2)*sqrt(-c*d*e + c^2*f) + 4*((c*d^2*e^2 - 3*c^2*
d*e*f + 2*c^3*f^2)*x^3 - (c^2*d*e^2 - c^3*e*f)*x)*sqrt(f*x^2 + e)
)/(d^2*x^4 + 2*c*d*x^2 + c^2)) - 2*(b^2*d*e + 2*(b^2*c - 2*a*b*d)
*f)*sqrt(-c*d*e + c^2*f)*arctan(sqrt(-f)*x/sqrt(f*x^2 + e)))/(sqr
t(-c*d*e + c^2*f)*d^2*sqrt(-f)*f), 1/2*(sqrt(c*d*e - c^2*f)*sqrt(
f*x^2 + e)*b^2*d*sqrt(-f)*x + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqr
t(-f)*f*arctan(1/2*((d*e - 2*c*f)*x^2 - c*e)/(sqrt(c*d*e - c^2*f)
*sqrt(f*x^2 + e)*x)) - (b^2*d*e + 2*(b^2*c - 2*a*b*d)*f)*sqrt(c*d
```

```
*e - c^2*f)*arctan(sqrt(-f)*x/sqrt(f*x^2 + e)))/(sqrt(c*d*e - c^2
*f)*d^2*sqrt(-f)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx^2)^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**2/(d*x**2+c)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**2/((c + d*x**2)*sqrt(e + f*x**2)), x)
```

GIAC/XCAS [A] time = 0.272211, size = 248, normalized size = 1.49

$$\frac{\sqrt{fx^2+e}\,b^2x}{2\,df} - \frac{\left(b^2c^2\sqrt{f} - 2\,abcd\sqrt{f} + a^2d^2\sqrt{f}\right)\arctan\left(\frac{\left(\sqrt{f}x-\sqrt{fx^2+e}\right)^2d+2\,cf-de}{2\,\sqrt{-c^2f^2+cdfe}}\right)}{\sqrt{-c^2f^2+cdfe}\,d^2} + \frac{\left(2\,b^2cf^{\frac{3}{2}} - 4\,abdf^{\frac{3}{2}} + b^2d\sqrt{f}e\right)\ln\left(\left(\sqrt{f}x-\sqrt{fx^2+e}\right)^2\right)}{4\,d^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^2/((d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(f*x^2 + e)*b^2*x/(d*f) - (b^2*c^2*sqrt(f) - 2*a*b*c*d*sq
rt(f) + a^2*d^2*sqrt(f))*arctan(1/2*((sqrt(f)*x - sqrt(f*x^2 + e)
)^2*d + 2*c*f - d*e)/sqrt(-c^2*f^2 + c*d*f*e))/(sqrt(-c^2*f^2 + c
*d*f*e)*d^2) + 1/4*(2*b^2*c*f^(3/2) - 4*a*b*d*f^(3/2) + b^2*d*sqr
t(f)*e)*ln((sqrt(f)*x - sqrt(f*x^2 + e))^2)/(d^2*f^2)
```

3.60 $\int \frac{a+bx^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=91

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d\sqrt{f}}-\frac{(bc-ad)\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}d\sqrt{de-cf}}$$

```
[Out] -(((b*c - a*d)*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2
])])/(Sqrt[c]*d*Sqrt[d*e - c*f])) + (b*ArcTanh[(Sqrt[f]*x)/Sqrt[e
 + f*x^2]])/(d*Sqrt[f])
```

Rubi [A] time = 0.198055, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d\sqrt{f}}-\frac{(bc-ad)\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}d\sqrt{de-cf}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] -(((b*c - a*d)*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2
])])/(Sqrt[c]*d*Sqrt[d*e - c*f])) + (b*ArcTanh[(Sqrt[f]*x)/Sqrt[e
 + f*x^2]])/(d*Sqrt[f])
```

Rubi in Sympy [A] time = 26.6409, size = 78, normalized size = 0.86

$$\frac{b\operatorname{atanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d\sqrt{f}}+\frac{(ad-bc)\operatorname{atanh}\left(\frac{x\sqrt{cf-de}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}d\sqrt{cf-de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/(d*x**2+c)/(f*x**2+e)**(1/2),x)
```

```
[Out] b*atanh(sqrt(f)*x/sqrt(e + f*x**2))/(d*sqrt(f)) + (a*d - b*c)*ata
nh(x*sqrt(c*f - d*e)/(sqrt(c)*sqrt(e + f*x**2)))/(sqrt(c)*d*sqrt(
c*f - d*e))
```

Mathematica [A] time = 0.147799, size = 93, normalized size = 1.02

$$\frac{(ad-bc)\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}d\sqrt{de-cf}}+\frac{b\log\left(\sqrt{f}\sqrt{e+fx^2}+fx\right)}{d\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] ((-(b*c) + a*d)*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^
2])])/(Sqrt[c]*d*Sqrt[d*e - c*f]) + (b*Log[f*x + Sqrt[f]*Sqrt[e +
 f*x^2]])/(d*Sqrt[f])
```

Maple [B] time = 0.017, size = 646, normalized size = 7.1

$$\frac{b}{d}\ln\left(x\sqrt{f}+\sqrt{fx^2+e}\right)\frac{1}{\sqrt{f}}$$
$$+\frac{a}{2}\ln\left(1\left(-2\frac{cf-de}{d}-2\frac{f\sqrt{-cd}}{d}\left(x+\frac{\sqrt{-cd}}{d}\right)+2\sqrt{-\frac{cf-de}{d}}\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^2f-2\frac{f\sqrt{-cd}}{d}\left(x+\frac{\sqrt{-cd}}{d}\right)-\frac{cf-de}{d}}\right)\left(x\right.\right.$$
$$-\frac{bc}{2d}\ln\left(1\left(-2\frac{cf-de}{d}-2\frac{f\sqrt{-cd}}{d}\left(x+\frac{\sqrt{-cd}}{d}\right)+2\sqrt{-\frac{cf-de}{d}}\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^2f-2\frac{f\sqrt{-cd}}{d}\left(x+\frac{\sqrt{-cd}}{d}\right)-\frac{cf-de}{d}}\right)\right.$$
$$-\frac{a}{2}\ln\left(1\left(-2\frac{cf-de}{d}+2\frac{f\sqrt{-cd}}{d}\left(x-\frac{\sqrt{-cd}}{d}\right)+2\sqrt{-\frac{cf-de}{d}}\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2f+2\frac{f\sqrt{-cd}}{d}\left(x-\frac{\sqrt{-cd}}{d}\right)-\frac{cf-de}{d}}\right)\left(\right.\right.$$
$$+\frac{bc}{2d}\ln\left(1\left(-2\frac{cf-de}{d}+2\frac{f\sqrt{-cd}}{d}\left(x-\frac{\sqrt{-cd}}{d}\right)+2\sqrt{-\frac{cf-de}{d}}\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2f+2\frac{f\sqrt{-cd}}{d}\left(x-\frac{\sqrt{-cd}}{d}\right)-\frac{cf-de}{d}}\right)\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x)
```

```
[Out] b/d*ln(x*f^(1/2)+(f*x^2+e)^(1/2))/f^(1/2)+1/2/(-c*d)^(1/2)/(-(c*f
-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d-2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1
/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*f-2*f*(-c*d)^
(1/2)/d*(x+(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x+(-c*d)^(1/2)/d)
)*a-1/2/(-c*d)^(1/2)/d/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d-2*
f*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x+(-
c*d)^(1/2)/d)^2*f-2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)-(c*f-d*e)
/d)^(1/2))/(x+(-c*d)^(1/2)/d))*b*c-1/2/(-c*d)^(1/2)/(-(c*f-d*e)/d
)^(1/2)*ln((-2*(c*f-d*e)/d+2*f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+
2*(-(c*f-d*e)/d)^(1/2)*((x-(-c*d)^(1/2)/d)^2*f+2*f*(-c*d)^(1/2)/d
```

```
*(x-(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x-(-c*d)^(1/2)/d))*a+1/2
/(-c*d)^(1/2)/d/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d+2*f*(-c*d
)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x-(-c*d)^(1
/2)/d)^2*f+2*f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/
2))/(x-(-c*d)^(1/2)/d))*b*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/((d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.07409, size = 1, normalized size = 0.01

$$\left[\frac{2\sqrt{-cde+c^2f}\,b\log\left(-2\sqrt{fx^2+e}\,fx-\left(2fx^2+e\right)\sqrt{f}\right)-(bc-ad)\sqrt{f}\log\left(\frac{\left(\left(d^2e^2-8cdef+8c^2f^2\right)x^4+c^2e^2-2\left(3cde^2-4c^2ef\right)x^2\right)\sqrt{\cdot}}{d^2\cdots}\right.}{4\sqrt{-cde+c^2f}\,d\sqrt{f}}\right.$$

$$-\frac{(bc-ad)\sqrt{f}\arctan\left(\frac{(de-2cf)x^2-ce}{2\sqrt{cde-c^2f}\sqrt{fx^2+ex}}\right)-\sqrt{cde-c^2f}\,b\log\left(-2\sqrt{fx^2+e}\,fx-\left(2fx^2+e\right)\sqrt{f}\right)}{2\sqrt{cde-c^2f}\,d\sqrt{f}},\ \frac{4\sqrt{-cde+c^2f}\,b\arctan\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/((d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-c*d*e + c^2*f)*b*log(-2*sqrt(f*x^2 + e)*f*x - (2*f*
x^2 + e)*sqrt(f)) - (b*c - a*d)*sqrt(f)*log((((d^2*e^2 - 8*c*d*e*
f + 8*c^2*f^2)*x^4 + c^2*e^2 - 2*(3*c*d*e^2 - 4*c^2*e*f)*x^2)*sqr
t(-c*d*e + c^2*f) + 4*((c*d^2*e^2 - 3*c^2*d*e*f + 2*c^3*f^2)*x^3
- (c^2*d*e^2 - c^3*e*f)*x)*sqrt(f*x^2 + e))/(d^2*x^4 + 2*c*d*x^2
+ c^2)))/(sqrt(-c*d*e + c^2*f)*d*sqrt(f)), -1/2*((b*c - a*d)*sqrt
(f)*arctan(1/2*((d*e - 2*c*f)*x^2 - c*e)/(sqrt(c*d*e - c^2*f)*sqr
t(f*x^2 + e)*x)) - sqrt(c*d*e - c^2*f)*b*log(-2*sqrt(f*x^2 + e)*f
*x - (2*f*x^2 + e)*sqrt(f)))/(sqrt(c*d*e - c^2*f)*d*sqrt(f)), 1/4
*(4*sqrt(-c*d*e + c^2*f)*b*arctan(sqrt(-f)*x/sqrt(f*x^2 + e)) - (
b*c - a*d)*sqrt(-f)*log((((d^2*e^2 - 8*c*d*e*f + 8*c^2*f^2)*x^4 +
 c^2*e^2 - 2*(3*c*d*e^2 - 4*c^2*e*f)*x^2)*sqrt(-c*d*e + c^2*f) +
4*((c*d^2*e^2 - 3*c^2*d*e*f + 2*c^3*f^2)*x^3 - (c^2*d*e^2 - c^3*e
*f)*x)*sqrt(f*x^2 + e))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/(sqrt(-c*d*
e + c^2*f)*d*sqrt(-f)), 1/2*(2*sqrt(c*d*e - c^2*f)*b*arctan(sqrt(
```

```
-f)*x/sqrt(f*x^2 + e)) - (b*c - a*d)*sqrt(-f)*arctan(1/2*((d*e -
2*c*f)*x^2 - c*e)/(sqrt(c*d*e - c^2*f)*sqrt(f*x^2 + e)*x)))/(sqrt
(c*d*e - c^2*f)*d*sqrt(-f))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + bx^2}{(c + dx^2)\sqrt{e + fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/(d*x**2+c)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)/((c + d*x**2)*sqrt(e + f*x**2)), x)
```

GIAC/XCAS [A] time = 0.269784, size = 159, normalized size = 1.75

$$\frac{\left(bc\sqrt{f} - ad\sqrt{f}\right)\arctan\left(\frac{\left(\sqrt{f}x-\sqrt{fx^2+e}\right)^2 d+2\,cf-de}{2\,\sqrt{-c^2f^2+cdfe}}\right)}{\sqrt{-c^2f^2 + cdfe}d} - \frac{b\ln\left(\left(\sqrt{f}x - \sqrt{fx^2 + e}\right)^2\right)}{2\,d\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)/((d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="giac")
```

```
[Out] (b*c*sqrt(f) - a*d*sqrt(f))*arctan(1/2*((sqrt(f)*x - sqrt(f*x^2 +
 e))^2*d + 2*c*f - d*e)/sqrt(-c^2*f^2 + c*d*f*e))/(sqrt(-c^2*f^2
+ c*d*f*e)*d) - 1/2*b*ln((sqrt(f)*x - sqrt(f*x^2 + e))^2)/(d*sqrt
(f))
```

3.61 $\int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=49

$$\frac{\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}\sqrt{de-cf}}$$

```
[Out] ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])]/(Sqrt[c]*Sq
rt[d*e - c*f])
```

Rubi [A] time = 0.0801177, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$

$$\frac{\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}\sqrt{de-cf}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])]/(Sqrt[c]*Sq
rt[d*e - c*f])
```

Rubi in Sympy [A] time = 11.5189, size = 42, normalized size = 0.86

$$\frac{\operatorname{atanh}\left(\frac{x\sqrt{cf-de}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}\sqrt{cf-de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d*x**2+c)/(f*x**2+e)**(1/2),x)
```

```
[Out] atanh(x*sqrt(c*f - d*e)/(sqrt(c)*sqrt(e + f*x**2)))/(sqrt(c)*sqrt
(c*f - d*e))
```

Mathematica [A] time = 0.0387733, size = 49, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}\sqrt{de-cf}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])]/(Sqrt[c]*Sq
rt[d*e - c*f])
```

Maple [B] time = 0.015, size = 306, normalized size = 6.2

$$-\frac{1}{2}\ln\left(1\left(-2\frac{cf-de}{d}+2\frac{f\sqrt{-cd}}{d}\left(x-\frac{\sqrt{-cd}}{d}\right)+2\sqrt{-\frac{cf-de}{d}}\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2 f+2\frac{f\sqrt{-cd}}{d}\left(x-\frac{\sqrt{-cd}}{d}\right)-\frac{cf-de}{d}}\right)\left(x\right.\right.$$
$$+\frac{1}{2}\ln\left(1\left(-2\frac{cf-de}{d}-2\frac{f\sqrt{-cd}}{d}\left(x+\frac{\sqrt{-cd}}{d}\right)+2\sqrt{-\frac{cf-de}{d}}\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^2 f-2\frac{f\sqrt{-cd}}{d}\left(x+\frac{\sqrt{-cd}}{d}\right)-\frac{cf-de}{d}}\right)\left(x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d*x^2+c)/(f*x^2+e)^(1/2),x)
```

```
[Out] -1/2/(-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d+2*f*(-c
*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x-(-c*d)^
(1/2)/d)^2*f+2*f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(
1/2))/(x-(-c*d)^(1/2)/d))+1/2/(-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*l
n((-2*(c*f-d*e)/d-2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)+2*(-(c*f-
d*e)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*f-2*f*(-c*d)^(1/2)/d*(x+(-c*d
)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x+(-c*d)^(1/2)/d))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.264382, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\frac{\left(\left(d^2e^2-8\,cdef+8\,c^2f^2\right)x^4+c^2e^2-2\left(3\,cde^2-4\,c^2ef\right)x^2\right)\sqrt{-cde+c^2f}+4\left(\left(cd^2e^2-3\,c^2def+2\,c^3f^2\right)x^3-\left(c^2de^2-c^3ef\right)x\right)\sqrt{fx^2+e}}{d^2x^4+2\,cdx^2+c^2}\right)}{4\sqrt{-cde+c^2f}}, \frac{\arctan\left(\frac{(de-2}{2\sqrt{cde-}}\right.}{2\sqrt{cde-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="fricas")
```

```
[Out] [1/4*log((((d^2*e^2 - 8*c*d*e*f + 8*c^2*f^2)*x^4 + c^2*e^2 - 2*(3
*c*d*e^2 - 4*c^2*e*f)*x^2)*sqrt(-c*d*e + c^2*f) + 4*((c*d^2*e^2 -
 3*c^2*d*e*f + 2*c^3*f^2)*x^3 - (c^2*d*e^2 - c^3*e*f)*x)*sqrt(f*x
^2 + e))/(d^2*x^4 + 2*c*d*x^2 + c^2))/sqrt(-c*d*e + c^2*f), 1/2*a
rctan(1/2*((d*e - 2*c*f)*x^2 - c*e)/(sqrt(c*d*e - c^2*f)*sqrt(f*x
^2 + e)*x))/sqrt(c*d*e - c^2*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c+dx^2\right)\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x**2+c)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(1/((c + d*x**2)*sqrt(e + f*x**2)), x)
```

GIAC/XCAS [A] time = 0.251006, size = 100, normalized size = 2.04

$$-\frac{\sqrt{f}\arctan\left(\frac{\left(\sqrt{f}x-\sqrt{fx^2+e}\right)^2d+2\,cf-de}{2\sqrt{-c^2f^2+cdfe}}\right)}{\sqrt{-c^2f^2+cdfe}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="giac")
```

```
[Out] -sqrt(f)*arctan(1/2*((sqrt(f)*x - sqrt(f*x^2 + e))^2*d + 2*c*f -
d*e)/sqrt(-c^2*f^2 + c*d*f*e))/sqrt(-c^2*f^2 + c*d*f*e)
```

3.62 $\int \frac{1}{(a+bx^2)(c+dx^2)\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=122

$$\frac{b\tan^{-1}\left(\frac{x\sqrt{be-af}}{\sqrt{a}\sqrt{e+fx^2}}\right)}{\sqrt{a}(bc-ad)\sqrt{be-af}}-\frac{d\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}(bc-ad)\sqrt{de-cf}}$$

```
[Out] (b*ArcTan[(Sqrt[b*e - a*f]*x)/(Sqrt[a]*Sqrt[e + f*x^2])])/(Sqrt[a
]*(b*c - a*d)*Sqrt[b*e - a*f]) - (d*ArcTan[(Sqrt[d*e - c*f]*x)/(S
qrt[c]*Sqrt[e + f*x^2])])/(Sqrt[c]*(b*c - a*d)*Sqrt[d*e - c*f])
```

Rubi [A] time = 0.344605, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{b\tan^{-1}\left(\frac{x\sqrt{be-af}}{\sqrt{a}\sqrt{e+fx^2}}\right)}{\sqrt{a}(bc-ad)\sqrt{be-af}}-\frac{d\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}(bc-ad)\sqrt{de-cf}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)*(c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (b*ArcTan[(Sqrt[b*e - a*f]*x)/(Sqrt[a]*Sqrt[e + f*x^2])])/(Sqrt[a
]*(b*c - a*d)*Sqrt[b*e - a*f]) - (d*ArcTan[(Sqrt[d*e - c*f]*x)/(S
qrt[c]*Sqrt[e + f*x^2])])/(Sqrt[c]*(b*c - a*d)*Sqrt[d*e - c*f])
```

Rubi in Sympy [A] time = 41.8518, size = 104, normalized size = 0.85

$$\frac{d\,\mathrm{atanh}\left(\frac{x\sqrt{cf-de}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}\,(ad-bc)\,\sqrt{cf-de}}-\frac{b\,\mathrm{atanh}\left(\frac{x\sqrt{af-be}}{\sqrt{a}\sqrt{e+fx^2}}\right)}{\sqrt{a}\,(ad-bc)\,\sqrt{af-be}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)/(d*x**2+c)/(f*x**2+e)**(1/2),x)
```

```
[Out] d*atanh(x*sqrt(c*f - d*e)/(sqrt(c)*sqrt(e + f*x**2)))/(sqrt(c)*(a
*d - b*c)*sqrt(c*f - d*e)) - b*atanh(x*sqrt(a*f - b*e)/(sqrt(a)*s
qrt(e + f*x**2)))/(sqrt(a)*(a*d - b*c)*sqrt(a*f - b*e))
```

Mathematica [A] time = 0.276808, size = 113, normalized size = 0.93

$$\frac{\frac{b\tan^{-1}\left(\frac{x\sqrt{be-af}}{\sqrt{a}\sqrt{e+fx^2}}\right)}{\sqrt{a}\sqrt{be-af}}-\frac{d\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}\sqrt{de-cf}}}{bc-ad}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)*(c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] ((b*ArcTan[(Sqrt[b*e - a*f]*x)/(Sqrt[a]*Sqrt[e + f*x^2])])/(Sqrt[
a]*Sqrt[b*e - a*f]) - (d*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt
[e + f*x^2])])/(Sqrt[c]*Sqrt[d*e - c*f]))/(b*c - a*d)
```

Maple [B] time = 0.056, size = 782, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x)
```

```
[Out] -1/2*b*d^2/(b*(-c*d)^(1/2)+(-a*b)^(1/2)*d)/(b*(-c*d)^(1/2)-(-a*b)
^(1/2)*d)/(-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d+2*
f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x-(-
c*d)^(1/2)/d)^2*f+2*f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)-(c*f-d*e)
/d)^(1/2))/(x-(-c*d)^(1/2)/d))+1/2*b*d^2/(b*(-c*d)^(1/2)+(-a*b)^(
1/2)*d)/(b*(-c*d)^(1/2)-(-a*b)^(1/2)*d)/(-c*d)^(1/2)/(-(c*f-d*e)/
d)^(1/2)*ln((-2*(c*f-d*e)/d-2*f*(-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)
+2*(-(c*f-d*e)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*f-2*f*(-c*d)^(1/2)/
d*(x+(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x+(-c*d)^(1/2)/d))+1/2*
b^2*d/(-a*b)^(1/2)/(b*(-c*d)^(1/2)+(-a*b)^(1/2)*d)/(b*(-c*d)^(1/2
)-(-a*b)^(1/2)*d)/(-(a*f-b*e)/b)^(1/2)*ln((-2*(a*f-b*e)/b+2*f*(-a
*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*f-b*e)/b)^(1/2)*((x-1/b*(
-a*b)^(1/2))^2*f+2*f*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*f-b*e
)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))-1/2*b^2*d/(-a*b)^(1/2)/(b*(-c*d
)^(1/2)+(-a*b)^(1/2)*d)/(b*(-c*d)^(1/2)-(-a*b)^(1/2)*d)/(-(a*f-b*
e)/b)^(1/2)*ln((-2*(a*f-b*e)/b-2*f*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(
1/2))+2*(-(a*f-b*e)/b)^(1/2)*((x+1/b*(-a*b)^(1/2))^2*f-2*f*(-a*b)
^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*f-b*e)/b)^(1/2))/(x+1/b*(-a*b)^(
1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)(dx^2+c)\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [A] time = 92.5344, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-c*d*e + c^2*f)*b*log((((b^2*e^2 - 8*a*b*e*f + 8*a^2*
f^2)*x^4 + a^2*e^2 - 2*(3*a*b*e^2 - 4*a^2*e*f)*x^2)*sqrt(-a*b*e +
 a^2*f) - 4*((a*b^2*e^2 - 3*a^2*b*e*f + 2*a^3*f^2)*x^3 - (a^2*b*e
^2 - a^3*e*f)*x)*sqrt(f*x^2 + e))/(b^2*x^4 + 2*a*b*x^2 + a^2)) +
sqrt(-a*b*e + a^2*f)*d*log((((d^2*e^2 - 8*c*d*e*f + 8*c^2*f^2)*x^
4 + c^2*e^2 - 2*(3*c*d*e^2 - 4*c^2*e*f)*x^2)*sqrt(-c*d*e + c^2*f)
 + 4*((c*d^2*e^2 - 3*c^2*d*e*f + 2*c^3*f^2)*x^3 - (c^2*d*e^2 - c^
3*e*f)*x)*sqrt(f*x^2 + e))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/(sqrt(-a
*b*e + a^2*f)*sqrt(-c*d*e + c^2*f)*(b*c - a*d)), 1/4*(2*sqrt(-c*d
*e + c^2*f)*b*arctan(1/2*((b*e - 2*a*f)*x^2 - a*e)/(sqrt(a*b*e -
a^2*f)*sqrt(f*x^2 + e)*x)) - sqrt(a*b*e - a^2*f)*d*log((((d^2*e^2
 - 8*c*d*e*f + 8*c^2*f^2)*x^4 + c^2*e^2 - 2*(3*c*d*e^2 - 4*c^2*e*
f)*x^2)*sqrt(-c*d*e + c^2*f) + 4*((c*d^2*e^2 - 3*c^2*d*e*f + 2*c^
3*f^2)*x^3 - (c^2*d*e^2 - c^3*e*f)*x)*sqrt(f*x^2 + e))/(d^2*x^4 +
 2*c*d*x^2 + c^2)))/(sqrt(a*b*e - a^2*f)*sqrt(-c*d*e + c^2*f)*(b*
c - a*d)), -1/4*(2*sqrt(-a*b*e + a^2*f)*d*arctan(1/2*((d*e - 2*c*
f)*x^2 - c*e)/(sqrt(c*d*e - c^2*f)*sqrt(f*x^2 + e)*x)) + sqrt(c*d
*e - c^2*f)*b*log((((b^2*e^2 - 8*a*b*e*f + 8*a^2*f^2)*x^4 + a^2*e
^2 - 2*(3*a*b*e^2 - 4*a^2*e*f)*x^2)*sqrt(-a*b*e + a^2*f) - 4*((a*
b^2*e^2 - 3*a^2*b*e*f + 2*a^3*f^2)*x^3 - (a^2*b*e^2 - a^3*e*f)*x)
*sqrt(f*x^2 + e))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(sqrt(-a*b*e + a^
2*f)*sqrt(c*d*e - c^2*f)*(b*c - a*d)), 1/2*(sqrt(c*d*e - c^2*f)*b
*arctan(1/2*((b*e - 2*a*f)*x^2 - a*e)/(sqrt(a*b*e - a^2*f)*sqrt(f
*x^2 + e)*x)) - sqrt(a*b*e - a^2*f)*d*arctan(1/2*((d*e - 2*c*f)*x
^2 - c*e)/(sqrt(c*d*e - c^2*f)*sqrt(f*x^2 + e)*x)))/(sqrt(a*b*e -
 a^2*f)*sqrt(c*d*e - c^2*f)*(b*c - a*d))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^2)(c+dx^2)\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)/(d*x**2+c)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*(c + d*x**2)*sqrt(e + f*x**2)), x)
```

GIAC/XCAS [A] time = 0.267234, size = 234, normalized size = 1.92

$$-f^{\frac{3}{2}}\left(\frac{b\arctan\left(\frac{\left(\sqrt{f}x-\sqrt{fx^2+e}\right)^2b+2\,af-be}{2\sqrt{-a^2f^2+abfe}}\right)}{\sqrt{-a^2f^2+abfe}(bcf-adf)}-\frac{d\arctan\left(\frac{\left(\sqrt{f}x-\sqrt{fx^2+e}\right)^2d+2\,cf-de}{2\sqrt{-c^2f^2+cdfe}}\right)}{\sqrt{-c^2f^2+cdfe}(bcf-adf)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="giac")
```

```
[Out] -f^(3/2)*(b*arctan(1/2*((sqrt(f)*x - sqrt(f*x^2 + e))^2*b + 2*a*f
 - b*e)/sqrt(-a^2*f^2 + a*b*f*e))/(sqrt(-a^2*f^2 + a*b*f*e)*(b*c*
f - a*d*f)) - d*arctan(1/2*((sqrt(f)*x - sqrt(f*x^2 + e))^2*d + 2
*c*f - d*e)/sqrt(-c^2*f^2 + c*d*f*e))/(sqrt(-c^2*f^2 + c*d*f*e)*(
b*c*f - a*d*f)))
```

3.63 $\int \frac{1}{(a+bx^2)^2(c+dx^2)\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=203

$$\frac{b\left(4a^2df-2abcf-3abde+b^2ce\right)\tan^{-1}\left(\frac{x\sqrt{be-af}}{\sqrt{a}\sqrt{e+fx^2}}\right)}{2a^{3/2}(bc-ad)^2(be-af)^{3/2}}+\frac{b^2x\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(bc-ad)(be-af)}+\frac{d^2\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}(bc-ad)^2\sqrt{de-cf}}$$

```
[Out] (b^2*x*Sqrt[e + f*x^2])/(2*a*(b*c - a*d)*(b*e - a*f)*(a + b*x^2))
 + (b*(b^2*c*e - 3*a*b*d*e - 2*a*b*c*f + 4*a^2*d*f)*ArcTan[(Sqrt[
b*e - a*f]*x)/(Sqrt[a]*Sqrt[e + f*x^2])])/(2*a^(3/2)*(b*c - a*d)^
2*(b*e - a*f)^(3/2)) + (d^2*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*S
qrt[e + f*x^2])])/(Sqrt[c]*(b*c - a*d)^2*Sqrt[d*e - c*f])
```

Rubi [A] time = 0.7523, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{b\left(4a^2df-2abcf-3abde+b^2ce\right)\tan^{-1}\left(\frac{x\sqrt{be-af}}{\sqrt{a}\sqrt{e+fx^2}}\right)}{2a^{3/2}(bc-ad)^2(be-af)^{3/2}}+\frac{b^2x\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(bc-ad)(be-af)}+\frac{d^2\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}(bc-ad)^2\sqrt{de-cf}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^2*(c + d*x^2)*Sqrt[e + f*x^2]),x]
```


```
[Out] (b^2*x*Sqrt[e + f*x^2])/(2*a*(b*c - a*d)*(b*e - a*f)*(a + b*x^2))
 + (b*(b^2*c*e - 3*a*b*d*e - 2*a*b*c*f + 4*a^2*d*f)*ArcTan[(Sqrt[
b*e - a*f]*x)/(Sqrt[a]*Sqrt[e + f*x^2])])/(2*a^(3/2)*(b*c - a*d)^
2*(b*e - a*f)^(3/2)) + (d^2*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*S
qrt[e + f*x^2])])/(Sqrt[c]*(b*c - a*d)^2*Sqrt[d*e - c*f])
```

Rubi in Sympy [A] time = 97.7256, size = 182, normalized size = 0.9

$$\frac{d^2 \operatorname{atanh}\left(\frac{x\sqrt{cf-de}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}\,(ad-bc)^2\,\sqrt{cf-de}}+\frac{b^2x\sqrt{e+fx^2}}{2a\,(a+bx^2)\,(ad-bc)\,(af-be)}-\frac{b\left(4a^2df-2abcf-3abde+b^2ce\right)\operatorname{atanh}\left(\frac{x\sqrt{af-be}}{\sqrt{a}\sqrt{e+fx^2}}\right)}{2a^{\frac{3}{2}}\,(ad-bc)^2\,(af-be)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**2/(d*x**2+c)/(f*x**2+e)**(1/2),x)
```

```
[Out] d**2*atanh(x*sqrt(c*f - d*e)/(sqrt(c)*sqrt(e + f*x**2)))/(sqrt(c)
*(a*d - b*c)**2*sqrt(c*f - d*e)) + b**2*x*sqrt(e + f*x**2)/(2*a*(
a + b*x**2)*(a*d - b*c)*(a*f - b*e)) - b*(4*a**2*d*f - 2*a*b*c*f
- 3*a*b*d*e + b**2*c*e)*atanh(x*sqrt(a*f - b*e)/(sqrt(a)*sqrt(e +
 f*x**2)))/(2*a**(3/2)*(a*d - b*c)**2*(a*f - b*e)**(3/2))
```

Mathematica [A] time = 0.986749, size = 203, normalized size = 1.

$$\frac{1}{2}\left(\frac{b\left(4a^2df-ab(2cf+3de)+b^2ce\right)\tan^{-1}\left(\frac{x\sqrt{be-af}}{\sqrt{a}\sqrt{e+fx^2}}\right)}{a^{3/2}(bc-ad)^2(be-af)^{3/2}}+\frac{b^2x\sqrt{e+fx^2}}{a\,(a+bx^2)\,(ad-bc)(af-be)}+\frac{2d^2\tan^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}(bc-ad)^2\sqrt{de-cf}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)^2*(c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] ((b^2*x*Sqrt[e + f*x^2])/(a*(-(b*c) + a*d)*(-(b*e) + a*f)*(a + b*
x^2)) + (b*(b^2*c*e + 4*a^2*d*f - a*b*(3*d*e + 2*c*f))*ArcTan[(Sq
rt[b*e - a*f]*x)/(Sqrt[a]*Sqrt[e + f*x^2])])/(a^(3/2)*(b*c - a*d)
^2*(b*e - a*f)^(3/2)) + (2*d^2*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c
]*Sqrt[e + f*x^2])])/(Sqrt[c]*(b*c - a*d)^2*Sqrt[d*e - c*f]))/2
```

Maple [B] time = 0.082, size = 1865, normalized size = 9.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x)
```

```
[Out] -1/2*b^2*d^4/(b*(-c*d)^(1/2)+(-a*b)^(1/2)*d)^2/(b*(-c*d)^(1/2)-(-
a*b)^(1/2)*d)^2/(-c*d)^(1/2)/(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e
)/d+2*f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*
((x-(-c*d)^(1/2)/d)^2*f+2*f*(-c*d)^(1/2)/d*(x-(-c*d)^(1/2)/d)-(c*
f-d*e)/d)^(1/2))/(x-(-c*d)^(1/2)/d))+1/2*b^2*d^4/(b*(-c*d)^(1/2)+
(-a*b)^(1/2)*d)^2/(b*(-c*d)^(1/2)-(-a*b)^(1/2)*d)^2/(-c*d)^(1/2)/
(-(c*f-d*e)/d)^(1/2)*ln((-2*(c*f-d*e)/d-2*f*(-c*d)^(1/2)/d*(x+(-c
*d)^(1/2)/d)+2*(-(c*f-d*e)/d)^(1/2)*((x+(-c*d)^(1/2)/d)^2*f-2*f*(
-c*d)^(1/2)/d*(x+(-c*d)^(1/2)/d)-(c*f-d*e)/d)^(1/2))/(x+(-c*d)^(1
/2)/d))+1/4*b^2*d/a/(b*(-c*d)^(1/2)+(-a*b)^(1/2)*d)/(b*(-c*d)^(1/
2)-(-a*b)^(1/2)*d)/(a*f-b*e)/(x-1/b*(-a*b)^(1/2))*((x-1/b*(-a*b)^
(1/2))^2*f+2*f*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*f-b*e)/b)^(
1/2)-1/4*b*d/a/(b*(-c*d)^(1/2)+(-a*b)^(1/2)*d)/(b*(-c*d)^(1/2)-(-
a*b)^(1/2)*d)*f*(-a*b)^(1/2)/(a*f-b*e)/(-(a*f-b*e)/b)^(1/2)*ln((-
2*(a*f-b*e)/b+2*f*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-(a*f-b*
e)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*f+2*f*(-a*b)^(1/2)/b*(x-1/b*(
-a*b)^(1/2))-(a*f-b*e)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))+1/4*b^2*d/
a/(b*(-c*d)^(1/2)+(-a*b)^(1/2)*d)/(b*(-c*d)^(1/2)-(-a*b)^(1/2)*d)
/(a*f-b*e)/(x+1/b*(-a*b)^(1/2))*((x+1/b*(-a*b)^(1/2))^2*f-2*f*(-a
*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*f-b*e)/b)^(1/2)+1/4*b*d/a/(b*
(-c*d)^(1/2)+(-a*b)^(1/2)*d)/(b*(-c*d)^(1/2)-(-a*b)^(1/2)*d)*f*(-
a*b)^(1/2)/(a*f-b*e)/(-(a*f-b*e)/b)^(1/2)*ln((-2*(a*f-b*e)/b-2*f*
(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*f-b*e)/b)^(1/2)*((x+1/
b*(-a*b)^(1/2))^2*f-2*f*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*f-
b*e)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+3/4*b^3*d^3/(-a*b)^(1/2)/(b*
(-c*d)^(1/2)+(-a*b)^(1/2)*d)^2/(b*(-c*d)^(1/2)-(-a*b)^(1/2)*d)^2/
(-(a*f-b*e)/b)^(1/2)*ln((-2*(a*f-b*e)/b+2*f*(-a*b)^(1/2)/b*(x-1/b
*(-a*b)^(1/2))+2*(-(a*f-b*e)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*f+2
*f*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))-(a*f-b*e)/b)^(1/2))/(x-1/b
*(-a*b)^(1/2)))-1/4*b^4*d^2/a/(-a*b)^(1/2)/(b*(-c*d)^(1/2)+(-a*b)
^(1/2)*d)^2/(b*(-c*d)^(1/2)-(-a*b)^(1/2)*d)^2/(-(a*f-b*e)/b)^(1/2
)*ln((-2*(a*f-b*e)/b+2*f*(-a*b)^(1/2)/b*(x-1/b*(-a*b)^(1/2))+2*(-
(a*f-b*e)/b)^(1/2)*((x-1/b*(-a*b)^(1/2))^2*f+2*f*(-a*b)^(1/2)/b*(
x-1/b*(-a*b)^(1/2))-(a*f-b*e)/b)^(1/2))/(x-1/b*(-a*b)^(1/2)))*c-3
/4*b^3*d^3/(-a*b)^(1/2)/(b*(-c*d)^(1/2)+(-a*b)^(1/2)*d)^2/(b*(-c*
d)^(1/2)-(-a*b)^(1/2)*d)^2/(-(a*f-b*e)/b)^(1/2)*ln((-2*(a*f-b*e)/
b-2*f*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))+2*(-(a*f-b*e)/b)^(1/2)*
((x+1/b*(-a*b)^(1/2))^2*f-2*f*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))
-(a*f-b*e)/b)^(1/2))/(x+1/b*(-a*b)^(1/2)))+1/4*b^4*d^2/a/(-a*b)^(
1/2)/(b*(-c*d)^(1/2)+(-a*b)^(1/2)*d)^2/(b*(-c*d)^(1/2)-(-a*b)^(1/
2)*d)^2/(-(a*f-b*e)/b)^(1/2)*ln((-2*(a*f-b*e)/b-2*f*(-a*b)^(1/2)/
b*(x+1/b*(-a*b)^(1/2))+2*(-(a*f-b*e)/b)^(1/2)*((x+1/b*(-a*b)^(1/2
))^2*f-2*f*(-a*b)^(1/2)/b*(x+1/b*(-a*b)^(1/2))-(a*f-b*e)/b)^(1/2)
)/(x+1/b*(-a*b)^(1/2)))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^2(dx^2+c)\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**2/(d*x**2+c)/(f*x**2+e)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 8.45473, size = 647, normalized size = 3.19

$$-\frac{1}{2}\left(\frac{2\,d^2\arctan\left(\frac{\left(\sqrt{f}x-\sqrt{fx^2+e}\right)^2 d+2\,cf-de}{2\sqrt{-c^2f^2+cdfe}}\right)}{(b^2c^2f^2-2\,abcdf^2+a^2d^2f^2)\sqrt{-c^2f^2+cdfe}}+\frac{\left(2\,ab^2cf-4\,a^2bdf-b^3ce+3\,ab^2de\right)\arctan\left(\frac{\left(\sqrt{f}x-\sqrt{fx^2+e}\right)^2}{2\sqrt{-a^2f^2+c}}\right.}{(a^2b^2c^2f^3-2\,a^3bcdf^3+a^4d^2f^3-ab^3c^2f^2e+2\,a^2b^2cdf^2e-a^3bd^2f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="giac")
```

```
[Out] -1/2*(2*d^2*arctan(1/2*((sqrt(f)*x - sqrt(f*x^2 + e))^2*d + 2*c*f
 - d*e)/sqrt(-c^2*f^2 + c*d*f*e))/((b^2*c^2*f^2 - 2*a*b*c*d*f^2 +
 a^2*d^2*f^2)*sqrt(-c^2*f^2 + c*d*f*e)) + (2*a*b^2*c*f - 4*a^2*b*
d*f - b^3*c*e + 3*a*b^2*d*e)*arctan(1/2*((sqrt(f)*x - sqrt(f*x^2
+ e))^2*b + 2*a*f - b*e)/sqrt(-a^2*f^2 + a*b*f*e))/((a^2*b^2*c^2*
f^3 - 2*a^3*b*c*d*f^3 + a^4*d^2*f^3 - a*b^3*c^2*f^2*e + 2*a^2*b^2
*c*d*f^2*e - a^3*b*d^2*f^2*e)*sqrt(-a^2*f^2 + a*b*f*e)) + 2*(2*(s
qrt(f)*x - sqrt(f*x^2 + e))^2*a*b*f - (sqrt(f)*x - sqrt(f*x^2 + e
))^2*b^2*e + b^2*e^2)/((a^2*b*c*f^3 - a^3*d*f^3 - a*b^2*c*f^2*e +
 a^2*b*d*f^2*e)*((sqrt(f)*x - sqrt(f*x^2 + e))^4*b + 4*(sqrt(f)*x
 - sqrt(f*x^2 + e))^2*a*f - 2*(sqrt(f)*x - sqrt(f*x^2 + e))^2*b*e
 + b*e^2)))*f^(5/2)
```

3.64 $\int \frac{(c+dx^2)^{5/2}\sqrt{e+fx^2}}{a+bx^2}\,dx$

Optimal. Leaf size=608

$$-\frac{\sqrt{e}\sqrt{c+dx^2}\left(15a^2d^2f^2-5abdf(7cf+de)+b^2\left(23c^2f^2+12cdef-2d^2e^2\right)\right)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15b^3f^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{de^{3/2}\sqrt{c+dx^2}\left(15a^2d^2f-40abcdf+b^2c(34cf-de)\right)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15b^3cf^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{e^{3/2}\sqrt{c+dx^2}(bc-ad)^3\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ab^3c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x\sqrt{c+dx^2}(bc-ad)(-3adf+4bcf+bde)}{3b^3\sqrt{e+fx^2}}$$
$$+\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}(bc-ad)}{3b^2}+\frac{dx\sqrt{c+dx^2}\left(\frac{3c^2f}{d}+7ce-\frac{2de^2}{f}\right)}{15b\sqrt{e+fx^2}}$$
$$+\frac{d^2x\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5bf}-\frac{2dx\sqrt{c+dx^2}\sqrt{e+fx^2}(de-3cf)}{15bf}$$

```
[Out] (d*(7*c*e - (2*d*e^2)/f + (3*c^2*f)/d)*x*Sqrt[c + d*x^2])/(15*b*S
qrt[e + f*x^2]) + ((b*c - a*d)*(b*d*e + 4*b*c*f - 3*a*d*f)*x*Sqrt
[c + d*x^2])/(3*b^3*Sqrt[e + f*x^2]) + (d*(b*c - a*d)*x*Sqrt[c +
d*x^2]*Sqrt[e + f*x^2])/(3*b^2) - (2*d*(d*e - 3*c*f)*x*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2])/(15*b*f) + (d^2*x*Sqrt[c + d*x^2]*(e + f*x
^2)^(3/2))/(5*b*f) - (Sqrt[e]*(15*a^2*d^2*f^2 - 5*a*b*d*f*(d*e +
7*c*f) + b^2*(-2*d^2*e^2 + 12*c*d*e*f + 23*c^2*f^2))*Sqrt[c + d*x
^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*
b^3*f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]
) + (d*e^(3/2)*(-40*a*b*c*d*f + 15*a^2*d^2*f + b^2*c*(-(d*e) + 34
*c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 -
 (d*e)/(c*f)])/(15*b^3*c*f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x
^2))]*Sqrt[e + f*x^2]) + ((b*c - a*d)^3*e^(3/2)*Sqrt[c + d*x^2]*E
llipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)
/(c*f)])/(a*b^3*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*S
qrt[e + f*x^2])
```

Rubi [A] time = 2.2298, antiderivative size = 776, normalized size of antiderivative = 1.28, number of steps used = 14, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$

$$\frac{de^{3/2}\sqrt{c+dx^2}(5bc-3ad)(bc-ad)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^3c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{e^{3/2}\sqrt{c+dx^2}(bc-ad)^3\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ab^3c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{x\sqrt{c+dx^2}(bc-ad)(-3adf+4bcf+bde)}{3b^3\sqrt{e+fx^2}} - \frac{\sqrt{e}\sqrt{c+dx^2}(bc-ad)(-3adf+4bcf+bde)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^3\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}(bc-ad)}{3b^2} + \frac{\sqrt{e}\sqrt{c+dx^2}\left(-3c^2f^2-7cdef+2d^2e^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15bf^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{dx\sqrt{c+dx^2}\left(\frac{3c^2f}{d}+7ce-\frac{2de^2}{f}\right)}{15b\sqrt{e+fx^2}} + \frac{d^2x\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5bf} - \frac{de^{3/2}\sqrt{c+dx^2}(de-9cf)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15bf^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} - \frac{2dx\sqrt{c+dx^2}\sqrt{e+fx^2}(de-3cf)}{15bf}$$

Antiderivative was successfully verified.

```
[In]  Int[((c + d*x^2)^(5/2)*Sqrt[e + f*x^2])/(a + b*x^2),x]
```

```
[Out] (d*(7*c*e - (2*d*e^2)/f + (3*c^2*f)/d)*x*Sqrt[c + d*x^2])/(15*b*S
qrt[e + f*x^2]) + ((b*c - a*d)*(b*d*e + 4*b*c*f - 3*a*d*f)*x*Sqrt
[c + d*x^2])/(3*b^3*Sqrt[e + f*x^2]) + (d*(b*c - a*d)*x*Sqrt[c +
d*x^2]*Sqrt[e + f*x^2])/(3*b^2) - (2*d*(d*e - 3*c*f)*x*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2])/(15*b*f) + (d^2*x*Sqrt[c + d*x^2]*(e + f*x
^2)^(3/2))/(5*b*f) - ((b*c - a*d)*Sqrt[e]*(b*d*e + 4*b*c*f - 3*a*
d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (
d*e)/(c*f)])/(3*b^3*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]
*Sqrt[e + f*x^2]) + (Sqrt[e]*(2*d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2)*
Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/
(c*f)])/(15*b*f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[
e + f*x^2]) + (d*(5*b*c - 3*a*d)*(b*c - a*d)*e^(3/2)*Sqrt[c + d*x
^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*b
^3*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2
]) - (d*e^(3/2)*(d*e - 9*c*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(S
qrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*b*f^(3/2)*Sqrt[(e*(c +
```

```
d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + ((b*c - a*d)^3*e^(3/2
)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/
Sqrt[e]], 1 - (d*e)/(c*f)])/(a*b^3*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))
/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(5/2)*(f*x**2+e)**(1/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 4.74015, size = 456, normalized size = 0.75

$$-iabde\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(15a^2d^2f^2-5abdf(7cf+de)+b^2\left(23c^2f^2+12cdef-2d^2e^2\right)\right)E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-ia\sqrt{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((c + d*x^2)^(5/2)*Sqrt[e + f*x^2])/(a + b*x^2),x]
```

```
[Out] ((-I)*a*b*d*e*(15*a^2*d^2*f^2 - 5*a*b*d*f*(d*e + 7*c*f) + b^2*(-2
*d^2*e^2 + 12*c*d*e*f + 23*c^2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 +
 (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*
(45*a^2*b*c*d^2*f^3 - 15*a^3*d^3*f^3 + 5*a*b^2*d*f*(d^2*e^2 - c*d
*e*f - 9*c^2*f^2) + b^3*(2*d^3*e^3 - 13*c*d^2*e^2*f + 11*c^2*d*e*
f^2 + 15*c^3*f^3))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Ellipt
icF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + f*(a*b^2*d*Sqrt[d/c]*x
*(c + d*x^2)*(e + f*x^2)*(11*b*c*f - 5*a*d*f + b*d*(e + 3*f*x^2))
 - (15*I)*(b*c - a*d)^3*f*(b*e - a*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1
+ (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f
)/(d*e)]))/(15*a*b^4*Sqrt[d/c]*f^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2
])
```

Maple [B] time = 0.056, size = 1891, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(5/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x)
```

```
[Out] -1/15*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*(15*((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^4*d^
3*f^3-45*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/
c)^(1/2),(c*f/d/e)^(1/2))*a^3*b*c*d^2*f^3+45*((d*x^2+c)/c)^(1/2)*
((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2
*b^2*c^2*d*f^3-5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elliptic
F(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b^2*d^3*e^2*f-15*((d*x^2+c)
/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^
(1/2))*a^3*b*d^3*e*f^2+5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*
EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b^2*d^3*e^2*f+5*((d
*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*
f/d/e)^(1/2))*a^2*b^2*c*d^2*e*f^2+45*((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)
^(1/2))*a^3*b*c*d^2*f^3+15*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2
)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^
3*b*d^3*e*f^2-45*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elliptic
Pi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*b^2*c^2*
d*f^3-15*(-d/c)^(1/2)*x^3*a*b^3*c*d^2*e*f^2+5*(-d/c)^(1/2)*x*a^2*
b^2*c*d^2*e*f^2-11*(-d/c)^(1/2)*x*a*b^3*c^2*d*e*f^2-(-d/c)^(1/2)*
x*a*b^3*c*d^2*e^2*f-15*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*El
lipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^4*d^
3*f^3+5*(-d/c)^(1/2)*x^5*a^2*b^2*d^3*f^3-3*(-d/c)^(1/2)*x^7*a*b^3
*d^3*f^3-14*(-d/c)^(1/2)*x^5*a*b^3*c*d^2*f^3-15*((d*x^2+c)/c)^(1/
2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(
1/2)/(-d/c)^(1/2))*b^4*c^3*e*f^2-15*((d*x^2+c)/c)^(1/2)*((f*x^2+e
)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^3*c^3*f^
3-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1
/2),(c*f/d/e)^(1/2))*a*b^3*d^3*e^3+2*((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^3*d^3*e
^3+15*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)
^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b^3*c^3*f^3-4*(-d/c)^
(1/2)*x^5*a*b^3*d^3*e*f^2+5*(-d/c)^(1/2)*x^3*a^2*b^2*c*d^2*f^3+5*
(-d/c)^(1/2)*x^3*a^2*b^2*d^3*e*f^2-11*(-d/c)^(1/2)*x^3*a*b^3*c^2*
d*f^3-(-d/c)^(1/2)*x^3*a*b^3*d^3*e^2*f-23*((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^3*
c^2*d*e*f^2-12*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(
x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^3*c*d^2*e^2*f-45*((d*x^2+c)/c
)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f
/e)^(1/2)/(-d/c)^(1/2))*a^2*b^2*c*d^2*e*f^2+45*((d*x^2+c)/c)^(1/2
)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1
/2)/(-d/c)^(1/2))*a*b^3*c^2*d*e*f^2-11*((d*x^2+c)/c)^(1/2)*((f*x^
2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^3*c^2
*d*e*f^2+13*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(
-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^3*c*d^2*e^2*f+35*((d*x^2+c)/c)^(
1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2)
)*a^2*b^2*c*d^2*e*f^2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)/b^4/f^2/(-d/
c)^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)/(b*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(5/2)*(f*x**2+e)**(1/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)/(b*x^2 + a), x)
```

3.65 $\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{a+bx^2}\,dx$

Optimal. Leaf size=400

$$\frac{de^{3/2}\sqrt{c+dx^2}(5bc-3ad)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^2c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{e^{3/2}\sqrt{c+dx^2}(bc-ad)^2\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ab^2c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x\sqrt{c+dx^2}(-3adf+4bcf+bde)}{3b^2\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}(-3adf+4bcf+bde)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^2\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b}$$

```
[Out] ((b*d*e + 4*b*c*f - 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*b^2*Sqrt[e + f
*x^2]) + (d*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*b) - (Sqrt[e]*(
b*d*e + 4*b*c*f - 3*a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt
[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*b^2*Sqrt[f]*Sqrt[(e*(c + d*
x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d*(5*b*c - 3*a*d)*e^(3
/2)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d
*e)/(c*f)])/(3*b^2*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))
]*Sqrt[e + f*x^2]) + ((b*c - a*d)^2*e^(3/2)*Sqrt[c + d*x^2]*Ellip
ticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*
f)])/(a*b^2*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[
e + f*x^2])
```

Rubi [A] time = 0.968117, antiderivative size = 400, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$

$$\frac{de^{3/2}\sqrt{c+dx^2}(5bc-3ad)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^2c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{e^{3/2}\sqrt{c+dx^2}(bc-ad)^2\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ab^2c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{x\sqrt{c+dx^2}(-3adf+4bcf+bde)}{3b^2\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{c+dx^2}(-3adf+4bcf+bde)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^2\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b}$$

Antiderivative was successfully verified.

```
[In]  Int[((c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(a + b*x^2),x]
```

```
[Out] ((b*d*e + 4*b*c*f - 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*b^2*Sqrt[e + f
*x^2]) + (d*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*b) - (Sqrt[e]*(
b*d*e + 4*b*c*f - 3*a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt
[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*b^2*Sqrt[f]*Sqrt[(e*(c + d*
x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d*(5*b*c - 3*a*d)*e^(3
/2)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d
*e)/(c*f)])/(3*b^2*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))
]*Sqrt[e + f*x^2]) + ((b*c - a*d)^2*e^(3/2)*Sqrt[c + d*x^2]*Ellip
ticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*
f)])/(a*b^2*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[
e + f*x^2])
```

Rubi in Sympy [A] time = 130.427, size = 355, normalized size = 0.89

$$\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b}+\frac{\sqrt{e}\sqrt{c+dx^2}\,(3adf-4bcf-bde)\,E\left(\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{3b^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}-\frac{x\sqrt{c+dx^2}\,(3adf-4bcf-bde)}{3b^2\sqrt{e+fx^2}}-\frac{de^{\frac{3}{2}}\sqrt{c+dx^2}\,(3ad-5bc)\,F\left(\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{3b^2c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}+\frac{e^{\frac{3}{2}}\sqrt{c+dx^2}\,(ad-bc)^2\left(1-\frac{be}{af};\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{ab^2c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(3/2)*(f*x**2+e)**(1/2)/(b*x**2+a),x)
```

```
[Out] d*x*sqrt(c + d*x**2)*sqrt(e + f*x**2)/(3*b) + sqrt(e)*sqrt(c + d*
x**2)*(3*a*d*f - 4*b*c*f - b*d*e)*elliptic_e(atan(sqrt(f)*x/sqrt(
e)), 1 - d*e/(c*f))/(3*b**2*sqrt(f)*sqrt(e*(c + d*x**2)/(c*(e + f
*x**2)))*sqrt(e + f*x**2)) - x*sqrt(c + d*x**2)*(3*a*d*f - 4*b*c*
f - b*d*e)/(3*b**2*sqrt(e + f*x**2)) - d*e**(3/2)*sqrt(c + d*x**2
)*(3*a*d - 5*b*c)*elliptic_f(atan(sqrt(f)*x/sqrt(e)), 1 - d*e/(c*
f))/(3*b**2*c*sqrt(f)*sqrt(e*(c + d*x**2)/(c*(e + f*x**2))))*sqrt(
e + f*x**2)) + e**(3/2)*sqrt(c + d*x**2)*(a*d - b*c)**2*elliptic_
pi(1 - b*e/(a*f), atan(sqrt(f)*x/sqrt(e)), 1 - d*e/(c*f))/(a*b**2
*c*sqrt(f)*sqrt(e*(c + d*x**2)/(c*(e + f*x**2))))*sqrt(e + f*x**2)
)
```

Mathematica [C] time = 2.57959, size = 346, normalized size = 0.86

$$-ia\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(3a^2d^2f^2-6abcdf^2+b^2\left(3c^2f^2+cdef-d^2e^2\right)\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+f\left(ab^2dx\sqrt{\frac{d}{c}}\left(c+dx^2\right)\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[((c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(a + b*x^2),x]
```

```
[Out] ((-I)*a*b*d*e*(b*d*e + 4*b*c*f - 3*a*d*f)*Sqrt[1 + (d*x^2)/c]*Sqr
t[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] -
 I*a*(-6*a*b*c*d*f^2 + 3*a^2*d^2*f^2 + b^2*(-(d^2*e^2) + c*d*e*f
+ 3*c^2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I
*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + f*(a*b^2*d*Sqrt[d/c]*x*(c +
 d*x^2)*(e + f*x^2) - (3*I)*(b*c - a*d)^2*(b*e - a*f)*Sqrt[1 + (d
*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sq
rt[d/c]*x], (c*f)/(d*e)]))/(3*a*b^3*Sqrt[d/c]*f*Sqrt[c + d*x^2]*S
qrt[e + f*x^2])
```

Maple [B] time = 0.028, size = 1059, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x)
```

```
[Out] 1/3*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*((-d/c)^(1/2)*x^5*a*b^2*d^2*f
^2+(-d/c)^(1/2)*x^3*a*b^2*c*d*f^2+(-d/c)^(1/2)*x^3*a*b^2*d^2*e*f+
3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2
),(c*f/d/e)^(1/2))*a^3*d^2*f^2-6*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e
)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*c*d*f^2+3
*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2)
,(c*f/d/e)^(1/2))*a*b^2*c^2*f^2+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c*d*e*f-((
d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c
*f/d/e)^(1/2))*a*b^2*d^2*e^2-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^
(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*d^2*e*f+4*(
(d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(
c*f/d/e)^(1/2))*a*b^2*c*d*e*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*d^2*e^2-3*((
d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b
*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^3*d^2*f^2+6*((d*x^2+c)/c)^(1/
2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(
1/2)/(-d/c)^(1/2))*a^2*b*c*d*f^2+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)
/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(
1/2))*a^2*b*d^2*e*f-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ell
```

```
ipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b^2*c
^2*f^2-6*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d
/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b^2*c*d*e*f+3*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c
/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^3*c^2*e*f+(-d/c)^(1/2)*x*a*b^2*
c*d*e*f)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)/b^3/(-d/c)^(1/2)/f/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)/(b*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx^2)^{\frac{3}{2}}\sqrt{e+fx^2}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(3/2)*(f*x**2+e)**(1/2)/(b*x**2+a),x)
```

```
[Out] Integral((c + d*x**2)**(3/2)*sqrt(e + f*x**2)/(a + b*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2 + c\right)^{\frac{3}{2}}\sqrt{fx^2 + e}}{bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)/(b*x^2 + a), x)
```

3.66 $\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{a+bx^2}\,dx$

Optimal. Leaf size=321

$$\frac{e^{3/2}\sqrt{c+dx^2}(bc-ad)\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{abc\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{de^{3/2}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{bc\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{fx\sqrt{c+dx^2}}{b\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

```
[Out] (f*x*Sqrt[c + d*x^2])/(b*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[f]*Sqrt
[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f
)])/(b*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (
d*e^(3/2)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]],
1 - (d*e)/(c*f)])/(b*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2
))]*Sqrt[e + f*x^2]) + ((b*c - a*d)*e^(3/2)*Sqrt[c + d*x^2]*Ellip
ticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*
f)])/(a*b*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
+ f*x^2])
```

Rubi [A] time = 0.644136, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$\frac{e^{3/2}\sqrt{c+dx^2}(bc-ad)\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{abc\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{de^{3/2}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{bc\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{fx\sqrt{c+dx^2}}{b\sqrt{e+fx^2}}-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2),x]
```


```
[Out] (f*x*Sqrt[c + d*x^2])/(b*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[f]*Sqrt
[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f
)])/(b*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (
d*e^(3/2)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]],
1 - (d*e)/(c*f)])/(b*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2
))]*Sqrt[e + f*x^2]) + ((b*c - a*d)*e^(3/2)*Sqrt[c + d*x^2]*Ellip
ticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*
f)])/(a*b*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
```

```
+ f*x^2])
```

Rubi in Sympy [A] time = 78.9471, size = 269, normalized size = 0.84

$$-\frac{\sqrt{c}\sqrt{d}\sqrt{e+fx^2}E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{b\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}}+\frac{dx\sqrt{e+fx^2}}{b\sqrt{c+dx^2}}$$
$$+\frac{de^{\frac{3}{2}}\sqrt{c+dx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{bc\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}-\frac{e^{\frac{3}{2}}\sqrt{c+dx^2}\,(ad-bc)\left(1-\frac{be}{af};\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{abc\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/(b*x**2+a),x)
```

```
[Out] -sqrt(c)*sqrt(d)*sqrt(e + f*x**2)*elliptic_e(atan(sqrt(d)*x/sqrt(
c)), -c*f/(d*e) + 1)/(b*sqrt(c*(e + f*x**2)/(e*(c + d*x**2)))*sqr
t(c + d*x**2)) + d*x*sqrt(e + f*x**2)/(b*sqrt(c + d*x**2)) + d*e*
*(3/2)*sqrt(c + d*x**2)*elliptic_f(atan(sqrt(f)*x/sqrt(e)), 1 - d
*e/(c*f))/(b*c*sqrt(f)*sqrt(e*(c + d*x**2)/(c*(e + f*x**2)))*sqrt
(e + f*x**2)) - e**(3/2)*sqrt(c + d*x**2)*(a*d - b*c)*elliptic_pi
(1 - b*e/(a*f), atan(sqrt(f)*x/sqrt(e)), 1 - d*e/(c*f))/(a*b*c*sq
rt(f)*sqrt(e*(c + d*x**2)/(c*(e + f*x**2)))*sqrt(e + f*x**2))
```

Mathematica [C] time = 0.442855, size = 184, normalized size = 0.57

$$-\frac{i\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(abdeE\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)\middle|\frac{cf}{de}\right)+(bc-ad)\left((be-af)\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)\middle|\frac{cf}{de}\right)+afF\left(i\sinh^{-1}\left(\sqrt{\cdots}\right.\right.\right.\right.}{ab^2\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2),x]
```

```
[Out] ((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(a*b*d*e*EllipticE[
I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (b*c - a*d)*(a*f*EllipticF
[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (b*e - a*f)*EllipticPi[(b
*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])))/(a*b^2*Sqrt[d/
c]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.021, size = 340, normalized size = 1.1

$$\frac{1}{(dfx^4+cfx^2+dex^2+ce)\,b^2a}\left(-EllipticF\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)a^2df+EllipticF\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)abcf+EllipticE\left(x\sqrt{-\frac{d}{c}},\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x)
```

```
[Out] (-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*d*f+EllipticF(x*(
-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c*f+EllipticE(x*(-d/c)^(1/2),(c*
f/d/e)^(1/2))*a*b*d*e+EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1
/2)/(-d/c)^(1/2))*a^2*d*f-EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e
)^(1/2)/(-d/c)^(1/2))*a*b*c*f-EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(
-f/e)^(1/2)/(-d/c)^(1/2))*a*b*d*e+EllipticPi(x*(-d/c)^(1/2),b*c/a
/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c*e)*(d*x^2+c)^(1/2)*(f*x^2+e)^
(1/2)*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)/(d*f*x^4+c*f*x^2+d*
e*x^2+c*e)/b^2/(-d/c)^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}\sqrt{fx^2+e}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/(b*x**2+a),x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt(e + f*x**2)/(a + b*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}\sqrt{fx^2+e}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a), x)
```

3.67 $\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=102

$$\frac{e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

```
[Out] (e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt
[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*Sqrt[f]*Sqrt[(e*(c + d*x^
2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.141206, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031

$$\frac{e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt
[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*Sqrt[f]*Sqrt[(e*(c + d*x^
2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi in Sympy [A] time = 19.4427, size = 82, normalized size = 0.8

$$\frac{e^{\frac{3}{2}}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**2+e)**(1/2)/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] e**(3/2)*sqrt(c + d*x**2)*elliptic_pi(1 - b*e/(a*f), atan(sqrt(f)
*x/sqrt(e)), 1 - d*e/(c*f))/(a*c*sqrt(f)*sqrt(e*(c + d*x**2)/(c*(
e + f*x**2)))*sqrt(e + f*x**2))
```

Mathematica [C] time = 0.193271, size = 143, normalized size = 1.4

$$-\frac{i\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left((be-af)\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+afF\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)\right)}{ab\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(a*f*EllipticF[I*Ar
cSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (b*e - a*f)*EllipticPi[(b*c)/(
a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(a*b*Sqrt[d/c]*Sqrt[
c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.028, size = 191, normalized size = 1.9

$$\frac{1}{ab\left(dfx^4+cfx^2+dex^2+ce\right)}\left(\mathit{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)af-\mathit{EllipticPi}\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},1\sqrt{-\frac{f}{e}}\frac{1}{\sqrt{-\frac{d}{c}}}\right)af+\mathit{EllipticPi}\left(x\sqrt{-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] (EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*f-EllipticPi(x*(-d/c
)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*f+EllipticPi(x*(-d/c
)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b*e)/b*((f*x^2+e)/e)^(
1/2)*((d*x^2+c)/c)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/(-d/c)
^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e + fx^2}}{(a + bx^2)\sqrt{c + dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e)**(1/2)/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(e + f*x**2)/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{fx^2 + e}}{(bx^2 + a)\sqrt{dx^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.68 $\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=209

$$\frac{be^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{d}\sqrt{e+fx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

```
[Out] -((Sqrt[d]*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
 1 - (c*f)/(d*e)])/(Sqrt[c]*(b*c - a*d)*Sqrt[c + d*x^2]*Sqrt[(c*(
e + f*x^2))/(e*(c + d*x^2))])) + (b*e^(3/2)*Sqrt[c + d*x^2]*Ellip
ticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*
f)])/(a*c*(b*c - a*d)*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2)
)]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.395867, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094

$$\frac{be^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{d}\sqrt{e+fx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[e + f*x^2]/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -((Sqrt[d]*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
 1 - (c*f)/(d*e)])/(Sqrt[c]*(b*c - a*d)*Sqrt[c + d*x^2]*Sqrt[(c*(
e + f*x^2))/(e*(c + d*x^2))])) + (b*e^(3/2)*Sqrt[c + d*x^2]*Ellip
ticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*
f)])/(a*c*(b*c - a*d)*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2)
)]*Sqrt[e + f*x^2])
```

Rubi in Sympy [A] time = 47.0877, size = 170, normalized size = 0.81

$$\frac{\sqrt{d}\sqrt{e+fx^2}E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{\sqrt{c}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}\,(ad-bc)}-\frac{be^{\frac{3}{2}}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**2+e)**(1/2)/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] sqrt(d)*sqrt(e + f*x**2)*elliptic_e(atan(sqrt(d)*x/sqrt(c)), -c*f
/(d*e) + 1)/(sqrt(c)*sqrt(c*(e + f*x**2)/(e*(c + d*x**2)))*sqrt(c
 + d*x**2)*(a*d - b*c)) - b*e**(3/2)*sqrt(c + d*x**2)*elliptic_pi
(1 - b*e/(a*f), atan(sqrt(f)*x/sqrt(e)), 1 - d*e/(c*f))/(a*c*sqrt
(f)*sqrt(e*(c + d*x**2)/(c*(e + f*x**2)))*sqrt(e + f*x**2)*(a*d -
 b*c))
```

Mathematica [C] time = 1.21845, size = 347, normalized size = 1.66

$$\frac{\sqrt{\frac{d}{c}}\left(ibce\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\,\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-iacf\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\,\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+ia\sqrt{\frac{dx^2}{c}+1}\cdots\right)}{ad\sqrt{c+dx^2}\sqrt{e+\cdots}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[e + f*x^2]/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d/c]*(a*d*Sqrt[d/c]*e*x + a*d*Sqrt[d/c]*f*x^3 + I*a*d*e*Sqr
t[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c
]*x], (c*f)/(d*e)] + I*a*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[
1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I
*b*c*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(
a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*c*f*Sqrt[1 + (d*
x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqr
t[d/c]*x], (c*f)/(d*e)]))/(a*d*(-(b*c) + a*d)*Sqrt[c + d*x^2]*Sqr
t[e + f*x^2])
```

Maple [A] time = 0.039, size = 390, normalized size = 1.9

$$\frac{1}{ac\left(ad-bc\right)\left(dfx^4+cfx^2+dex^2+ce\right)}\left(x^3adf\sqrt{-\frac{d}{c}}-EllipticF\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)acf\sqrt{\frac{fx^2+e}{e}}\sqrt{\frac{dx^2+c}{c}}+EllipticF\left(x\sqrt{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] (x^3*a*d*f*(-d/c)^(1/2)-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))
*a*c*f*((f*x^2+e)/e)^(1/2)*((d*x^2+c)/c)^(1/2)+EllipticF(x*(-d/c)
^(1/2),(c*f/d/e)^(1/2))*a*d*e*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)-EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e*((d*x^2+c)/c
)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f
/e)^(1/2)/(-d/c)^(1/2))*a*c*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
```

```
1/2)-EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))
*b*c*e*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+x*a*d*e*(-d/c)^(1/
2))*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/c/a/(-d/c)^(1/2)/(a*d-b*c)/(d
*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="maxima")

[Out] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**2+e)**(1/2)/(b*x**2+a)/(d*x**2+c)**(3/2),x)

[Out] Integral(sqrt(e + f*x**2)/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{f x^2 + e}}{(b x^2 + a)(d x^2 + c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

3.69 $\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=401

$$\frac{b^2e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{d}\sqrt{e+fx^2}(bc(5de-4cf)-ad(2de-cf))E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}\sqrt{c+dx^2}(bc-ad)^2(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{de^{3/2}\sqrt{f}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2\sqrt{e+fx^2}(bc-ad)(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{dx\sqrt{e+fx^2}}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}$$

```
[Out] -(d*x*Sqrt[e + f*x^2])/(3*c*(b*c - a*d)*(c + d*x^2)^(3/2)) - (Sqr
t[d]*(b*c*(5*d*e - 4*c*f) - a*d*(2*d*e - c*f))*Sqrt[e + f*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(3*c^(3/2)
*(b*c - a*d)^2*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(
e*(c + d*x^2))]) + (d*e^(3/2)*Sqrt[f]*Sqrt[c + d*x^2]*EllipticF[A
rcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*(b*c - a*d)*
(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]
) + (b^2*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcT
an[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)^2*Sqr
t[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 1.08284, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$\frac{b^2e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{d}\sqrt{e+fx^2}(bc(5de-4cf)-ad(2de-cf))E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}\sqrt{c+dx^2}(bc-ad)^2(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{de^{3/2}\sqrt{f}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2\sqrt{e+fx^2}(bc-ad)(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{dx\sqrt{e+fx^2}}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[e + f*x^2]/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -(d*x*Sqrt[e + f*x^2])/(3*c*(b*c - a*d)*(c + d*x^2)^(3/2)) - (Sqr
t[d]*(b*c*(5*d*e - 4*c*f) - a*d*(2*d*e - c*f))*Sqrt[e + f*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(3*c^(3/2)
*(b*c - a*d)^2*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(
e*(c + d*x^2))]) + (d*e^(3/2)*Sqrt[f]*Sqrt[c + d*x^2]*EllipticF[A
rcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*(b*c - a*d)*
(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]
) + (b^2*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcT
an[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)^2*Sqr
t[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi in Sympy [A] time = 127.482, size = 333, normalized size = 0.83

$$\frac{dx\sqrt{e+fx^2}}{3c\left(c+dx^2\right)^{\frac{3}{2}}(ad-bc)}+\frac{\sqrt{d}f\sqrt{e+fx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{3\sqrt{c}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}(ad-bc)(cf-de)}$$
$$+\frac{\sqrt{d}\sqrt{e+fx^2}\left(cf(ad-4bc)-de(2ad-5bc)\right)E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{3c^{\frac{3}{2}}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}(ad-bc)^2(cf-de)}$$
$$+\frac{b^2e^{\frac{3}{2}}\sqrt{c+dx^2}\left(1-\frac{be}{af};\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**2+e)**(1/2)/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] d*x*sqrt(e + f*x**2)/(3*c*(c + d*x**2)**(3/2)*(a*d - b*c)) + sqrt
(d)*f*sqrt(e + f*x**2)*elliptic_f(atan(sqrt(d)*x/sqrt(c)), -c*f/(
d*e) + 1)/(3*sqrt(c)*sqrt(c*(e + f*x**2)/(e*(c + d*x**2)))*sqrt(c
 + d*x**2)*(a*d - b*c)*(c*f - d*e)) + sqrt(d)*sqrt(e + f*x**2)*(c
*f*(a*d - 4*b*c) - d*e*(2*a*d - 5*b*c))*elliptic_e(atan(sqrt(d)*x
/sqrt(c)), -c*f/(d*e) + 1)/(3*c**(3/2)*sqrt(c*(e + f*x**2)/(e*(c
+ d*x**2)))*sqrt(c + d*x**2)*(a*d - b*c)**2*(c*f - d*e)) + b**2*e
**(3/2)*sqrt(c + d*x**2)*elliptic_pi(1 - b*e/(a*f), atan(sqrt(f)*
x/sqrt(e)), 1 - d*e/(c*f))/(a*c*sqrt(f)*sqrt(e*(c + d*x**2)/(c*(e
 + f*x**2)))*sqrt(e + f*x**2)*(a*d - b*c)**2)
```

Mathematica [C] time = 6.41633, size = 427, normalized size = 1.06

$$\frac{acx\left(\frac{d}{c}\right)^{3/2}\left(e+fx^2\right)\left(ad\left(2c^2f-3cde+cdfx^2-2d^2ex^2\right)+bc\left(-5c^2f+6cde-4cdfx^2+5d^2ex^2\right)\right)-3ibc^2\left(c+dx^2\right)\sqrt{\cdots}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[e + f*x^2]/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (a*c*(d/c)^(3/2)*x*(e + f*x^2)*(b*c*(6*c*d*e - 5*c^2*f + 5*d^2*e*
x^2 - 4*c*d*f*x^2) + a*d*(-3*c*d*e + 2*c^2*f - 2*d^2*e*x^2 + c*d*
f*x^2)) - I*a*d*e*(a*d*(2*d*e - c*f) + b*c*(-5*d*e + 4*c*f))*(c +
 d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSi
nh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*(-(d*e) + c*f)*(2*a*d^2*e + b
*c*(-5*d*e + 3*c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*
x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (3*I)*b*
c^2*(b*e - a*f)*(-(d*e) + c*f)*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sq
rt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x],
 (c*f)/(d*e)])/(3*a*c^2*Sqrt[d/c]*(b*c - a*d)^2*(-(d*e) + c*f)*(c
 + d*x^2)^(3/2)*Sqrt[e + f*x^2])
```

Maple [B] time = 0.055, size = 2068, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x)
```

```
[Out] 1/3*(-2*x^3*a^2*d^4*e^2*(-d/c)^(1/2)+x^5*a^2*c*d^3*f^2*(-d/c)^(1/
2)-8*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*b*c^2*d^2*e*
f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+4*EllipticE(x*(-d/c)^(1
/2),(c*f/d/e)^(1/2))*x^2*a*b*c^2*d^2*e*f*((d*x^2+c)/c)^(1/2)*((f*
x^2+e)/e)^(1/2)+3*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/
(-d/c)^(1/2))*x^2*a*b*c^2*d^2*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/
e)^(1/2)+5*x^3*a*b*c*d^3*e^2*(-d/c)^(1/2)+2*EllipticF(x*(-d/c)^(1
/2),(c*f/d/e)^(1/2))*x^2*a^2*c*d^3*e*f*((d*x^2+c)/c)^(1/2)*((f*x^
2+e)/e)^(1/2)+3*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*b
*c^3*d*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+5*EllipticF(x*
(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*b*c*d^3*e^2*((d*x^2+c)/c)^(1/
2)*((f*x^2+e)/e)^(1/2)-EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*
x^2*a^2*c*d^3*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-5*Ellip
ticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*b*c*d^3*e^2*((d*x^2+c)
/c)^(1/2)*((f*x^2+e)/e)^(1/2)-3*EllipticPi(x*(-d/c)^(1/2),b*c/a/d
,(-f/e)^(1/2)/(-d/c)^(1/2))*x^2*a*b*c^3*d*f^2*((d*x^2+c)/c)^(1/2)
*((f*x^2+e)/e)^(1/2)+3*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(
1/2)/(-d/c)^(1/2))*x^2*b^2*c^3*d*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)-8*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c^3*d
*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+4*EllipticE(x*(-d/c)
^(1/2),(c*f/d/e)^(1/2))*a*b*c^3*d*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2
+e)/e)^(1/2)-5*x*a*b*c^3*d*e*f*(-d/c)^(1/2)-2*x^5*a^2*d^4*e*f*(-d
/c)^(1/2)+2*x^3*a^2*c^2*d^2*f^2*(-d/c)^(1/2)-3*x*a^2*c*d^3*e^2*(-
d/c)^(1/2)+2*x*a^2*c^2*d^2*e*f*(-d/c)^(1/2)+6*x*a*b*c^2*d^2*e^2*(
-d/c)^(1/2)-2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^2*d
^4*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticE(x*(-d/
c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^2*d^4*e^2*((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)-2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*
c*d^3*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+3*EllipticF(x*(
```

```
-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c^4*f^2*((d*x^2+c)/c)^(1/2)*((f*
x^2+e)/e)^(1/2)+2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c
*d^3*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-3*EllipticPi(x*(
-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b*c^4*f^2*((d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+3*EllipticPi(x*(-d/c)^(1/2),b*c
/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c^4*e*f*((d*x^2+c)/c)^(1/2)*(
(f*x^2+e)/e)^(1/2)-3*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/
2)/(-d/c)^(1/2))*b^2*c^3*d*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^
(1/2)-4*x^5*a*b*c^2*d^2*f^2*(-d/c)^(1/2)-2*x^3*a^2*c*d^3*e*f*(-d/
c)^(1/2)-5*x^3*a*b*c^3*d*f^2*(-d/c)^(1/2)-3*EllipticPi(x*(-d/c)^(
1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*x^2*b^2*c^2*d^2*e^2*((d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticF(x*(-d/c)^(1/2),(c*
f/d/e)^(1/2))*a^2*c^2*d^2*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)+5*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c^2*d^2*e^2*
((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticE(x*(-d/c)^(1/2),
(c*f/d/e)^(1/2))*a^2*c^2*d^2*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e
)^(1/2)-5*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c^2*d^2*e
^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+5*x^5*a*b*c*d^3*e*f*(-
d/c)^(1/2)+2*x^3*a*b*c^2*d^2*e*f*(-d/c)^(1/2)+3*EllipticPi(x*(-d/
c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b*c^3*d*e*f*((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2))/(f*x^2+e)^(1/2)/(a*d-b*c)^2/c^2
/(c*f-d*e)/(-d/c)^(1/2)/a/(d*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(5/2)),x, algorithm="maxima")

[Out] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(5/2)),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**2+e)**(1/2)/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{fx^2 + e}}{(bx^2 + a)(dx^2 + c)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(5/2)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

3.70 $\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx$

Optimal. Leaf size=630

$$\frac{b^3e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{b^2\sqrt{d}\sqrt{e+fx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)^3\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$+\frac{de^{3/2}\sqrt{f}\sqrt{c+dx^2}(bc(9de-11cf)-2ad(2de-3cf))F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15c^3\sqrt{e+fx^2}(bc-ad)^2(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{dx\sqrt{e+fx^2}(bc(9de-8cf)-ad(4de-3cf))}{15c^2\left(c+dx^2\right)^{3/2}(bc-ad)^2(de-cf)}$$
$$+\frac{\sqrt{d}\sqrt{e+fx^2}\left(ad\left(3c^2f^2-13cdef+8d^2e^2\right)-2bc\left(4c^2f^2-14cdef+9d^2e^2\right)\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}\sqrt{c+dx^2}(bc-ad)^2(de-cf)^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$-\frac{dx\sqrt{e+fx^2}}{5c\left(c+dx^2\right)^{5/2}(bc-ad)}$$

```
[Out] -(d*x*Sqrt[e + f*x^2])/(5*c*(b*c - a*d)*(c + d*x^2)^(5/2)) - (d*(
b*c*(9*d*e - 8*c*f) - a*d*(4*d*e - 3*c*f))*x*Sqrt[e + f*x^2])/(15
*c^2*(b*c - a*d)^2*(d*e - c*f)*(c + d*x^2)^(3/2)) - (b^2*Sqrt[d]*
Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/
(d*e)])/(Sqrt[c]*(b*c - a*d)^3*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2
))/(e*(c + d*x^2))]) + (Sqrt[d]*(a*d*(8*d^2*e^2 - 13*c*d*e*f + 3*
c^2*f^2) - 2*b*c*(9*d^2*e^2 - 14*c*d*e*f + 4*c^2*f^2))*Sqrt[e + f
*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(1
5*c^(5/2)*(b*c - a*d)^2*(d*e - c*f)^2*Sqrt[c + d*x^2]*Sqrt[(c*(e
+ f*x^2))/(e*(c + d*x^2))]) + (d*e^(3/2)*Sqrt[f]*(b*c*(9*d*e - 11
*c*f) - 2*a*d*(2*d*e - 3*c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(
Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*c^3*(b*c - a*d)^2*(d*e
 - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
+ (b^3*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan
[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)^3*Sqrt[
f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 2.04757, antiderivative size = 630, normalized size of antiderivative = 1., number of

steps used = 9, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{b^3e^{3/2}\sqrt{c+dx^2}\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{b^2\sqrt{d}\sqrt{e+fx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)^3\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$+\frac{de^{3/2}\sqrt{f}\sqrt{c+dx^2}(bc(9de-11cf)-2ad(2de-3cf))F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15c^3\sqrt{e+fx^2}(bc-ad)^2(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{dx\sqrt{e+fx^2}(bc(9de-8cf)-ad(4de-3cf))}{15c^2\left(c+dx^2\right)^{3/2}(bc-ad)^2(de-cf)}$$
$$+\frac{\sqrt{d}\sqrt{e+fx^2}\left(ad\left(3c^2f^2-13cdef+8d^2e^2\right)-2bc\left(4c^2f^2-14cdef+9d^2e^2\right)\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}\sqrt{c+dx^2}(bc-ad)^2(de-cf)^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$-\frac{dx\sqrt{e+fx^2}}{5c\left(c+dx^2\right)^{5/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[e + f*x^2]/((a + b*x^2)*(c + d*x^2)^(7/2)),x]
```

```
[Out] -(d*x*Sqrt[e + f*x^2])/(5*c*(b*c - a*d)*(c + d*x^2)^(5/2)) - (d*(
b*c*(9*d*e - 8*c*f) - a*d*(4*d*e - 3*c*f))*x*Sqrt[e + f*x^2])/(15
*c^2*(b*c - a*d)^2*(d*e - c*f)*(c + d*x^2)^(3/2)) - (b^2*Sqrt[d]*
Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/
(d*e)])/(Sqrt[c]*(b*c - a*d)^3*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2
))/(e*(c + d*x^2))]) + (Sqrt[d]*(a*d*(8*d^2*e^2 - 13*c*d*e*f + 3*
c^2*f^2) - 2*b*c*(9*d^2*e^2 - 14*c*d*e*f + 4*c^2*f^2))*Sqrt[e + f
*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(1
5*c^(5/2)*(b*c - a*d)^2*(d*e - c*f)^2*Sqrt[c + d*x^2]*Sqrt[(c*(e
+ f*x^2))/(e*(c + d*x^2))]) + (d*e^(3/2)*Sqrt[f]*(b*c*(9*d*e - 11
*c*f) - 2*a*d*(2*d*e - 3*c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(
Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*c^3*(b*c - a*d)^2*(d*e
 - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
+ (b^3*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan
[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)^3*Sqrt[
f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**2+e)**(1/2)/(b*x**2+a)/(d*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 5.48753, size = 584, normalized size = 0.93

$$-adx\sqrt{\frac{d}{c}}\left(e+fx^2\right)\left(\left(c+dx^2\right)^2\left(a^2d^2\left(3c^2f^2-13cdef+8d^2e^2\right)+abcd\left(-11c^2f^2+41cdef-26d^2e^2\right)+b^2c^2\left(23c^2f^2-58\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[e + f*x^2]/((a + b*x^2)*(c + d*x^2)^(7/2)),x]
```

```
[Out] (-(a*d*Sqrt[d/c]*x*(e + f*x^2)*(3*c^2*(b*c - a*d)^2*(d*e - c*f)^2
 + c*(b*c - a*d)*(-(d*e) + c*f)*(a*d*(4*d*e - 3*c*f) + b*c*(-9*d*
e + 8*c*f))*(c + d*x^2) + (a*b*c*d*(-26*d^2*e^2 + 41*c*d*e*f - 11
*c^2*f^2) + a^2*d^2*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2) + b^2*c^
2*(33*d^2*e^2 - 58*c*d*e*f + 23*c^2*f^2))*(c + d*x^2)^2)) - I*(c
+ d*x^2)^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(a*d*e*(a*b*c*
d*(-26*d^2*e^2 + 41*c*d*e*f - 11*c^2*f^2) + a^2*d^2*(8*d^2*e^2 -
13*c*d*e*f + 3*c^2*f^2) + b^2*c^2*(33*d^2*e^2 - 58*c*d*e*f + 23*c
^2*f^2))*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (d*e -
c*f)*(-(a*(2*a*b*c*d^2*e*(13*d*e - 14*c*f) + a^2*d^3*e*(-8*d*e +
9*c*f) + b^2*c^2*(-33*d^2*e^2 + 49*c*d*e*f - 15*c^2*f^2))*Ellipti
cF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]) + 15*b^2*c^3*(b*e - a*f)
*(-(d*e) + c*f)*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (
c*f)/(d*e)])))/(15*a*c^3*Sqrt[d/c]*(b*c - a*d)^3*(d*e - c*f)^2*(c
 + d*x^2)^(5/2)*Sqrt[e + f*x^2])
```

Maple [B] time = 0.087, size = 6245, normalized size = 9.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(7/2)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(7/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(7/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**2+e)**(1/2)/(b*x**2+a)/(d*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(7/2)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(7/2)), x)
```

3.71 $\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{a+bx^2}\,dx$

Optimal. Leaf size=659

$$-\frac{\sqrt{e}\sqrt{c+dx^2}\left(15a^2d^2f^2-20abdf(cf+de)+3b^2\left(c^2f^2+9cdef+d^2e^2\right)\right)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15b^3d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{e^{3/2}\sqrt{c+dx^2}\left(15a^2d^2f-5abd(5cf+3de)+3b^2c(3cf+8de)\right)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15b^3c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{e^{3/2}\sqrt{c+dx^2}(bc-ad)^2(be-af)\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ab^3c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{f^2x\sqrt{c+dx^2}(bc-ad)^2}{b^3d\sqrt{e+fx^2}}$$
$$+\frac{fx\sqrt{c+dx^2}\sqrt{e+fx^2}(bc-ad)}{3b^2}+\frac{2fx\sqrt{c+dx^2}(bc-ad)(2de-cf)}{3b^2d\sqrt{e+fx^2}}$$
$$+\frac{x\sqrt{c+dx^2}\left(-2c^2f^2+7cdef+3d^2e^2\right)}{15bd\sqrt{e+fx^2}}+\frac{fx\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{5b}+\frac{2x\sqrt{c+dx^2}\sqrt{e+fx^2}(3de-cf)}{15b}$$

```
[Out] ((b*c - a*d)^2*f^2*x*Sqrt[c + d*x^2])/(b^3*d*Sqrt[e + f*x^2]) + (
2*(b*c - a*d)*f*(2*d*e - c*f)*x*Sqrt[c + d*x^2])/(3*b^2*d*Sqrt[e
+ f*x^2]) + ((3*d^2*e^2 + 7*c*d*e*f - 2*c^2*f^2)*x*Sqrt[c + d*x^2
])/(15*b*d*Sqrt[e + f*x^2]) + ((b*c - a*d)*f*x*Sqrt[c + d*x^2]*Sq
rt[e + f*x^2])/(3*b^2) + (2*(3*d*e - c*f)*x*Sqrt[c + d*x^2]*Sqrt[
e + f*x^2])/(15*b) + (f*x*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(5*b
) - (Sqrt[e]*(15*a^2*d^2*f^2 - 20*a*b*d*f*(d*e + c*f) + 3*b^2*(d^
2*e^2 + 9*c*d*e*f + c^2*f^2))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(S
qrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*b^3*d*Sqrt[f]*Sqrt[(e*(
c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (e^(3/2)*(15*a^2*
d^2*f + 3*b^2*c*(8*d*e + 3*c*f) - 5*a*b*d*(3*d*e + 5*c*f))*Sqrt[c
 + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)]
)/(15*b^3*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
+ f*x^2]) + ((b*c - a*d)^2*e^(3/2)*(b*e - a*f)*Sqrt[c + d*x^2]*El
lipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/
(c*f)])/(a*b^3*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sq
rt[e + f*x^2])
```

Rubi [A] time = 2.18842, antiderivative size = 784, normalized size of antiderivative = 1.19, number of steps used = 14, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$

$$\frac{c^{3/2}\sqrt{e+fx^2}(bc-ad)(be-af)^2\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab^3\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$+\frac{fx\sqrt{c+dx^2}(bc-ad)(-3adf+bcf+4bde)}{3b^3d\sqrt{e+fx^2}}$$
$$+\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(bc-ad)(5be-3af)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^3\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(bc-ad)(-3adf+bcf+4bde)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^3d\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{fx\sqrt{c+dx^2}\sqrt{e+fx^2}(bc-ad)}{3b^2}+\frac{x\sqrt{c+dx^2}\left(-2c^2f^2+7cdef+3d^2e^2\right)}{15bd\sqrt{e+fx^2}}$$
$$-\frac{\sqrt{e}\sqrt{c+dx^2}\left(-2c^2f^2+7cdef+3d^2e^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15bd\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{e^{3/2}\sqrt{c+dx^2}(9de-cf)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15b\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{fx\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}{5b}+\frac{2x\sqrt{c+dx^2}\sqrt{e+fx^2}(3de-cf)}{15b}$$

Antiderivative was successfully verified.

```
[In]  Int[((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] ((b*c - a*d)*f*(4*b*d*e + b*c*f - 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*
b^3*d*Sqrt[e + f*x^2]) + ((3*d^2*e^2 + 7*c*d*e*f - 2*c^2*f^2)*x*S
qrt[c + d*x^2])/(15*b*d*Sqrt[e + f*x^2]) + ((b*c - a*d)*f*x*Sqrt[
c + d*x^2]*Sqrt[e + f*x^2])/(3*b^2) + (2*(3*d*e - c*f)*x*Sqrt[c +
 d*x^2]*Sqrt[e + f*x^2])/(15*b) + (f*x*(c + d*x^2)^(3/2)*Sqrt[e +
 f*x^2])/(5*b) - ((b*c - a*d)*Sqrt[e]*Sqrt[f]*(4*b*d*e + b*c*f -
3*a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1
 - (d*e)/(c*f)])/(3*b^3*d*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*S
qrt[e + f*x^2]) - (Sqrt[e]*(3*d^2*e^2 + 7*c*d*e*f - 2*c^2*f^2)*Sq
rt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c
*f)])/(15*b*d*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[
e + f*x^2]) + ((b*c - a*d)*Sqrt[e]*Sqrt[f]*(5*b*e - 3*a*f)*Sqrt[c
 + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)]
)/(3*b^3*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) +
 (e^(3/2)*(9*d*e - c*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]
*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*b*Sqrt[f]*Sqrt[(e*(c + d*x^2)
)/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (c^(3/2)*(b*c - a*d)*(b*e -
```

```
 a*f)^2*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[
d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(a*b^3*Sqrt[d]*e*Sqrt[c + d*x^2
]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(3/2)*(f*x**2+e)**(3/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 4.50967, size = 445, normalized size = 0.68

$$-iabe\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(15a^2d^2f^2-20abdf(cf+de)+3b^2\left(c^2f^2+9cdef+d^2e^2\right)\right)E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-ia\sqrt{\frac{dx^2}{c}+$$

Antiderivative was successfully verified.

```
[In]  Integrate[((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] ((-I)*a*b*e*(15*a^2*d^2*f^2 - 20*a*b*d*f*(d*e + c*f) + 3*b^2*(d^2
*e^2 + 9*c*d*e*f + c^2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)
/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*(-15*a^3
*d^2*f^3 + 15*a^2*b*d*f^2*(d*e + 2*c*f) - 3*b^3*e*(d^2*e^2 + c*d*
e*f - 7*c^2*f^2) + 5*a*b^2*f*(d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2))*S
qrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d
/c]*x], (c*f)/(d*e)] + f*(a*b^2*Sqrt[d/c]*x*(c + d*x^2)*(e + f*x^
2)*(-5*a*d*f + 3*b*(2*d*e + 2*c*f + d*f*x^2)) - (15*I)*(b*c - a*d
)^2*(b*e - a*f)^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Ellipti
cPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(15*a*b^4
*Sqrt[d/c]*f*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [B] time = 0.032, size = 1939, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x)
```

```
[Out] 1/15*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*(3*(-d/c)^(1/2)*x^7*a*b^3*d^
2*f^3+30*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/
c)^(1/2),(c*f/d/e)^(1/2))*a^3*b*c*d*f^3+3*((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^3*
c^2*e*f^2-30*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x
*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^3*b*c*d*f^3-30
*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2
),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^3*b*d^2*e*f^2+15*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,
(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*b^2*d^2*e^2*f-30*((d*x^2+c)/c)^(1/
2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(
1/2)/(-d/c)^(1/2))*a*b^3*c^2*e*f^2+15*((d*x^2+c)/c)^(1/2)*((f*x^2
+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^3*b*d^2*
e*f^2+5*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c
)^(1/2),(c*f/d/e)^(1/2))*a^2*b^2*d^2*e^2*f+21*((d*x^2+c)/c)^(1/2)
*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*
b^3*c^2*e*f^2+15*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elliptic
E(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^3*b*d^2*e*f^2-20*((d*x^2+c)/c
)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1
/2))*a^2*b^2*d^2*e^2*f-5*(-d/c)^(1/2)*x*a^2*b^2*c*d*e*f^2+6*(-d/c
)^(1/2)*x*a*b^3*c*d*e^2*f+15*(-d/c)^(1/2)*x^3*a*b^3*c*d*e*f^2-5*(
-d/c)^(1/2)*x^5*a^2*b^2*d^2*f^3+6*(-d/c)^(1/2)*x^3*a*b^3*c^2*f^3-
15*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/
2),(c*f/d/e)^(1/2))*a^4*d^2*f^3+15*((d*x^2+c)/c)^(1/2)*((f*x^2+e)
/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(
1/2))*a^4*d^2*f^3+9*(-d/c)^(1/2)*x^5*a*b^3*c*d*f^3+9*(-d/c)^(1/2)
*x^5*a*b^3*d^2*e*f^2-5*(-d/c)^(1/2)*x^3*a^2*b^2*c*d*f^3-5*(-d/c)^
(1/2)*x^3*a^2*b^2*d^2*e*f^2+6*(-d/c)^(1/2)*x^3*a*b^3*d^2*e^2*f-15
*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2)
,(c*f/d/e)^(1/2))*a^2*b^2*c^2*f^3-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e
)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^3*d^2*e^
3+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1
/2),(c*f/d/e)^(1/2))*a*b^3*d^2*e^3+15*((d*x^2+c)/c)^(1/2)*((f*x^2
+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c
)^(1/2))*a^2*b^2*c^2*f^3+15*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/
2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b
^4*c^2*e^2*f+6*(-d/c)^(1/2)*x*a*b^3*c^2*e*f^2-30*((d*x^2+c)/c)^(1
/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^
(1/2)/(-d/c)^(1/2))*a*b^3*c*d*e^2*f+27*((d*x^2+c)/c)^(1/2)*((f*x^
2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^3*c*d
*e^2*f+60*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-
d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*b^2*c*d*e*f^2-3
*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2)
,(c*f/d/e)^(1/2))*a*b^3*c*d*e^2*f-20*((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b^2*c*d
*e*f^2-35*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d
/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b^2*c*d*e*f^2)/f/(d*f*x^4+c*f*x^2+
d*e*x^2+c*e)/b^4/(-d/c)^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}\left(fx^2+e\right)^{\frac{3}{2}}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)/(b*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(3/2)*(f*x**2+e)**(3/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}\left(fx^2+e\right)^{\frac{3}{2}}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)/(b*x^2 + a), x)
```

3.72 $\int \frac{\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{a+bx^2} dx$

Optimal. Leaf size=403

$$\frac{c^{3/2}\sqrt{e+fx^2}(be-af)^2\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab^2\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{fx\sqrt{c+dx^2}(-3adf+bcf+4bde)}{3b^2d\sqrt{e+fx^2}}+\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(5be-3af)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^2\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(-3adf+bcf+4bde)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^2d\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b}$$

```
[Out] (f*(4*b*d*e + b*c*f - 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*b^2*d*Sqrt[e
 + f*x^2]) + (f*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*b) - (Sqrt[
e]*Sqrt[f]*(4*b*d*e + b*c*f - 3*a*d*f)*Sqrt[c + d*x^2]*EllipticE[
ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*b^2*d*Sqrt[(e*(
c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (Sqrt[e]*Sqrt[f]*
(5*b*e - 3*a*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt
[e]], 1 - (d*e)/(c*f)])/(3*b^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2
))]*Sqrt[e + f*x^2]) + (c^(3/2)*(b*e - a*f)^2*Sqrt[e + f*x^2]*Ell
ipticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(
d*e)])/(a*b^2*Sqrt[d]*e*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(
c + d*x^2))])
```

Rubi [A] time = 0.944809, antiderivative size = 403, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$

$$\frac{c^{3/2}\sqrt{e+fx^2}(be-af)^2\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab^2\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{fx\sqrt{c+dx^2}(-3adf+bcf+4bde)}{3b^2d\sqrt{e+fx^2}}+\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(5be-3af)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^2\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(-3adf+bcf+4bde)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^2d\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] (f*(4*b*d*e + b*c*f - 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*b^2*d*Sqrt[e
 + f*x^2]) + (f*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*b) - (Sqrt[
e]*Sqrt[f]*(4*b*d*e + b*c*f - 3*a*d*f)*Sqrt[c + d*x^2]*EllipticE[
ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*b^2*d*Sqrt[(e*(
c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (Sqrt[e]*Sqrt[f]*
(5*b*e - 3*a*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt
[e]], 1 - (d*e)/(c*f)])/(3*b^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2
))]*Sqrt[e + f*x^2]) + (c^(3/2)*(b*e - a*f)^2*Sqrt[e + f*x^2]*Ell
ipticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(
d*e)])/(a*b^2*Sqrt[d]*e*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(
c + d*x^2))])
```

Rubi in Sympy [A] time = 131.911, size = 360, normalized size = 0.89

$$\frac{fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b} - \frac{c^{\frac{3}{2}}f\sqrt{e+fx^2}\,(3af-5be)\,F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{3b^2\sqrt{d}e\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}} + \frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}\,(3adf-bcf-4bde)\,E\left(\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{3b^2d\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{fx\sqrt{c+dx^2}\,(3adf-bcf-4bde)}{3b^2d\sqrt{e+fx^2}} + \frac{c^{\frac{3}{2}}\sqrt{e+fx^2}\,(af-be)^2\,\Pi\left(1-\frac{bc}{ad};\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{ab^2\sqrt{d}e\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(1/2)*(f*x**2+e)**(3/2)/(b*x**2+a),x)
```

```
[Out] f*x*sqrt(c + d*x**2)*sqrt(e + f*x**2)/(3*b) - c**(3/2)*f*sqrt(e +
 f*x**2)*(3*a*f - 5*b*e)*elliptic_f(atan(sqrt(d)*x/sqrt(c)), -c*f
/(d*e) + 1)/(3*b**2*sqrt(d)*e*sqrt(c*(e + f*x**2)/(e*(c + d*x**2)
))*sqrt(c + d*x**2)) + sqrt(e)*sqrt(f)*sqrt(c + d*x**2)*(3*a*d*f
- b*c*f - 4*b*d*e)*elliptic_e(atan(sqrt(f)*x/sqrt(e)), 1 - d*e/(c
*f))/(3*b**2*d*sqrt(e*(c + d*x**2)/(c*(e + f*x**2)))*sqrt(e + f*x
**2)) - f*x*sqrt(c + d*x**2)*(3*a*d*f - b*c*f - 4*b*d*e)/(3*b**2*
d*sqrt(e + f*x**2)) + c**(3/2)*sqrt(e + f*x**2)*(a*f - b*e)**2*el
liptic_pi(1 - b*c/(a*d), atan(sqrt(d)*x/sqrt(c)), -c*f/(d*e) + 1)
/(a*b**2*sqrt(d)*e*sqrt(c*(e + f*x**2)/(e*(c + d*x**2)))*sqrt(c +
 d*x**2))
```

Mathematica [C] time = 2.92614, size = 739, normalized size = 1.83

$$3ia^3df^2\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\,\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)\middle|\frac{cf}{de}\right) - ia\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(3a^2df^2-3abf(cf+de)+b^2e(4cf-de)\right)F$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] (a*b^2*c*Sqrt[d/c]*e*f*x + a*b^2*d*Sqrt[d/c]*e*f*x^3 + a*b^2*c*Sq
rt[d/c]*f^2*x^3 + a*b^2*d*Sqrt[d/c]*f^2*x^5 - I*a*b*e*(4*b*d*e +
b*c*f - 3*a*d*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Elliptic
E[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*(3*a^2*d*f^2 - 3*a*b
*f*(d*e + c*f) + b^2*e*(-(d*e) + 4*c*f))*Sqrt[1 + (d*x^2)/c]*Sqrt
[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] -
(3*I)*b^3*c*e^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticP
i[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (3*I)*a*b^2
*d*e^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(
a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (6*I)*a*b^2*c*e*f*Sq
rt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*A
rcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (6*I)*a^2*b*d*e*f*Sqrt[1 + (d
*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sq
rt[d/c]*x], (c*f)/(d*e)] - (3*I)*a^2*b*c*f^2*Sqrt[1 + (d*x^2)/c]*
Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x
], (c*f)/(d*e)] + (3*I)*a^3*d*f^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f
*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d
*e)])/(3*a*b^3*Sqrt[d/c]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [B] time = 0.027, size = 1028, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x)
```

```
[Out] 1/3*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*((-d/c)^(1/2)*x^5*a*b^2*d*f^2
+(-d/c)^(1/2)*x^3*a*b^2*c*f^2+(-d/c)^(1/2)*x^3*a*b^2*d*e*f+3*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f
/d/e)^(1/2))*a^3*d*f^2-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*
EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*c*f^2-3*((d*x^2+c
)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)
^(1/2))*a^2*b*d*e*f+4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ell
ipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c*e*f-((d*x^2+c)/c)^
(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2
))*a*b^2*d*e^2-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elliptic
E(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*d*e*f+((d*x^2+c)/c)^(1/2)
*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*
b^2*c*e*f+4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(
-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*d*e^2-3*((d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/
(-d/c)^(1/2))*a^3*d*f^2+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)
*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2
*b*c*f^2+6*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(
-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*b*d*e*f-6*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c
```

```
/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b^2*c*e*f-3*((d*x^2+c)/c)^(1/2)
*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/
2)/(-d/c)^(1/2))*a*b^2*d*e^2+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^
(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2)
)*b^3*c*e^2+(-d/c)^(1/2)*x*a*b^2*c*e*f)/(d*f*x^4+c*f*x^2+d*e*x^2+
c*e)/b^3/(-d/c)^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}\left(fx^2+e\right)^{\frac{3}{2}}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(b*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}\left(e+fx^2\right)^{\frac{3}{2}}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*(f*x**2+e)**(3/2)/(b*x**2+a),x)
```

```
[Out] Integral(sqrt(c + d*x**2)*(e + f*x**2)**(3/2)/(a + b*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}\left(fx^2+e\right)^{\frac{3}{2}}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(b*x^2 + a), x)
```

3.73 $\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=328

$$\frac{e^{3/2}\sqrt{c+dx^2}(be-af)\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{abc\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{bc\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{\sqrt{e}f^{3/2}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{bd\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{f^2x\sqrt{c+dx^2}}{bd\sqrt{e+fx^2}}$$

```
[Out] (f^2*x*Sqrt[c + d*x^2])/(b*d*Sqrt[e + f*x^2]) - (Sqrt[e]*f^(3/2)*
Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/
(c*f)])/(b*d*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2
]) + (e^(3/2)*Sqrt[f]*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x
)/Sqrt[e]], 1 - (d*e)/(c*f)])/(b*c*Sqrt[(e*(c + d*x^2))/(c*(e + f
*x^2))]*Sqrt[e + f*x^2]) + (e^(3/2)*(b*e - a*f)*Sqrt[c + d*x^2]*E
llipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)
/(c*f)])/(a*b*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqr
t[e + f*x^2])
```

Rubi [A] time = 0.654585, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$\frac{e^{3/2}\sqrt{c+dx^2}(be-af)\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{abc\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{bc\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{\sqrt{e}f^{3/2}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{bd\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{f^2x\sqrt{c+dx^2}}{bd\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x^2)^(3/2)/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (f^2*x*Sqrt[c + d*x^2])/(b*d*Sqrt[e + f*x^2]) - (Sqrt[e]*f^(3/2)*
Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/
(c*f)])/(b*d*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2
]) + (e^(3/2)*Sqrt[f]*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x
)/Sqrt[e]], 1 - (d*e)/(c*f)])/(b*c*Sqrt[(e*(c + d*x^2))/(c*(e + f
*x^2))]*Sqrt[e + f*x^2]) + (e^(3/2)*(b*e - a*f)*Sqrt[c + d*x^2]*E
llipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)
/(c*f)])/(a*b*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqr
```

```
t[e + f*x^2])
```

Rubi in Sympy [A] time = 79.32, size = 269, normalized size = 0.82

$$-\frac{\sqrt{c}f\sqrt{e+fx^2}E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{b\sqrt{d}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}}+\frac{\sqrt{c}f\sqrt{e+fx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{b\sqrt{d}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}}+\frac{fx\sqrt{e+fx^2}}{b\sqrt{c+dx^2}}-\frac{e^{\frac{3}{2}}\sqrt{c+dx^2}\,(af-be)\left(1-\frac{be}{af};\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{abc\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**2+e)**(3/2)/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] -sqrt(c)*f*sqrt(e + f*x**2)*elliptic_e(atan(sqrt(d)*x/sqrt(c)), -
c*f/(d*e) + 1)/(b*sqrt(d)*sqrt(c*(e + f*x**2)/(e*(c + d*x**2)))*s
qrt(c + d*x**2)) + sqrt(c)*f*sqrt(e + f*x**2)*elliptic_f(atan(sqr
t(d)*x/sqrt(c)), -c*f/(d*e) + 1)/(b*sqrt(d)*sqrt(c*(e + f*x**2)/(
e*(c + d*x**2)))*sqrt(c + d*x**2)) + f*x*sqrt(e + f*x**2)/(b*sqrt
(c + d*x**2)) - e**(3/2)*sqrt(c + d*x**2)*(a*f - b*e)*elliptic_pi
(1 - b*e/(a*f), atan(sqrt(f)*x/sqrt(e)), 1 - d*e/(c*f))/(a*b*c*sq
rt(f)*sqrt(e*(c + d*x**2)/(c*(e + f*x**2)))*sqrt(e + f*x**2))
```

Mathematica [C] time = 0.344124, size = 184, normalized size = 0.56

$$-\frac{i\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(abefE\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+(be-af)\left((be-af)\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+afF\left(i\sinh^{-1}\left(\sqrt{\right.\right.\right.\right.}{ab^2\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(a*b*e*f*EllipticE[
I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (b*e - a*f)*(a*f*EllipticF
[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (b*e - a*f)*EllipticPi[(b
*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])))/(a*b^2*Sqrt[d/
c]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.031, size = 300, normalized size = 0.9

$$\frac{1}{ab^2\left(dfx^4+cfx^2+dex^2+ce\right)}\left(-EllipticF\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)a^2f^2+EllipticF\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)abef+EllipticE\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{c}{d}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x)
```

```
[Out] (-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*f^2+EllipticF(x*(
-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*e*f+EllipticE(x*(-d/c)^(1/2),(c*
f/d/e)^(1/2))*a*b*e*f+EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1
/2)/(-d/c)^(1/2))*a^2*f^2-2*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f
/e)^(1/2)/(-d/c)^(1/2))*a*b*e*f+EllipticPi(x*(-d/c)^(1/2),b*c/a/d
,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*e^2)*((f*x^2+e)/e)^(1/2)*((d*x^2+
c)/c)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/(-d/c)^(1/2)/b^2/(d
*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(fx^2+e\right)^{\frac{3}{2}}}{(bx^2+a)\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="maxima")
```

```
[Out] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e + fx^{2}\right)^{\frac{3}{2}}}{\left(a + bx^{2}\right)\sqrt{c + dx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**2+e)**(3/2)/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((e + f*x**2)**(3/2)/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx^{2} + e\right)^{\frac{3}{2}}}{\left(bx^{2} + a\right)\sqrt{dx^{2} + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*sqrt(d*x^2 + c)),x, algorithm="giac")
```

```
[Out] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.74 $\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal. Leaf size=224

$$\frac{e^{3/2}\sqrt{c+dx^2}(be-af)\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{e+fx^2}(de-cf)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

```
[Out] -(((d*e - c*f)*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[
c]], 1 - (c*f)/(d*e)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[c + d*x^
2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])) + (e^(3/2)*(b*e - a*f)
*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/S
qrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)*Sqrt[f]*Sqrt[(e*(c +
d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.407764, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094

$$\frac{e^{3/2}\sqrt{c+dx^2}(be-af)\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{e+fx^2}(de-cf)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(((d*e - c*f)*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[
c]], 1 - (c*f)/(d*e)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[c + d*x^
2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])) + (e^(3/2)*(b*e - a*f)
*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/S
qrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)*Sqrt[f]*Sqrt[(e*(c +
d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi in Sympy [A] time = 50.4669, size = 182, normalized size = 0.81

$$-\frac{\sqrt{e+fx^2}\,(cf-de)\,E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}\,(ad-bc)}+\frac{e^{\frac{3}{2}}\sqrt{c+dx^2}\,(af-be)\left(1-\frac{be}{af};\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**2+e)**(3/2)/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] -sqrt(e + f*x**2)*(c*f - d*e)*elliptic_e(atan(sqrt(d)*x/sqrt(c)),
 -c*f/(d*e) + 1)/(sqrt(c)*sqrt(d)*sqrt(c*(e + f*x**2)/(e*(c + d*x
**2)))*sqrt(c + d*x**2)*(a*d - b*c)) + e**(3/2)*sqrt(c + d*x**2)*
(a*f - b*e)*elliptic_pi(1 - b*e/(a*f), atan(sqrt(f)*x/sqrt(e)), 1
 - d*e/(c*f))/(a*c*sqrt(f)*sqrt(e*(c + d*x**2)/(c*(e + f*x**2)))*
sqrt(e + f*x**2)*(a*d - b*c))
```

Mathematica [C] time = 2.20672, size = 492, normalized size = 2.2

$$\sqrt{\frac{d}{c}}\left(ia^2cf^2\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+ib^2ce^2\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+abde^2x\ldots\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d/c]*(a*b*d*Sqrt[d/c]*e^2*x - a*b*c*Sqrt[d/c]*e*f*x + a*b*d
*Sqrt[d/c]*e*f*x^3 - a*b*c*Sqrt[d/c]*f^2*x^3 - I*a*b*e*(-(d*e) +
c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[
Sqrt[d/c]*x], (c*f)/(d*e)] + I*a*(-(a*c*f^2) + b*e*(-(d*e) + 2*c*
f))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[S
qrt[d/c]*x], (c*f)/(d*e)] + I*b^2*c*e^2*Sqrt[1 + (d*x^2)/c]*Sqrt[
1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c
*f)/(d*e)] - (2*I)*a*b*c*e*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)
/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]
+ I*a^2*c*f^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[
(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(a*b*d*(-(b*c
) + a*d)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [B] time = 0.043, size = 630, normalized size = 2.8

$$\frac{1}{abc\,(ad-bc)\,(dfx^4+cfx^2+dex^2+ce)}\left(-x^3abcf^2\sqrt{-\frac{d}{c}}+x^3abdef\sqrt{-\frac{d}{c}}+EllipticF\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)a^2cf^2\sqrt{\frac{dx^2+c}{c}}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x)
```

```
[Out] (-x^3*a*b*c*f^2*(-d/c)^(1/2)+x^3*a*b*d*e*f*(-d/c)^(1/2)+EllipticF
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c*f^2*((d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)-2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b
```

```
*c*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticF(x*(-d/c)
^(1/2),(c*f/d/e)^(1/2))*a*b*d*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/
e)^(1/2)+EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c*e*f*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticE(x*(-d/c)^(1/2),(c*f
/d/e)^(1/2))*a*b*d*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-El
lipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*c*
f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticPi(x*(-d/c)
^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b*c*e*f*((d*x^2+c)/c)
^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/
e)^(1/2)/(-d/c)^(1/2))*b^2*c*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e
)^(1/2)-x*a*b*c*e*f*(-d/c)^(1/2)+x*a*b*d*e^2*(-d/c)^(1/2))*(d*x^2
+c)^(1/2)*(f*x^2+e)^(1/2)/b/a/(-d/c)^(1/2)/c/(a*d-b*c)/(d*f*x^4+c
*f*x^2+d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx^2+e\right)^{\frac{3}{2}}}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="maxima
```

```
[Out] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="fricas
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**2+e)**(3/2)/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx^2+e\right)^{\frac{3}{2}}}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

3.75 $\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=391

$$-\frac{\sqrt{e+fx^2}(bc(5de-cf)-2ad(cf+de))E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}\sqrt{d}\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2\sqrt{e+fx^2}(bc-ad)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{be^{3/2}\sqrt{c+dx^2}(be-af)\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x\sqrt{e+fx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}$$

```
[Out] -((d*e - c*f)*x*Sqrt[e + f*x^2])/(3*c*(b*c - a*d)*(c + d*x^2)^(3/
2)) - ((b*c*(5*d*e - c*f) - 2*a*d*(d*e + c*f))*Sqrt[e + f*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(3*c^(3/2)
*Sqrt[d]*(b*c - a*d)^2*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c
 + d*x^2))]) + (e^(3/2)*Sqrt[f]*Sqrt[c + d*x^2]*EllipticF[ArcTan[
(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*(b*c - a*d)*Sqrt[(
e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (b*e^(3/2)*(b*
e - a*f)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt
[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)^2*Sqrt[f]*Sqr
t[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 1.1467, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$-\frac{\sqrt{e+fx^2}(bc(5de-cf)-2ad(cf+de))E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}\sqrt{d}\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2\sqrt{e+fx^2}(bc-ad)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{be^{3/2}\sqrt{c+dx^2}(be-af)\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x\sqrt{e+fx^2}(de-cf)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -((d*e - c*f)*x*Sqrt[e + f*x^2])/(3*c*(b*c - a*d)*(c + d*x^2)^(3/
2)) - ((b*c*(5*d*e - c*f) - 2*a*d*(d*e + c*f))*Sqrt[e + f*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(3*c^(3/2)
*Sqrt[d]*(b*c - a*d)^2*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c
 + d*x^2))]) + (e^(3/2)*Sqrt[f]*Sqrt[c + d*x^2]*EllipticF[ArcTan[
(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*(b*c - a*d)*Sqrt[(
e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (b*e^(3/2)*(b*
e - a*f)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt
[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)^2*Sqrt[f]*Sqr
t[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi in Sympy [A] time = 148.299, size = 338, normalized size = 0.86

$$-\frac{x\sqrt{e+fx^2}\,(cf-de)}{3c\,(c+dx^2)^{\frac{3}{2}}\,(ad-bc)}-\frac{e^{\frac{3}{2}}\sqrt{f}\sqrt{c+dx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{3c^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}\,(ad-bc)}+\frac{\sqrt{e+fx^2}\left(2acdf+2ad^2e+bc^2f-5bcde\right)E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{3c^{\frac{3}{2}}\sqrt{d}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}\,(ad-bc)^2}-\frac{be^{\frac{3}{2}}\sqrt{c+dx^2}\,(af-be)\left(1-\frac{be}{af};\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}\,(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**2+e)**(3/2)/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] -x*sqrt(e + f*x**2)*(c*f - d*e)/(3*c*(c + d*x**2)**(3/2)*(a*d - b
*c)) - e**(3/2)*sqrt(f)*sqrt(c + d*x**2)*elliptic_f(atan(sqrt(f)*
x/sqrt(e)), 1 - d*e/(c*f))/(3*c**2*sqrt(e*(c + d*x**2)/(c*(e + f*
x**2)))*sqrt(e + f*x**2)*(a*d - b*c)) + sqrt(e + f*x**2)*(2*a*c*d
*f + 2*a*d**2*e + b*c**2*f - 5*b*c*d*e)*elliptic_e(atan(sqrt(d)*x
/sqrt(c)), -c*f/(d*e) + 1)/(3*c**(3/2)*sqrt(d)*sqrt(c*(e + f*x**2
)/(e*(c + d*x**2)))*sqrt(c + d*x**2)*(a*d - b*c)**2) - b*e**(3/2)
*sqrt(c + d*x**2)*(a*f - b*e)*elliptic_pi(1 - b*e/(a*f), atan(sqr
t(f)*x/sqrt(e)), 1 - d*e/(c*f))/(a*c*sqrt(f)*sqrt(e*(c + d*x**2)/
(c*(e + f*x**2)))*sqrt(e + f*x**2)*(a*d - b*c)**2)
```

Mathematica [C] time = 3.24876, size = 999, normalized size = 2.55

$$abc^3\left(\frac{d}{c}\right)^{3/2}f^2x^5+2a^2cd^2\sqrt{\frac{d}{c}}f^2x^5+2a^2d^3\sqrt{\frac{d}{c}}efx^5-5abcd^2\sqrt{\frac{d}{c}}efx^5+2a^2d^3\sqrt{\frac{d}{c}}e^2x^3-5abcd^2\sqrt{\frac{d}{c}}e^2x^3+a^2c^3\left(\frac{d}{c}\right)^{3/2}f^2$$

Antiderivative was successfully verified.

```
[In]  Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (3*a^2*c*d^2*Sqrt[d/c]*e^2*x - 6*a*b*c^3*(d/c)^(3/2)*e^2*x + 2*a*
b*c^3*Sqrt[d/c]*e*f*x + a^2*c^3*(d/c)^(3/2)*e*f*x - 5*a*b*c*d^2*S
qrt[d/c]*e^2*x^3 + 2*a^2*d^3*Sqrt[d/c]*e^2*x^3 + 5*a^2*c*d^2*Sqrt
[d/c]*e*f*x^3 - 5*a*b*c^3*(d/c)^(3/2)*e*f*x^3 + 2*a*b*c^3*Sqrt[d/
c]*f^2*x^3 + a^2*c^3*(d/c)^(3/2)*f^2*x^3 - 5*a*b*c*d^2*Sqrt[d/c]*
e*f*x^5 + 2*a^2*d^3*Sqrt[d/c]*e*f*x^5 + 2*a^2*c*d^2*Sqrt[d/c]*f^2
*x^5 + a*b*c^3*(d/c)^(3/2)*f^2*x^5 + I*a*e*(b*c*(-5*d*e + c*f) +
2*a*d*(d*e + c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^
2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*(-(d*e
) + c*f)*(5*b*c*e - 2*a*d*e - 3*a*c*f)*(c + d*x^2)*Sqrt[1 + (d*x^
2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)
/(d*e)] - (3*I)*b^2*c^3*e^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/
e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] +
 (6*I)*a*b*c^3*e*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Ellipt
icPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (3*I)*a^
2*c^3*f^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c
)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (3*I)*b^2*c^2*d*e
^2*x^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(
a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (6*I)*a*b*c^2*d*e*f*
x^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d
), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (3*I)*a^2*c^2*d*f^2*x^2
*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d),
I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(3*a*c^2*Sqrt[d/c]*(b*c - a
*d)^2*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])
```

Maple [B] time = 0.055, size = 1879, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x)
```

```
[Out] -1/3*(-3*x*a^2*c*d^2*e^2*(-d/c)^(1/2)-2*x^3*a^2*d^3*e^2*(-d/c)^(1
/2)-x^5*a*b*c^2*d*f^2*(-d/c)^(1/2)-5*x^3*a^2*c*d^2*e*f*(-d/c)^(1/
2)+5*x^3*a*b*c*d^2*e^2*(-d/c)^(1/2)-x*a^2*c^2*d*e*f*(-d/c)^(1/2)-
2*x*a*b*c^3*e*f*(-d/c)^(1/2)+6*x*a*b*c^2*d*e^2*(-d/c)^(1/2)-2*Ell
ipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^2*d^3*e^2*((d*x^2+c)
/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e
)^(1/2))*x^2*a^2*d^3*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-
2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c*d^2*e^2*((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticE(x*(-d/c)^(1/2),(c*f/
d/e)^(1/2))*a^2*c*d^2*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)
+3*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c^3*f^2*((d*x^2+
c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-3*EllipticPi(x*(-d/c)^(1/2),b*c/a
/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*c^3*f^2*((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)-3*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)
```

```
/(-d/c)^(1/2))*b^2*c^3*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2
)-2*x^5*a^2*c*d^2*f^2*(-d/c)^(1/2)-2*x^5*a^2*d^3*e*f*(-d/c)^(1/2)
-x^3*a^2*c^2*d*f^2*(-d/c)^(1/2)-2*x^3*a*b*c^3*f^2*(-d/c)^(1/2)+El
lipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c^3*e*f*((d*x^2+c)/c)
^(1/2)*((f*x^2+e)/e)^(1/2)+5*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(
1/2))*x^2*a*b*c*d^2*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2
*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^2*c*d^2*e*f*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-5*EllipticE(x*(-d/c)^(1/2),(c
*f/d/e)^(1/2))*x^2*a*b*c*d^2*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e
)^(1/2)-5*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*b*c^2*d
*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticE(x*(-d/c)^(
1/2),(c*f/d/e)^(1/2))*x^2*a*b*c^2*d*e*f*((d*x^2+c)/c)^(1/2)*((f*x
^2+e)/e)^(1/2)+6*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(
-d/c)^(1/2))*x^2*a*b*c^2*d*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^
(1/2)+5*x^5*a*b*c*d^2*e*f*(-d/c)^(1/2)+5*x^3*a*b*c^2*d*e*f*(-d/c)
^(1/2)+3*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^2*c^2*d*
f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-3*EllipticPi(x*(-d/c)
^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*x^2*a^2*c^2*d*f^2*((d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-3*EllipticPi(x*(-d/c)^(1/2),b*
c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*x^2*b^2*c^2*d*e^2*((d*x^2+c)/c)^
(1/2)*((f*x^2+e)/e)^(1/2)-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2
))*a^2*c^2*d*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-5*Ellipt
icF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c^3*e*f*((d*x^2+c)/c)^(1/
2)*((f*x^2+e)/e)^(1/2)+5*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2)
)*a*b*c^2*d*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*Ellipti
cE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c^2*d*e*f*((d*x^2+c)/c)^(1
/2)*((f*x^2+e)/e)^(1/2)-5*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2
))*a*b*c^2*d*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+6*Ellipt
icPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b*c^3*e*
f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticF(x*(-d/c)^(1/2
),(c*f/d/e)^(1/2))*x^2*a^2*c*d^2*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2))/(f*x^2+e)^(1/2)/(a*d-b*c)^2/c^2/(-d/c)^(1/2)/a/(d*x^
2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(5/2)),x, algorithm="maxima
```

```
[Out] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(5/2)),x, algorithm="fricas
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x**2+e)**(3/2)/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx^2+e\right)^{\frac{3}{2}}}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(5/2)),x, algorithm="giac")
```

```
[Out] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

3.76 $\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx$

Optimal. Leaf size=639

$$\frac{b^2e^{3/2}\sqrt{c+dx^2}(be-af)\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(3bc(3de-2cf)-ad(4de-cf))F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15c^3\sqrt{e+fx^2}(bc-ad)^2(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{x\sqrt{e+fx^2}(3bc(3de-cf)-2ad(cf+2de))}{15c^2\left(c+dx^2\right)^{3/2}(bc-ad)^2}+\frac{\sqrt{e+fx^2}\left(ad\left(-2c^2f^2-3cdef+8d^2e^2\right)-3bc\left(c^2f^2-6cdef+6d^2e^2\right)\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}\sqrt{d}\sqrt{c+dx^2}(bc-ad)^2(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{x\sqrt{e+fx^2}(de-cf)}{5c\left(c+dx^2\right)^{5/2}(bc-ad)}-\frac{b\sqrt{d}\sqrt{e+fx^2}(be-af)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)^3\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

```
[Out] -((d*e - c*f)*x*Sqrt[e + f*x^2])/(5*c*(b*c - a*d)*(c + d*x^2)^(5/
2)) - ((3*b*c*(3*d*e - c*f) - 2*a*d*(2*d*e + c*f))*x*Sqrt[e + f*x
^2])/(15*c^2*(b*c - a*d)^2*(c + d*x^2)^(3/2)) - (b*Sqrt[d]*(b*e -
 a*f)*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(c*f)/(d*e)])/(Sqrt[c]*(b*c - a*d)^3*Sqrt[c + d*x^2]*Sqrt[(c*(e +
 f*x^2))/(e*(c + d*x^2))]) + ((a*d*(8*d^2*e^2 - 3*c*d*e*f - 2*c^2
*f^2) - 3*b*c*(6*d^2*e^2 - 6*c*d*e*f + c^2*f^2))*Sqrt[e + f*x^2]*
EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(15*c^(5
/2)*Sqrt[d]*(b*c - a*d)^2*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e
+ f*x^2))/(e*(c + d*x^2))]) + (e^(3/2)*Sqrt[f]*(3*b*c*(3*d*e - 2*
c*f) - a*d*(4*d*e - c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[
f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*c^3*(b*c - a*d)^2*(d*e - c*
f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (b^2*
e^(3/2)*(b*e - a*f)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), A
rcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)^3*
Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 2.15564, antiderivative size = 639, normalized size of antiderivative = 1., number of

steps used = 9, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{b^2e^{3/2}\sqrt{c+dx^2}(be-af)\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(3bc(3de-2cf)-ad(4de-cf))F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15c^3\sqrt{e+fx^2}(bc-ad)^2(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{x\sqrt{e+fx^2}(3bc(3de-cf)-2ad(cf+2de))}{15c^2\left(c+dx^2\right)^{3/2}(bc-ad)^2}$$
$$+\frac{\sqrt{e+fx^2}\left(ad\left(-2c^2f^2-3cdef+8d^2e^2\right)-3bc\left(c^2f^2-6cdef+6d^2e^2\right)\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}\sqrt{d}\sqrt{c+dx^2}(bc-ad)^2(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$-\frac{x\sqrt{e+fx^2}(de-cf)}{5c\left(c+dx^2\right)^{5/2}(bc-ad)}-\frac{b\sqrt{d}\sqrt{e+fx^2}(be-af)E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)^3\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(7/2)),x]
```

```
[Out] -((d*e - c*f)*x*Sqrt[e + f*x^2])/(5*c*(b*c - a*d)*(c + d*x^2)^(5/
2)) - ((3*b*c*(3*d*e - c*f) - 2*a*d*(2*d*e + c*f))*x*Sqrt[e + f*x
^2])/(15*c^2*(b*c - a*d)^2*(c + d*x^2)^(3/2)) - (b*Sqrt[d]*(b*e -
 a*f)*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(c*f)/(d*e)])/(Sqrt[c]*(b*c - a*d)^3*Sqrt[c + d*x^2]*Sqrt[(c*(e +
 f*x^2))/(e*(c + d*x^2))]) + ((a*d*(8*d^2*e^2 - 3*c*d*e*f - 2*c^2
*f^2) - 3*b*c*(6*d^2*e^2 - 6*c*d*e*f + c^2*f^2))*Sqrt[e + f*x^2]*
EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(15*c^(5
/2)*Sqrt[d]*(b*c - a*d)^2*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e
+ f*x^2))/(e*(c + d*x^2))]) + (e^(3/2)*Sqrt[f]*(3*b*c*(3*d*e - 2*
c*f) - a*d*(4*d*e - c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[
f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*c^3*(b*c - a*d)^2*(d*e - c*
f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (b^2*
e^(3/2)*(b*e - a*f)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), A
rcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)^3*
Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((f*x**2+e)**(3/2)/(b*x**2+a)/(d*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 5.03297, size = 570, normalized size = 0.89

$$-ax\sqrt{\frac{d}{c}}\left(e+fx^2\right)\left(\left(c+dx^2\right)^2\left(a^2d^2\left(-2c^2f^2-3cdef+8d^2e^2\right)+2abcd\left(7c^2f^2+3cdef-13d^2e^2\right)+3b^2c^2\left(c^2f^2-11cdef\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(7/2)),x]
```

```
[Out] (-(a*Sqrt[d/c]*x*(e + f*x^2)*(3*c^2*(b*c - a*d)^2*(d*e - c*f)^2 +
 c*(b*c - a*d)*(-(d*e) + c*f)*(3*b*c*(-3*d*e + c*f) + 2*a*d*(2*d*
e + c*f))*(c + d*x^2) + (a^2*d^2*(8*d^2*e^2 - 3*c*d*e*f - 2*c^2*f
^2) + 3*b^2*c^2*(11*d^2*e^2 - 11*c*d*e*f + c^2*f^2) + 2*a*b*c*d*(
-13*d^2*e^2 + 3*c*d*e*f + 7*c^2*f^2))*(c + d*x^2)^2)) + I*(c + d*
x^2)^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(a*e*(-3*b^2*c^2*(
11*d^2*e^2 - 11*c*d*e*f + c^2*f^2) + a^2*d^2*(-8*d^2*e^2 + 3*c*d*
e*f + 2*c^2*f^2) - 2*a*b*c*d*(-13*d^2*e^2 + 3*c*d*e*f + 7*c^2*f^2
))*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (d*e - c*f)*(
a*(3*b^2*c^2*e*(11*d*e - 8*c*f) + a^2*d^2*e*(8*d*e + c*f) + a*b*c
*(-26*d^2*e^2 - 7*c*d*e*f + 15*c^2*f^2))*EllipticF[I*ArcSinh[Sqrt
[d/c]*x], (c*f)/(d*e)] - 15*b*c^3*(b*e - a*f)^2*EllipticPi[(b*c)/
(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])))/(15*a*c^3*Sqrt[d/c
]*(b*c - a*d)^3*(d*e - c*f)*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2])
```

Maple [B] time = 0.11, size = 6211, normalized size = 9.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx^2+e\right)^{\frac{3}{2}}}{(bx^2+a)(dx^2+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(7/2)),x, algorithm="maxima
```

```
[Out] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(7/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(7/2)),x, algorithm="fricas
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e)**(3/2)/(b*x**2+a)/(d*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx^2 + e\right)^{\frac{3}{2}}}{(bx^2 + a)(dx^2 + c)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(7/2)),x, algorithm="giac")
```

```
[Out] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(7/2)), x)
```

3.77 $\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=621

$$\frac{c^{3/2}\sqrt{e+fx^2}(bc-ad)^2\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab^2\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{dx\sqrt{c+dx^2}(bc-ad)}{b^2\sqrt{e+fx^2}}+\frac{d\sqrt{e}\sqrt{c+dx^2}(bc-ad)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b^2\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{d\sqrt{e}\sqrt{c+dx^2}(bc-ad)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b^2\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{d^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3bf}-\frac{d\sqrt{e}\sqrt{c+dx^2}(de-3cf)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3bf^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{2d\sqrt{e}\sqrt{c+dx^2}(de-2cf)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3bf^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{2dx\sqrt{c+dx^2}(de-2cf)}{3bf\sqrt{e+fx^2}}$$

```
[Out] (d*(b*c - a*d)*x*Sqrt[c + d*x^2])/(b^2*Sqrt[e + f*x^2]) - (2*d*(d
*e - 2*c*f)*x*Sqrt[c + d*x^2])/(3*b*f*Sqrt[e + f*x^2]) + (d^2*x*S
qrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*b*f) - (d*(b*c - a*d)*Sqrt[e]*
Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/
(c*f)])/(b^2*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
 + f*x^2]) + (2*d*Sqrt[e]*(d*e - 2*c*f)*Sqrt[c + d*x^2]*EllipticE
[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*b*f^(3/2)*Sqrt
[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d*(b*c - a*
d)*Sqrt[e]*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]],
 1 - (d*e)/(c*f)])/(b^2*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^
2))]*Sqrt[e + f*x^2]) - (d*Sqrt[e]*(d*e - 3*c*f)*Sqrt[c + d*x^2]*
EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*b*f^(
3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (c^
(3/2)*(b*c - a*d)^2*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), A
rcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(a*b^2*Sqrt[d]*e*Sq
rt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

Rubi [A] time = 1.43776, antiderivative size = 621, normalized size of antiderivative = 1., number of

steps used = 12, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{c^{3/2}\sqrt{e+fx^2}(bc-ad)^2\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab^2\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{dx\sqrt{c+dx^2}(bc-ad)}{b^2\sqrt{e+fx^2}}+\frac{d\sqrt{e}\sqrt{c+dx^2}(bc-ad)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b^2\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{d\sqrt{e}\sqrt{c+dx^2}(bc-ad)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b^2\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{d^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3bf}-\frac{d\sqrt{e}\sqrt{c+dx^2}(de-3cf)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3bf^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{2d\sqrt{e}\sqrt{c+dx^2}(de-2cf)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3bf^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{2dx\sqrt{c+dx^2}(de-2cf)}{3bf\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^(5/2)/((a + b*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (d*(b*c - a*d)*x*Sqrt[c + d*x^2])/(b^2*Sqrt[e + f*x^2]) - (2*d*(d
*e - 2*c*f)*x*Sqrt[c + d*x^2])/(3*b*f*Sqrt[e + f*x^2]) + (d^2*x*S
qrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*b*f) - (d*(b*c - a*d)*Sqrt[e]*
Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/
(c*f)])/(b^2*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
 + f*x^2]) + (2*d*Sqrt[e]*(d*e - 2*c*f)*Sqrt[c + d*x^2]*EllipticE
[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*b*f^(3/2)*Sqrt
[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d*(b*c - a*
d)*Sqrt[e]*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]],
 1 - (d*e)/(c*f)])/(b^2*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^
2))]*Sqrt[e + f*x^2]) - (d*Sqrt[e]*(d*e - 3*c*f)*Sqrt[c + d*x^2]*
EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*b*f^(
3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (c^
(3/2)*(b*c - a*d)^2*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), A
rcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(a*b^2*Sqrt[d]*e*Sq
rt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

Rubi in Sympy [A] time = 153.277, size = 552, normalized size = 0.89

$$\frac{c^{\frac{3}{2}}\sqrt{d}\sqrt{e+fx^2}\,(3cf-de)\,F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{3bef\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}} - \frac{2\sqrt{c}d^{\frac{3}{2}}\sqrt{e+fx^2}\,(2cf-de)\,E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{3bf^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}} + \frac{d^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3bf} + \frac{2d^2x\sqrt{e+fx^2}\,(2cf-de)}{3bf^2\sqrt{c+dx^2}} - \frac{c^{\frac{3}{2}}\sqrt{d}\sqrt{e+fx^2}\,(ad-bc)\,F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{b^2e\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}} + \frac{\sqrt{c}d^{\frac{3}{2}}\sqrt{e+fx^2}\,(ad-bc)\,E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{b^2f\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}} - \frac{d^2x\sqrt{e+fx^2}\,(ad-bc)}{b^2f\sqrt{c+dx^2}} + \frac{c^{\frac{3}{2}}\sqrt{e+fx^2}\,(ad-bc)^2\left(1-\frac{bc}{ad};\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{ab^2\sqrt{d}e\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(5/2)/(b*x**2+a)/(f*x**2+e)**(1/2),x)
```

```
[Out] c**(3/2)*sqrt(d)*sqrt(e + f*x**2)*(3*c*f - d*e)*elliptic_f(atan(s
qrt(d)*x/sqrt(c)), -c*f/(d*e) + 1)/(3*b*e*f*sqrt(c*(e + f*x**2)/(
e*(c + d*x**2)))*sqrt(c + d*x**2)) - 2*sqrt(c)*d**(3/2)*sqrt(e +
f*x**2)*(2*c*f - d*e)*elliptic_e(atan(sqrt(d)*x/sqrt(c)), -c*f/(d
*e) + 1)/(3*b*f**2*sqrt(c*(e + f*x**2)/(e*(c + d*x**2)))*sqrt(c +
 d*x**2)) + d**2*x*sqrt(c + d*x**2)*sqrt(e + f*x**2)/(3*b*f) + 2*
d**2*x*sqrt(e + f*x**2)*(2*c*f - d*e)/(3*b*f**2*sqrt(c + d*x**2))
 - c**(3/2)*sqrt(d)*sqrt(e + f*x**2)*(a*d - b*c)*elliptic_f(atan(
sqrt(d)*x/sqrt(c)), -c*f/(d*e) + 1)/(b**2*e*sqrt(c*(e + f*x**2)/(
e*(c + d*x**2)))*sqrt(c + d*x**2)) + sqrt(c)*d**(3/2)*sqrt(e + f*
x**2)*(a*d - b*c)*elliptic_e(atan(sqrt(d)*x/sqrt(c)), -c*f/(d*e)
+ 1)/(b**2*f*sqrt(c*(e + f*x**2)/(e*(c + d*x**2)))*sqrt(c + d*x**
2)) - d**2*x*sqrt(e + f*x**2)*(a*d - b*c)/(b**2*f*sqrt(c + d*x**2
)) + c**(3/2)*sqrt(e + f*x**2)*(a*d - b*c)**2*elliptic_pi(1 - b*c
/(a*d), atan(sqrt(d)*x/sqrt(c)), -c*f/(d*e) + 1)/(a*b**2*sqrt(d)*
e*sqrt(c*(e + f*x**2)/(e*(c + d*x**2)))*sqrt(c + d*x**2))
```

Mathematica [C] time = 2.2309, size = 350, normalized size = 0.56

$$-iad\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(3a^2d^2f^2+3abdf(de-3cf)+b^2\left(9c^2f^2-8cdef+2d^2e^2\right)\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)\middle|\frac{cf}{de}\right)+f\left(ab^2cdx\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^(5/2)/((a + b*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] ((-I)*a*b*d^2*e*(-2*b*d*e + 7*b*c*f - 3*a*d*f)*Sqrt[1 + (d*x^2)/c
]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*
e)] - I*a*d*(3*a^2*d^2*f^2 + 3*a*b*d*f*(d*e - 3*c*f) + b^2*(2*d^2
*e^2 - 8*c*d*e*f + 9*c^2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^
2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + f*(a*b^2*c
*d*(d/c)^(3/2)*x*(c + d*x^2)*(e + f*x^2) - (3*I)*(b*c - a*d)^3*f*
Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I
*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(3*a*b^3*Sqrt[d/c]*f^2*Sqrt
[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.036, size = 988, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x)
```

```
[Out] 1/3*((-d/c)^(1/2)*x^5*a*b^2*d^3*f^2+(-d/c)^(1/2)*x^3*a*b^2*c*d^2*
f^2+(-d/c)^(1/2)*x^3*a*b^2*d^3*e*f+3*((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^3*d^3*f^2
-9*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/
2),(c*f/d/e)^(1/2))*a^2*b*c*d^2*f^2+3*((d*x^2+c)/c)^(1/2)*((f*x^2
+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*d^3*
e*f+9*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^
(1/2),(c*f/d/e)^(1/2))*a*b^2*c^2*d*f^2-8*((d*x^2+c)/c)^(1/2)*((f*
x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c
*d^2*e*f+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-
d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*d^3*e^2-3*((d*x^2+c)/c)^(1/2)*(
(f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*
b*d^3*e*f+7*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(
-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c*d^2*e*f-2*((d*x^2+c)/c)^(1/2
)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a
*b^2*d^3*e^2-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi
(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^3*d^3*f^2+9*
((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2)
,b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*b*c*d^2*f^2-9*((d*x^2+c)/
c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-
f/e)^(1/2)/(-d/c)^(1/2))*a*b^2*c^2*d*f^2+3*((d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/
(-d/c)^(1/2))*b^3*c^3*f^2+(-d/c)^(1/2)*x*a*b^2*c*d^2*e*f)*(f*x^2+
e)^(1/2)*(d*x^2+c)^(1/2)/a/(-d/c)^(1/2)/f^2/b^3/(d*f*x^4+c*f*x^2+
d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}}}{(bx^2+a)\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*sqrt(f*x^2 + e)),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*sqrt(f*x^2 + e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(5/2)/(b*x**2+a)/(f*x**2+e)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{5}{2}}}{(bx^2+a)\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*sqrt(f*x^2 + e)),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

3.78 $\int \frac{\left(c+dx^2\right)^{3/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=319

$$\frac{c^{3/2}\sqrt{e+fx^2}(bc-ad)\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{dx\sqrt{c+dx^2}}{b\sqrt{e+fx^2}}+\frac{d\sqrt{e}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{d\sqrt{e}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

```
[Out] (d*x*Sqrt[c + d*x^2])/(b*Sqrt[e + f*x^2]) - (d*Sqrt[e]*Sqrt[c + d
*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(b
*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) +
 (d*Sqrt[e]*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]]
, 1 - (d*e)/(c*f)])/(b*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2
))]*Sqrt[e + f*x^2]) + (c^(3/2)*(b*c - a*d)*Sqrt[e + f*x^2]*Ellip
ticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*
e)])/(a*b*Sqrt[d]*e*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c +
d*x^2))])
```

Rubi [A] time = 0.615722, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$\frac{c^{3/2}\sqrt{e+fx^2}(bc-ad)\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{dx\sqrt{c+dx^2}}{b\sqrt{e+fx^2}}+\frac{d\sqrt{e}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{d\sqrt{e}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^(3/2)/((a + b*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (d*x*Sqrt[c + d*x^2])/(b*Sqrt[e + f*x^2]) - (d*Sqrt[e]*Sqrt[c + d
*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(b
*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) +
 (d*Sqrt[e]*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]]
, 1 - (d*e)/(c*f)])/(b*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2
))]*Sqrt[e + f*x^2]) + (c^(3/2)*(b*c - a*d)*Sqrt[e + f*x^2]*Ellip
ticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*
e)])/(a*b*Sqrt[d]*e*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c +
d*x^2))])
```

Rubi in Sympy [A] time = 80.4529, size = 272, normalized size = 0.85

$$\frac{c^{\frac{3}{2}}\sqrt{d}\sqrt{e+fx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{be\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}}-\frac{\sqrt{c}d^{\frac{3}{2}}\sqrt{e+fx^2}E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{bf\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}}+\frac{d^2x\sqrt{e+fx^2}}{bf\sqrt{c+dx^2}}-\frac{c^{\frac{3}{2}}\sqrt{e+fx^2}\,(ad-bc)\left(1-\frac{bc}{ad};\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{ab\sqrt{d}e\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(3/2)/(b*x**2+a)/(f*x**2+e)**(1/2),x)
```

```
[Out] c**(3/2)*sqrt(d)*sqrt(e + f*x**2)*elliptic_f(atan(sqrt(d)*x/sqrt(
c)), -c*f/(d*e) + 1)/(b*e*sqrt(c*(e + f*x**2)/(e*(c + d*x**2)))*s
qrt(c + d*x**2)) - sqrt(c)*d**(3/2)*sqrt(e + f*x**2)*elliptic_e(a
tan(sqrt(d)*x/sqrt(c)), -c*f/(d*e) + 1)/(b*f*sqrt(c*(e + f*x**2)/
(e*(c + d*x**2)))*sqrt(c + d*x**2)) + d**2*x*sqrt(e + f*x**2)/(b*
f*sqrt(c + d*x**2)) - c**(3/2)*sqrt(e + f*x**2)*(a*d - b*c)*ellip
tic_pi(1 - b*c/(a*d), atan(sqrt(d)*x/sqrt(c)), -c*f/(d*e) + 1)/(a
*b*sqrt(d)*e*sqrt(c*(e + f*x**2)/(e*(c + d*x**2)))*sqrt(c + d*x**
2))
```

Mathematica [C] time = 0.36431, size = 197, normalized size = 0.62

$$-\frac{i\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(abd^2eE\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-ad(adf-2bcf+bde)F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+f(bc-ad)^2\left(\frac{bc}{ad};\right.\right.}{ab^2f\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^(3/2)/((a + b*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(a*b*d^2*e*Elliptic
E[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - a*d*(b*d*e - 2*b*c*f + a
*d*f)*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (b*c - a*d
)^2*f*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)
]))/(a*b^2*Sqrt[d/c]*f*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.031, size = 341, normalized size = 1.1

$$\frac{1}{afb^2\,(dfx^4+cfx^2+dex^2+ce)}\left(-EllipticF\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)a^2d^2f+2\,EllipticF\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)abcdf-EllipticF\left(x\sqrt{-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x)
```

```
[Out] (-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*d^2*f+2*EllipticF
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c*d*f-EllipticF(x*(-d/c)^(1/
2),(c*f/d/e)^(1/2))*a*b*d^2*e+EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^
(1/2))*a*b*d^2*e+EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(
-d/c)^(1/2))*a^2*d^2*f-2*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)
^(1/2)/(-d/c)^(1/2))*a*b*c*d*f+EllipticPi(x*(-d/c)^(1/2),b*c/a/d,
(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c^2*f)*((f*x^2+e)/e)^(1/2)*((d*x^2
+c)/c)^(1/2)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/f/a/(-d/c)^(1/2)/b^2
/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(dx^2+c\right)^{\frac{3}{2}}}{(bx^2+a)\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*sqrt(f*x^2 + e)),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*sqrt(f*x^2 + e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^2\right)^{\frac{3}{2}}}{\left(a+bx^2\right)\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(3/2)/(b*x**2+a)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/((a + b*x**2)*sqrt(e + f*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{\left(bx^2+a\right)\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*sqrt(f*x^2 + e)),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

3.79 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=102

$$\frac{c^{3/2}\sqrt{e+fx^2}\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

```
[Out] (c^(3/2)*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt
[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(a*Sqrt[d]*e*Sqrt[c + d*x^2]*S
qrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

Rubi [A] time = 0.136921, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031

$$\frac{c^{3/2}\sqrt{e+fx^2}\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c + d*x^2]/((a + b*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (c^(3/2)*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt
[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(a*Sqrt[d]*e*Sqrt[c + d*x^2]*S
qrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

Rubi in Sympy [A] time = 19.4795, size = 82, normalized size = 0.8

$$\frac{c^{\frac{3}{2}}\sqrt{e+fx^2}\left(1-\frac{bc}{ad};\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{a\sqrt{d}e\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(1/2)/(b*x**2+a)/(f*x**2+e)**(1/2),x)
```

```
[Out] c**(3/2)*sqrt(e + f*x**2)*elliptic_pi(1 - b*c/(a*d), atan(sqrt(d)
*x/sqrt(c)), -c*f/(d*e) + 1)/(a*sqrt(d)*e*sqrt(c*(e + f*x**2)/(e*
(c + d*x**2)))*sqrt(c + d*x**2))
```

Mathematica [C] time = 0.182323, size = 143, normalized size = 1.4

$$-\frac{i\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left((bc-ad)\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+adF\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)\right)}{ab\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[c + d*x^2]/((a + b*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(a*d*EllipticF[I*Ar
cSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (b*c - a*d)*EllipticPi[(b*c)/(
a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(a*b*Sqrt[d/c]*Sqrt[
c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.029, size = 191, normalized size = 1.9

$$\frac{1}{ab\left(dfx^4+cfx^2+dex^2+ce\right)}\left(\mathit{EllipticF}\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)ad-\mathit{EllipticPi}\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},1\sqrt{-\frac{f}{e}}\frac{1}{\sqrt{-\frac{d}{c}}}\right)ad+\mathit{EllipticPi}\left(x\sqrt{-\frac{d}{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x)
```

```
[Out] (EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d-EllipticPi(x*(-d/c
)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*d+EllipticPi(x*(-d/c
)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b*c)/b*((f*x^2+e)/e)^(
1/2)*((d*x^2+c)/c)^(1/2)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/a/(-d/c)
^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{(bx^2+a)\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*sqrt(f*x^2 + e)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*sqrt(f*x^2 + e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/((a + b*x**2)*sqrt(e + f*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{(bx^2+a)\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*sqrt(f*x^2 + e)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

3.80 $\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=100

$$\frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

```
[Out] (Sqrt[-c]*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c
)/(a*d), ArcSin[(Sqrt[d]*x)/Sqrt[-c]], (c*f)/(d*e)])/(a*Sqrt[d]*S
qrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.479151, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (Sqrt[-c]*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c
)/(a*d), ArcSin[(Sqrt[d]*x)/Sqrt[-c]], (c*f)/(d*e)])/(a*Sqrt[d]*S
qrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Rubi in Sympy [A] time = 50.4011, size = 173, normalized size = 1.73

$$\frac{\sqrt{c}f\sqrt{e+fx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{\sqrt{d}e\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}\,(af-be)}-\frac{be^{\frac{3}{2}}\sqrt{c+dx^2}\left(1-\frac{be}{af};\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}\,(af-be)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] sqrt(c)*f*sqrt(e + f*x**2)*elliptic_f(atan(sqrt(d)*x/sqrt(c)), -c
*f/(d*e) + 1)/(sqrt(d)*e*sqrt(c*(e + f*x**2)/(e*(c + d*x**2)))*sq
rt(c + d*x**2)*(a*f - b*e)) - b*e**(3/2)*sqrt(c + d*x**2)*ellipti
c_pi(1 - b*e/(a*f), atan(sqrt(f)*x/sqrt(e)), 1 - d*e/(c*f))/(a*c*
sqrt(f)*sqrt(e*(c + d*x**2)/(c*(e + f*x**2)))*sqrt(e + f*x**2)*(a
```

```
*f - b*e))
```

Mathematica [C] time = 0.150235, size = 101, normalized size = 1.01

$$-\frac{i\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)}{a\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a
*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(a*Sqrt[d/c]*Sqrt[c +
d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.03, size = 118, normalized size = 1.2

$$\frac{1}{a(dfx^4+cfx^2+dex^2+ce)}\mathit{EllipticPi}\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},1\sqrt{-\frac{f}{e}}\frac{1}{\sqrt{-\frac{d}{c}}}\right)\sqrt{\frac{fx^2+e}{e}}\sqrt{\frac{dx^2+c}{c}}\sqrt{fx^2+e}\sqrt{dx^2+c}\frac{1}{\sqrt{-\frac{d}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*((f*
x^2+e)/e)^(1/2)*((d*x^2+c)/c)^(1/2)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/
2)/a/(-d/c)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.81 $\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=344

$$\frac{b^2c^{3/2}\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{d^{3/2}\sqrt{e+fx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{d\sqrt{e}\sqrt{c+dx^2}(adf-2bcf+bde)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}(bc-ad)^2(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

```
[Out] -((d^(3/2)*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
 1 - (c*f)/(d*e)])/(Sqrt[c]*(b*c - a*d)*(d*e - c*f)*Sqrt[c + d*x^
2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])) - (d*Sqrt[e]*(b*d*e -
2*b*c*f + a*d*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqr
t[e]], 1 - (d*e)/(c*f)])/(c*(b*c - a*d)^2*Sqrt[f]*(d*e - c*f)*Sqr
t[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (b^2*c^(3/2
)*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/
Sqrt[c]], 1 - (c*f)/(d*e)])/(a*Sqrt[d]*(b*c - a*d)^2*e*Sqrt[c + d
*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

Rubi [A] time = 0.70765, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$

$$\frac{b^2c^{3/2}\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{d^{3/2}\sqrt{e+fx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{d\sqrt{e}\sqrt{c+dx^2}(adf-2bcf+bde)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}(bc-ad)^2(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```


```
[Out] -((d^(3/2)*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
 1 - (c*f)/(d*e)])/(Sqrt[c]*(b*c - a*d)*(d*e - c*f)*Sqrt[c + d*x^
2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])) - (d*Sqrt[e]*(b*d*e -
2*b*c*f + a*d*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqr
t[e]], 1 - (d*e)/(c*f)])/(c*(b*c - a*d)^2*Sqrt[f]*(d*e - c*f)*Sqr
t[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (b^2*c^(3/2
)*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/
Sqrt[c]], 1 - (c*f)/(d*e)])/(a*Sqrt[d]*(b*c - a*d)^2*e*Sqrt[c + d
*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

Rubi in Sympy [A] time = 94.8511, size = 289, normalized size = 0.84

$$\frac{d\sqrt{e}\sqrt{c+dx^2}\,(adf-2bcf+bde)\,F\left(\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}\,(ad-bc)^2\,(cf-de)} - \frac{d^{\frac{3}{2}}\sqrt{e+fx^2}E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{\sqrt{c}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}\,(ad-bc)\,(cf-de)} + \frac{b^2c^{\frac{3}{2}}\sqrt{e+fx^2}\left(1-\frac{bc}{ad};\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{a\sqrt{d}e\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}\,(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)/(d*x**2+c)**(3/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] d*sqrt(e)*sqrt(c + d*x**2)*(a*d*f - 2*b*c*f + b*d*e)*elliptic_f(a
tan(sqrt(f)*x/sqrt(e)), 1 - d*e/(c*f))/(c*sqrt(f)*sqrt(e*(c + d*x
**2)/(c*(e + f*x**2)))*sqrt(e + f*x**2)*(a*d - b*c)**2*(c*f - d*e
)) - d**(3/2)*sqrt(e + f*x**2)*elliptic_e(atan(sqrt(d)*x/sqrt(c))
, -c*f/(d*e) + 1)/(sqrt(c)*sqrt(c*(e + f*x**2)/(e*(c + d*x**2)))*
sqrt(c + d*x**2)*(a*d - b*c)*(c*f - d*e)) + b**2*c**(3/2)*sqrt(e
+ f*x**2)*elliptic_pi(1 - b*c/(a*d), atan(sqrt(d)*x/sqrt(c)), -c*
f/(d*e) + 1)/(a*sqrt(d)*e*sqrt(c*(e + f*x**2)/(e*(c + d*x**2)))*s
qrt(c + d*x**2)*(a*d - b*c)**2)
```

Mathematica [C] time = 1.21527, size = 365, normalized size = 1.06

$$\frac{\sqrt{\frac{d}{c}}\left(-ibc^2f\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\,\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)\middle|\frac{cf}{de}\right)+ibcde\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\,\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)\middle|\frac{cf}{de}\right)+iad^2e\sqrt{\cdots}\right.}{ad\sqrt{c+dx^2}\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (Sqrt[d/c]*(a*c*d*(d/c)^(3/2)*e*x + a*c*d*(d/c)^(3/2)*f*x^3 + I*a
*d^2*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSin
h[Sqrt[d/c]*x], (c*f)/(d*e)] + I*a*d*(-(d*e) + c*f)*Sqrt[1 + (d*x
^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f
)/(d*e)] + I*b*c*d*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Elli
pticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*b*c^
2*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d
), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(a*d*(-(b*c) + a*d)*(d*
e - c*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.046, size = 413, normalized size = 1.2

$$\frac{1}{(ad-bc)\,ac\,(cf-de)\,(dfx^4+cfx^2+dex^2+ce)}\left(-x^3ad^2f\sqrt{-\frac{d}{c}}+EllipticF\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)acdf\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{fx^2+e}{e}}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] (-x^3*a*d^2*f*(-d/c)^(1/2)+EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/
2))*a*c*d*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticF(x*(
-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^2*e*((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)+EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^2*e*((d
*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticPi(x*(-d/c)^(1/2),b*
c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b*c^2*f*((d*x^2+c)/c)^(1/2)*((f*
x^2+e)/e)^(1/2)+EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-
d/c)^(1/2))*b*c*d*e*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-x*a*d
^2*e*(-d/c)^(1/2))*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/c/a/(a*d-b*c)/
(-d/c)^(1/2)/(c*f-d*e)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)),x, algorithm="maxima
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)),x, algorithm="fricas
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{\frac{3}{2}}\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)/(d*x**2+c)**(3/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*(c + d*x**2)**(3/2)*sqrt(e + f*x**2)), x
)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

3.82 $\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=435

$$\frac{b^2\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\,\Pi\left(\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}(bc-ad)^2}-\frac{d^{3/2}\sqrt{e+fx^2}(bc(5de-7cf)-2ad(de-2cf))E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}\sqrt{c+dx^2}(bc-ad)^2(de-cf)^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{d\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(ad(de-3cf)-2bc(2de-3cf))F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2\sqrt{e+fx^2}(bc-ad)^2(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{d^2x\sqrt{e+fx^2}}{3c\left(c+dx^2\right)^{3/2}(bc-ad)(de-cf)}$$

```
[Out] -(d^2*x*Sqrt[e + f*x^2])/(3*c*(b*c - a*d)*(d*e - c*f)*(c + d*x^2)
^(3/2)) - (d^(3/2)*(b*c*(5*d*e - 7*c*f) - 2*a*d*(d*e - 2*c*f))*Sq
rt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d
*e)])/(3*c^(3/2)*(b*c - a*d)^2*(d*e - c*f)^2*Sqrt[c + d*x^2]*Sqrt
[(c*(e + f*x^2))/(e*(c + d*x^2))]) - (d*Sqrt[e]*Sqrt[f]*(a*d*(d*e
 - 3*c*f) - 2*b*c*(2*d*e - 3*c*f))*Sqrt[c + d*x^2]*EllipticF[ArcT
an[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*(b*c - a*d)^2*(
d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2
]) + (b^2*Sqrt[-c]*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Ellipt
icPi[(b*c)/(a*d), ArcSin[(Sqrt[d]*x)/Sqrt[-c]], (c*f)/(d*e)])/(a*
Sqrt[d]*(b*c - a*d)^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Rubi [A] time = 1.69787, antiderivative size = 435, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$

$$\frac{b^2\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\,\Pi\left(\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}(bc-ad)^2}-\frac{d^{3/2}\sqrt{e+fx^2}(bc(5de-7cf)-2ad(de-2cf))E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}\sqrt{c+dx^2}(bc-ad)^2(de-cf)^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{d\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(ad(de-3cf)-2bc(2de-3cf))F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2\sqrt{e+fx^2}(bc-ad)^2(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{d^2x\sqrt{e+fx^2}}{3c\left(c+dx^2\right)^{3/2}(bc-ad)(de-cf)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] -(d^2*x*Sqrt[e + f*x^2])/(3*c*(b*c - a*d)*(d*e - c*f)*(c + d*x^2)
^(3/2)) - (d^(3/2)*(b*c*(5*d*e - 7*c*f) - 2*a*d*(d*e - 2*c*f))*Sq
rt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d
*e)])/(3*c^(3/2)*(b*c - a*d)^2*(d*e - c*f)^2*Sqrt[c + d*x^2]*Sqrt
[(c*(e + f*x^2))/(e*(c + d*x^2))]) - (d*Sqrt[e]*Sqrt[f]*(a*d*(d*e
 - 3*c*f) - 2*b*c*(2*d*e - 3*c*f))*Sqrt[c + d*x^2]*EllipticF[ArcT
an[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*(b*c - a*d)^2*(
d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2
]) + (b^2*Sqrt[-c]*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Ellipt
icPi[(b*c)/(a*d), ArcSin[(Sqrt[d]*x)/Sqrt[-c]], (c*f)/(d*e)])/(a*
Sqrt[d]*(b*c - a*d)^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)/(d*x**2+c)**(5/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 6.34525, size = 433, normalized size = 1.

$$-3ib^2c^2\left(c+dx^2\right)\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(de-cf)^2\,\Pi\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+acdx\left(\frac{d}{c}\right)^{3/2}\left(e+fx^2\right)\left(ad\left(-5c^2f+cd\left(3e-\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (a*c*d*(d/c)^(3/2)*x*(e + f*x^2)*(b*c*(-6*c*d*e + 8*c^2*f - 5*d^2
*e*x^2 + 7*c*d*f*x^2) + a*d*(-5*c^2*f + 2*d^2*e*x^2 + c*d*(3*e -
4*f*x^2))) + I*a*d^2*e*(2*a*d*(d*e - 2*c*f) + b*c*(-5*d*e + 7*c*f
))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[
I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*a*d*(-(d*e) + c*f)*(a*d*
(2*d*e - 3*c*f) + b*c*(-5*d*e + 6*c*f))*(c + d*x^2)*Sqrt[1 + (d*x
^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f
)/(d*e)] - (3*I)*b^2*c^2*(d*e - c*f)^2*(c + d*x^2)*Sqrt[1 + (d*x^
2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[
```

```
d/c]*x], (c*f)/(d*e)])/(3*a*c^2*Sqrt[d/c]*(b*c - a*d)^2*(d*e - c*
f)^2*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])
```

Maple [B] time = 0.068, size = 2062, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] 1/3*(-7*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c^3*d^2*e*f
*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-5*EllipticF(x*(-d/c)^(1/
2),(c*f/d/e)^(1/2))*x^2*a^2*c*d^4*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2
+e)/e)^(1/2)-6*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*b*
c^3*d^2*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-5*EllipticF(x
*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*b*c*d^4*e^2*((d*x^2+c)/c)^(1
/2)*((f*x^2+e)/e)^(1/2)+4*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2
))*x^2*a^2*c*d^4*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+5*El
lipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*b*c*d^4*e^2*((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-4*x^5*a^2*c*d^4*f^2*(-d/c)^(1/2)
+11*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*b*c^2*d^3*e*f
*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-7*EllipticE(x*(-d/c)^(1/
2),(c*f/d/e)^(1/2))*x^2*a*b*c^2*d^3*e*f*((d*x^2+c)/c)^(1/2)*((f*x
^2+e)/e)^(1/2)+2*x^3*a^2*d^5*e^2*(-d/c)^(1/2)-6*EllipticPi(x*(-d/
c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*x^2*b^2*c^3*d^2*e*f*(
(d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+11*EllipticF(x*(-d/c)^(1/2
),(c*f/d/e)^(1/2))*a*b*c^3*d^2*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)
/e)^(1/2)+x^3*a*b*c^2*d^3*e*f*(-d/c)^(1/2)+8*x*a*b*c^3*d^2*e*f*(-
d/c)^(1/2)+3*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^2*c^
2*d^3*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+3*EllipticPi(x*
(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*x^2*b^2*c^4*d*f^2
*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+3*EllipticPi(x*(-d/c)^(1
/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*x^2*b^2*c^2*d^3*e^2*((d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-5*EllipticF(x*(-d/c)^(1/2),(c*f
/d/e)^(1/2))*a^2*c^2*d^3*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1
/2)-6*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c^4*d*f^2*((d
*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-5*EllipticF(x*(-d/c)^(1/2),(
c*f/d/e)^(1/2))*a*b*c^2*d^3*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)+4*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c^2*d^3*e*
f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+5*EllipticE(x*(-d/c)^(1
/2),(c*f/d/e)^(1/2))*a*b*c^2*d^3*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+
e)/e)^(1/2)-6*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/
c)^(1/2))*b^2*c^4*d*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-5
*x^5*a*b*c*d^4*e*f*(-d/c)^(1/2)+2*x^5*a^2*d^5*e*f*(-d/c)^(1/2)-5*
x^3*a^2*c^2*d^3*f^2*(-d/c)^(1/2)+3*x*a^2*c*d^4*e^2*(-d/c)^(1/2)+3
*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2
*c^5*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-5*x^3*a*b*c*d^4*
e^2*(-d/c)^(1/2)-5*x*a^2*c^2*d^3*e*f*(-d/c)^(1/2)-6*x*a*b*c^2*d^3
*e^2*(-d/c)^(1/2)+2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2
*a^2*d^5*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*EllipticE(
x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^2*d^5*e^2*((d*x^2+c)/c)^(1/
```

```
2)*((f*x^2+e)/e)^(1/2)+3*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2)
)*a^2*c^3*d^2*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*Ellip
ticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c*d^4*e^2*((d*x^2+c)/c)^
(1/2)*((f*x^2+e)/e)^(1/2)-2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1
/2))*a^2*c*d^4*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+3*Elli
pticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c^3*
d^2*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+7*x^5*a*b*c^2*d^3
*f^2*(-d/c)^(1/2)-x^3*a^2*c*d^4*e*f*(-d/c)^(1/2)+8*x^3*a*b*c^3*d^
2*f^2*(-d/c)^(1/2))/(f*x^2+e)^(1/2)/(a*d-b*c)^2/(-d/c)^(1/2)/a/(c
*f-d*e)^2/c^2/(d*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)),x, algorithm="maxima

[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)),x, algorithm="fricas

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(5/2)/(f*x**2+e)**(1/2),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)(dx^2 + c)^{\frac{5}{2}}\sqrt{fx^2 + e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)), x)
```

3.83 $\int \frac{\left(c+dx^2\right)^{5/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=980

$$\frac{e^{3/2}\sqrt{dx^2+c}\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)(bc-ad)^3}{abc\sqrt{f}(be-af)^2\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}$$
$$-\frac{\sqrt{e}(bde+4bcf-3adf)\sqrt{dx^2+c}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)(bc-ad)}{3b\sqrt{f}(be-af)^2\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}$$
$$+\frac{d(5bc-3ad)e^{3/2}\sqrt{dx^2+c}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)(bc-ad)}{3bc\sqrt{f}(be-af)^2\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}$$
$$+\frac{dx\sqrt{dx^2+c}\sqrt{fx^2+e}(bc-ad)}{3(be-af)^2}+\frac{(bde+4bcf-3adf)x\sqrt{dx^2+c}(bc-ad)}{3b(be-af)^2\sqrt{fx^2+e}}$$
$$-\frac{\left(be\left(6d^2e^2-7cdfe-c^2f^2\right)-af\left(8d^2e^2-13cdfe+3c^2f^2\right)\right)\sqrt{dx^2+c}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3\sqrt{e}f^{3/2}(be-af)^2\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}$$
$$-\frac{\sqrt{e}\left(2adf(2de-3cf)-b\left(3d^2e^2-2cdfe-3c^2f^2\right)\right)\sqrt{dx^2+c}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3f^{3/2}(be-af)^2\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}$$
$$+\frac{d(af(4de-3cf)-be(3de-2cf))x\sqrt{dx^2+c}\sqrt{fx^2+e}}{3ef(be-af)^2}+\frac{(de-cf)x\left(dx^2+c\right)^{3/2}}{e(be-af)\sqrt{fx^2+e}}$$
$$+\frac{\left(be\left(6d^2e^2-7cdfe-c^2f^2\right)-af\left(8d^2e^2-13cdfe+3c^2f^2\right)\right)x\sqrt{dx^2+c}}{3ef(be-af)^2\sqrt{fx^2+e}}$$

```
[Out] ((b*c - a*d)*(b*d*e + 4*b*c*f - 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*b*
(b*e - a*f)^2*Sqrt[e + f*x^2]) + ((b*e*(6*d^2*e^2 - 7*c*d*e*f - c
^2*f^2) - a*f*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2))*x*Sqrt[c + d*
x^2])/(3*e*f*(b*e - a*f)^2*Sqrt[e + f*x^2]) + ((d*e - c*f)*x*(c +
 d*x^2)^(3/2))/(e*(b*e - a*f)*Sqrt[e + f*x^2]) + (d*(b*c - a*d)*x
*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*(b*e - a*f)^2) + (d*(a*f*(4*
d*e - 3*c*f) - b*e*(3*d*e - 2*c*f))*x*Sqrt[c + d*x^2]*Sqrt[e + f*
x^2])/(3*e*f*(b*e - a*f)^2) - ((b*c - a*d)*Sqrt[e]*(b*d*e + 4*b*c
*f - 3*a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e
]], 1 - (d*e)/(c*f)])/(3*b*Sqrt[f]*(b*e - a*f)^2*Sqrt[(e*(c + d*x
^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - ((b*e*(6*d^2*e^2 - 7*c*d
*e*f - c^2*f^2) - a*f*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2))*Sqrt[
c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)
])/(3*Sqrt[e]*f^(3/2)*(b*e - a*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e +
f*x^2))]*Sqrt[e + f*x^2]) + (d*(5*b*c - 3*a*d)*(b*c - a*d)*e^(3/2
)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e
)/(c*f)])/(3*b*c*Sqrt[f]*(b*e - a*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e
```

```
 + f*x^2))]*Sqrt[e + f*x^2]) - (Sqrt[e]*(2*a*d*f*(2*d*e - 3*c*f)
- b*(3*d^2*e^2 - 2*c*d*e*f - 3*c^2*f^2))*Sqrt[c + d*x^2]*Elliptic
F[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*f^(3/2)*(b*e
- a*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) +
 ((b*c - a*d)^3*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f
), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*b*c*Sqrt[f]*
(b*e - a*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^
2])
```

Rubi [A] time = 3.20434, antiderivative size = 980, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$

$$\frac{e^{3/2}\sqrt{dx^2+c}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)(bc-ad)^3}{abc\sqrt{f}(be-af)^2\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}} - \frac{\sqrt{e}(bde+4bcf-3adf)\sqrt{dx^2+c}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)(bc-ad)}{3b\sqrt{f}(be-af)^2\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}} + \frac{d(5bc-3ad)e^{3/2}\sqrt{dx^2+c}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)(bc-ad)}{3bc\sqrt{f}(be-af)^2\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}} + \frac{dx\sqrt{dx^2+c}\sqrt{fx^2+e}(bc-ad)}{3(be-af)^2} + \frac{(bde+4bcf-3adf)x\sqrt{dx^2+c}(bc-ad)}{3b(be-af)^2\sqrt{fx^2+e}} - \frac{\left(be\left(6d^2e^2-7cdfe-c^2f^2\right)-af\left(8d^2e^2-13cdfe+3c^2f^2\right)\right)\sqrt{dx^2+c}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3\sqrt{e}f^{3/2}(be-af)^2\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}} - \frac{\sqrt{e}\left(2adf(2de-3cf)-b\left(3d^2e^2-2cdfe-3c^2f^2\right)\right)\sqrt{dx^2+c}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3f^{3/2}(be-af)^2\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}} + \frac{d(af(4de-3cf)-be(3de-2cf))x\sqrt{dx^2+c}\sqrt{fx^2+e}}{3ef(be-af)^2} + \frac{(de-cf)x\left(dx^2+c\right)^{3/2}}{e(be-af)\sqrt{fx^2+e}} + \frac{\left(be\left(6d^2e^2-7cdfe-c^2f^2\right)-af\left(8d^2e^2-13cdfe+3c^2f^2\right)\right)x\sqrt{dx^2+c}}{3ef(be-af)^2\sqrt{fx^2+e}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^(5/2)/((a + b*x^2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] ((b*c - a*d)*(b*d*e + 4*b*c*f - 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*b*
(b*e - a*f)^2*Sqrt[e + f*x^2]) + ((b*e*(6*d^2*e^2 - 7*c*d*e*f - c
^2*f^2) - a*f*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2))*x*Sqrt[c + d*
```

```
x^2])/(3*e*f*(b*e - a*f)^2*Sqrt[e + f*x^2]) + ((d*e - c*f)*x*(c +
 d*x^2)^(3/2))/(e*(b*e - a*f)*Sqrt[e + f*x^2]) + (d*(b*c - a*d)*x
*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*(b*e - a*f)^2) + (d*(a*f*(4*
d*e - 3*c*f) - b*e*(3*d*e - 2*c*f))*x*Sqrt[c + d*x^2]*Sqrt[e + f*
x^2])/(3*e*f*(b*e - a*f)^2) - ((b*c - a*d)*Sqrt[e]*(b*d*e + 4*b*c
*f - 3*a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e
]], 1 - (d*e)/(c*f)])/(3*b*Sqrt[f]*(b*e - a*f)^2*Sqrt[(e*(c + d*x
^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - ((b*e*(6*d^2*e^2 - 7*c*d
*e*f - c^2*f^2) - a*f*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2))*Sqrt[
c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)
])/(3*Sqrt[e]*f^(3/2)*(b*e - a*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e +
f*x^2))]*Sqrt[e + f*x^2]) + (d*(5*b*c - 3*a*d)*(b*c - a*d)*e^(3/2
)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e
)/(c*f)])/(3*b*c*Sqrt[f]*(b*e - a*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e
 + f*x^2))]*Sqrt[e + f*x^2]) - (Sqrt[e]*(2*a*d*f*(2*d*e - 3*c*f)
- b*(3*d^2*e^2 - 2*c*d*e*f - 3*c^2*f^2))*Sqrt[c + d*x^2]*Elliptic
F[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*f^(3/2)*(b*e
- a*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) +
 ((b*c - a*d)^3*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f
), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*b*c*Sqrt[f]*
(b*e - a*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^
2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(5/2)/(b*x**2+a)/(f*x**2+e)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 2.5149, size = 352, normalized size = 0.36

$$\frac{-f\left(ab^2x\sqrt{\frac{d}{c}}\left(c+dx^2\right)(de-cf)^2+ief\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(bc-ad)^3\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)\right)-iabde\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+}{ab^2ef^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^(5/2)/((a + b*x^2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] ((-I)*a*b*d*e*(-(a*d^2*e*f) + b*(2*d^2*e^2 - 2*c*d*e*f + c^2*f^2)
)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqr
t[d/c]*x], (c*f)/(d*e)] - I*a*d^2*e*(b*e - a*f)*(-2*b*d*e + 3*b*c
```

```
*f - a*d*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*A
rcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - f*(a*b^2*Sqrt[d/c]*(d*e - c*f
)^2*x*(c + d*x^2) + I*(b*c - a*d)^3*e*f*Sqrt[1 + (d*x^2)/c]*Sqrt[
1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c
*f)/(d*e)]))/(a*b^2*Sqrt[d/c]*e*f^2*(b*e - a*f)*Sqrt[c + d*x^2]*S
qrt[e + f*x^2])
```

Maple [A] time = 0.047, size = 1063, normalized size = 1.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x)
```

```
[Out] (x^3*a*b^2*c^2*d*f^3*(-d/c)^(1/2)-2*x^3*a*b^2*c*d^2*e*f^2*(-d/c)^
(1/2)+x^3*a*b^2*d^3*e^2*f*(-d/c)^(1/2)-EllipticF(x*(-d/c)^(1/2),(
c*f/d/e)^(1/2))*a^3*d^3*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)+3*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*c*d^2*e*f^
2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticF(x*(-d/c)^(1/2
),(c*f/d/e)^(1/2))*a^2*b*d^3*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)
/e)^(1/2)-3*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c*d^2
*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticF(x*(-d/
c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*d^3*e^3*((d*x^2+c)/c)^(1/2)*((f*x
^2+e)/e)^(1/2)+EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*d^
3*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticE(x*(-d/c
)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c^2*d*e*f^2*((d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)+2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b
^2*c*d^2*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*Elliptic
E(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*d^3*e^3*((d*x^2+c)/c)^(1/
2)*((f*x^2+e)/e)^(1/2)+EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(
1/2)/(-d/c)^(1/2))*a^3*d^3*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e
)^(1/2)-3*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(
1/2))*a^2*b*c*d^2*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+3
*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b
^2*c^2*d*e*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticPi
(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^3*c^3*e*f^2*
((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+x*a*b^2*c^3*f^3*(-d/c)^(1
/2)-2*x*a*b^2*c^2*d*e*f^2*(-d/c)^(1/2)+x*a*b^2*c*d^2*e^2*f*(-d/c)
^(1/2))*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/e/f^2/a/(-d/c)^(1/2)/b^2/
(a*f-b*e)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2 + c\right)^{\frac{5}{2}}}{(bx^2 + a)(fx^2 + e)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*(f*x^2 + e)^(3/2)),x, algorithm="maxima

[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*(f*x^2 + e)^(3/2)),x, algorithm="fricas

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(5/2)/(b*x**2+a)/(f*x**2+e)**(3/2),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2 + c\right)^{\frac{5}{2}}}{(bx^2 + a)(fx^2 + e)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*(f*x^2 + e)^(3/2)),x, algorithm="giac")

[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

3.84 $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)(e+fx^2)^{3/2}} \, dx$

Optimal. Leaf size=223

$$\frac{c^{3/2}\sqrt{e+fx^2}(bc-ad)\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(be-af)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{\sqrt{c+dx^2}(de-cf)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{f}\sqrt{e+fx^2}(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

```
[Out] ((d*e - c*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]
], 1 - (d*e)/(c*f)])/(Sqrt[e]*Sqrt[f]*(b*e - a*f)*Sqrt[(e*(c + d*
x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (c^(3/2)*(b*c - a*d)*Sq
rt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt
[c]], 1 - (c*f)/(d*e)])/(a*Sqrt[d]*e*(b*e - a*f)*Sqrt[c + d*x^2]*
Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

Rubi [A] time = 0.415613, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$

$$\frac{c^{3/2}\sqrt{e+fx^2}(bc-ad)\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(be-af)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}+\frac{\sqrt{c+dx^2}(de-cf)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{f}\sqrt{e+fx^2}(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(c + d*x^2)^(3/2)/((a + b*x^2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] ((d*e - c*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]
], 1 - (d*e)/(c*f)])/(Sqrt[e]*Sqrt[f]*(b*e - a*f)*Sqrt[(e*(c + d*
x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (c^(3/2)*(b*c - a*d)*Sq
rt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt
[c]], 1 - (c*f)/(d*e)])/(a*Sqrt[d]*e*(b*e - a*f)*Sqrt[c + d*x^2]*
Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

Rubi in Sympy [A] time = 49.0305, size = 182, normalized size = 0.82

$$\frac{\sqrt{c+dx^2}\,(cf-de)\,E\left(\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}\,(af-be)}+\frac{c^{\frac{3}{2}}\sqrt{e+fx^2}\,(ad-bc)\,\Pi\left(1-\frac{bc}{ad};\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{a\sqrt{d}e\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}\,(af-be)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(3/2)/(b*x**2+a)/(f*x**2+e)**(3/2),x)
```

```
[Out] sqrt(c + d*x**2)*(c*f - d*e)*elliptic_e(atan(sqrt(f)*x/sqrt(e)),
1 - d*e/(c*f))/(sqrt(e)*sqrt(f)*sqrt(e*(c + d*x**2)/(c*(e + f*x**
2)))*sqrt(e + f*x**2)*(a*f - b*e)) + c**(3/2)*sqrt(e + f*x**2)*(a
*d - b*c)*elliptic_pi(1 - b*c/(a*d), atan(sqrt(d)*x/sqrt(c)), -c*
f/(d*e) + 1)/(a*sqrt(d)*e*sqrt(c*(e + f*x**2)/(e*(c + d*x**2)))*s
qrt(c + d*x**2)*(a*f - b*e))
```

Mathematica [C] time = 1.58882, size = 304, normalized size = 1.36

$$\frac{-iad^2e\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(be-af)F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+abfx\sqrt{\frac{d}{c}}\left(c+dx^2\right)(de-cf)-iabde\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(cf}{abef\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}(be-}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c + d*x^2)^(3/2)/((a + b*x^2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] (a*b*Sqrt[d/c]*f*(d*e - c*f)*x*(c + d*x^2) - I*a*b*d*e*(-(d*e) +
c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[
Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*d^2*e*(b*e - a*f)*Sqrt[1 + (d*x^
2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)
/(d*e)] - I*(b*c - a*d)^2*e*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2
)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]
)/(a*b*Sqrt[d/c]*e*f*(b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [B] time = 0.043, size = 594, normalized size = 2.7

$$\frac{1}{abef(af-be)(dfx^4+cfx^2+dex^2+ce)}\left(x^3abcdf^2\sqrt{-\frac{d}{c}}-x^3abd^2ef\sqrt{-\frac{d}{c}}+EllipticF\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)a^2d^2ef\sqrt{\frac{dx^2+}{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x)
```

```
[Out] (x^3*a*b*c*d*f^2*(-d/c)^(1/2)-x^3*a*b*d^2*e*f*(-d/c)^(1/2)+Ellipt
icF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*d^2*e*f*((d*x^2+c)/c)^(1/
2)*((f*x^2+e)/e)^(1/2)-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*
a*b*d^2*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticE(x*(
-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c*d*e*f*((d*x^2+c)/c)^(1/2)*((f*
x^2+e)/e)^(1/2)+EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*d^2
*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticPi(x*(-d/c)^
```

```
(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*d^2*e*f*((d*x^2+c)/c
)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(
-f/e)^(1/2)/(-d/c)^(1/2))*a*b*c*d*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2
+e)/e)^(1/2)-EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c
)^(1/2))*b^2*c^2*e*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+x*a*
b*c^2*f^2*(-d/c)^(1/2)-x*a*b*c*d*e*f*(-d/c)^(1/2))*(f*x^2+e)^(1/2
)*(d*x^2+c)^(1/2)/b/a/(-d/c)^(1/2)/e/f/(a*f-b*e)/(d*f*x^4+c*f*x^2
+d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*(f*x^2 + e)^(3/2)),x, algorithm="maxima

[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*(f*x^2 + e)^(3/2)),x, algorithm="fricas

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(3/2)/(b*x**2+a)/(f*x**2+e)**(3/2),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^2+c\right)^{\frac{3}{2}}}{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*(f*x^2 + e)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

3.85 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=209

$$\frac{bc^{3/2}\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(be-af)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{\sqrt{f}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{e+fx^2}(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

```
[Out] -((Sqrt[f]*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]],
 1 - (d*e)/(c*f)])/(Sqrt[e]*(b*e - a*f)*Sqrt[(e*(c + d*x^2))/(c*(
e + f*x^2))]*Sqrt[e + f*x^2])) + (b*c^(3/2)*Sqrt[e + f*x^2]*Ellip
ticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*
e)])/(a*Sqrt[d]*e*(b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2)
)/(e*(c + d*x^2))])
```

Rubi [A] time = 0.36228, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$

$$\frac{bc^{3/2}\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(be-af)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{\sqrt{f}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{e+fx^2}(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c + d*x^2]/((a + b*x^2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] -((Sqrt[f]*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]],
 1 - (d*e)/(c*f)])/(Sqrt[e]*(b*e - a*f)*Sqrt[(e*(c + d*x^2))/(c*(
e + f*x^2))]*Sqrt[e + f*x^2])) + (b*c^(3/2)*Sqrt[e + f*x^2]*Ellip
ticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*
e)])/(a*Sqrt[d]*e*(b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2)
)/(e*(c + d*x^2))])
```

Rubi in Sympy [A] time = 47.4031, size = 170, normalized size = 0.81

$$\frac{\sqrt{f}\sqrt{c+dx^2}E\left(\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}\,(af-be)}-\frac{bc^{\frac{3}{2}}\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{a\sqrt{d}e\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}\,(af-be)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+c)**(1/2)/(b*x**2+a)/(f*x**2+e)**(3/2),x)
```

```
[Out] sqrt(f)*sqrt(c + d*x**2)*elliptic_e(atan(sqrt(f)*x/sqrt(e)), 1 -
d*e/(c*f))/(sqrt(e)*sqrt(e*(c + d*x**2)/(c*(e + f*x**2)))*sqrt(e
+ f*x**2)*(a*f - b*e)) - b*c**(3/2)*sqrt(e + f*x**2)*elliptic_pi(
1 - b*c/(a*d), atan(sqrt(d)*x/sqrt(c)), -c*f/(d*e) + 1)/(a*sqrt(d
)*e*sqrt(c*(e + f*x**2)/(e*(c + d*x**2)))*sqrt(c + d*x**2)*(a*f -
 b*e))
```

Mathematica [C] time = 0.621166, size = 207, normalized size = 0.99

$$\frac{-ie\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(bc-ad)\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-iade\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-afx\sqrt{\frac{d}{c}}\,(c}{ae\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}(be-af)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[c + d*x^2]/((a + b*x^2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] (-(a*Sqrt[d/c]*f*x*(c + d*x^2)) - I*a*d*e*Sqrt[1 + (d*x^2)/c]*Sqr
t[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] -
 I*(b*c - a*d)*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Elliptic
Pi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(a*Sqrt[d/c
]*e*(b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.038, size = 285, normalized size = 1.4

$$\frac{1}{ae\left(af-be\right)\left(dfx^4+cfx^2+dex^2+ce\right)}\left(x^3adf\sqrt{-\frac{d}{c}}-EllipticE\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)ade\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{fx^2+e}{e}}+EllipticPi\left(x\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x)
```

```
[Out] (x^3*a*d*f*(-d/c)^(1/2)-EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))
*a*d*e*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticPi(x*(-d/c
)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*d*e*((d*x^2+c)/c)^(1
/2)*((f*x^2+e)/e)^(1/2)-EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^
(1/2)/(-d/c)^(1/2))*b*c*e*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)
+x*a*c*f*(-d/c)^(1/2))*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/e/a/(-d/c)
^(1/2)/(a*f-b*e)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + c}}{(bx^2 + a)(fx^2 + e)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*(f*x^2 + e)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*(f*x^2 + e)^(3/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + dx^2}}{(a + bx^2)\,(e + fx^2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(1/2)/(b*x**2+a)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/((a + b*x**2)*(e + f*x**2)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + c}}{(bx^2 + a)(fx^2 + e)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*(f*x^2 + e)^(3/2)),x, algorithm="giac")

[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

3.86 $\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=344

$$\frac{b^2e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{f^{3/2}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{e+fx^2}(be-af)(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(-adf-bcf+2bde)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{e+fx^2}(be-af)^2(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

```
[Out] (f^(3/2)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1
 - (d*e)/(c*f)])/(Sqrt[e]*(b*e - a*f)*(d*e - c*f)*Sqrt[(e*(c + d*
x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[f]*(2*b*d
*e - b*c*f - a*d*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/
Sqrt[e]], 1 - (d*e)/(c*f)])/(c*(b*e - a*f)^2*(d*e - c*f)*Sqrt[(e*
(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (b^2*e^(3/2)*Sqr
t[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[
e]], 1 - (d*e)/(c*f)])/(a*c*Sqrt[f]*(b*e - a*f)^2*Sqrt[(e*(c + d*
x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.710956, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156

$$\frac{b^2e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{f^{3/2}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{e+fx^2}(be-af)(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(-adf-bcf+2bde)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{e+fx^2}(be-af)^2(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

```
[Out] (f^(3/2)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1
 - (d*e)/(c*f)])/(Sqrt[e]*(b*e - a*f)*(d*e - c*f)*Sqrt[(e*(c + d*
x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[f]*(2*b*d
*e - b*c*f - a*d*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/
Sqrt[e]], 1 - (d*e)/(c*f)])/(c*(b*e - a*f)^2*(d*e - c*f)*Sqrt[(e*
(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (b^2*e^(3/2)*Sqr
t[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[
e]], 1 - (d*e)/(c*f)])/(a*c*Sqrt[f]*(b*e - a*f)^2*Sqrt[(e*(c + d*
```

```
x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rubi in Sympy [A] time = 94.2393, size = 289, normalized size = 0.84

$$-\frac{\sqrt{c}f\sqrt{e+fx^2}\,(adf+bcf-2bde)\,F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{\sqrt{d}e\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}\,(af-be)^2\,(cf-de)}+\frac{f^{\frac{3}{2}}\sqrt{c+dx^2}E\left(\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}\,(af-be)\,(cf-de)}+\frac{b^2e^{\frac{3}{2}}\sqrt{c+dx^2}\left(1-\frac{be}{af};\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}\,(af-be)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)/(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] -sqrt(c)*f*sqrt(e + f*x**2)*(a*d*f + b*c*f - 2*b*d*e)*elliptic_f(
atan(sqrt(d)*x/sqrt(c)), -c*f/(d*e) + 1)/(sqrt(d)*e*sqrt(c*(e + f
*x**2)/(e*(c + d*x**2)))*sqrt(c + d*x**2)*(a*f - b*e)**2*(c*f - d
*e)) + f**(3/2)*sqrt(c + d*x**2)*elliptic_e(atan(sqrt(f)*x/sqrt(e
)), 1 - d*e/(c*f))/(sqrt(e)*sqrt(e*(c + d*x**2)/(c*(e + f*x**2)))
*sqrt(e + f*x**2)*(a*f - b*e)*(c*f - d*e)) + b**2*e**(3/2)*sqrt(c
 + d*x**2)*elliptic_pi(1 - b*e/(a*f), atan(sqrt(f)*x/sqrt(e)), 1
- d*e/(c*f))/(a*c*sqrt(f)*sqrt(e*(c + d*x**2)/(c*(e + f*x**2)))*s
qrt(e + f*x**2)*(a*f - b*e)**2)
```

Mathematica [C] time = 0.919241, size = 221, normalized size = 0.64

$$\frac{-ibe\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(cf-de)\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-iadef\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-af^2x\sqrt{\cdots}}{ae\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}(af-be)(de-cf)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

```
[Out] (-(a*Sqrt[d/c]*f^2*x*(c + d*x^2)) - I*a*d*e*f*Sqrt[1 + (d*x^2)/c]
*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e
)] - I*b*e*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]
*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(a
*Sqrt[d/c]*e*(-(b*e) + a*f)*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2])
```

Maple [A] time = 0.044, size = 303, normalized size = 0.9

$$\frac{1}{(af-be)\,ae\,(cf-de)\,(dfx^4+cfx^2+dex^2+ce)}\left(x^3adf^2\sqrt{-\frac{d}{c}}-\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{fx^2+e}{e}}\,EllipticE\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)adef-E$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

```
[Out] (x^3*a*d*f^2*(-d/c)^(1/2)-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)
*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e*f-EllipticPi(x*(
-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b*c*e*f*((d*x^2+c)
/c)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(
-f/e)^(1/2)/(-d/c)^(1/2))*b*d*e^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/
e)^(1/2)+x*a*c*f^2*(-d/c)^(1/2))*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/
a/e/(a*f-b*e)/(-d/c)^(1/2)/(c*f-d*e)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)),x, algorithm="maxima
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)),x, algorithm="fricas
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)/(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(3/2)), x
)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

3.87 $\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=539

$$\frac{b^3c^{3/2}\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(bc-ad)^2(be-af)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{b^2\sqrt{f}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{e+fx^2}(bc-ad)^2(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{d\sqrt{f}\sqrt{c+dx^2}\left(2bc^2f-ad(cf+de)\right)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{e}\sqrt{e+fx^2}(bc-ad)^2(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{d^2x}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(bc-ad)(de-cf)}$$
$$-\frac{d^2\sqrt{e}\sqrt{c+dx^2}(2adf-3bcf+bde)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}(bc-ad)^2(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

```
[Out] -((d^2*x)/(c*(b*c - a*d)*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x
^2])) - (b^2*Sqrt[f]*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)
/Sqrt[e]], 1 - (d*e)/(c*f)])/((b*c - a*d)^2*Sqrt[e]*(b*e - a*f)*S
qrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (d*Sqrt[f
]*(2*b*c^2*f - a*d*(d*e + c*f))*Sqrt[c + d*x^2]*EllipticE[ArcTan[
(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*(b*c - a*d)^2*Sqrt[e]*
(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^
2]) - (d^2*Sqrt[e]*(b*d*e - 3*b*c*f + 2*a*d*f)*Sqrt[c + d*x^2]*El
lipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*(b*c -
a*d)^2*Sqrt[f]*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))
]*Sqrt[e + f*x^2]) + (b^3*c^(3/2)*Sqrt[e + f*x^2]*EllipticPi[1 -
(b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(a*Sq
rt[d]*(b*c - a*d)^2*e*(b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*
x^2))/(e*(c + d*x^2))])
```

Rubi [A] time = 1.40994, antiderivative size = 539, normalized size of antiderivative = 1., number of

steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219

$$\frac{b^3c^{3/2}\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(bc-ad)^2(be-af)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}-\frac{b^2\sqrt{f}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{e+fx^2}(bc-ad)^2(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{d\sqrt{f}\sqrt{c+dx^2}\left(2bc^2f-ad(cf+de)\right)E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{e}\sqrt{e+fx^2}(bc-ad)^2(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{d^2x}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(bc-ad)(de-cf)}$$
$$-\frac{d^2\sqrt{e}\sqrt{c+dx^2}(2adf-3bcf+bde)F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}(bc-ad)^2(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] -((d^2*x)/(c*(b*c - a*d)*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x
^2])) - (b^2*Sqrt[f]*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)
/Sqrt[e]], 1 - (d*e)/(c*f)])/((b*c - a*d)^2*Sqrt[e]*(b*e - a*f)*S
qrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (d*Sqrt[f
]*(2*b*c^2*f - a*d*(d*e + c*f))*Sqrt[c + d*x^2]*EllipticE[ArcTan[
(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*(b*c - a*d)^2*Sqrt[e]*
(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^
2]) - (d^2*Sqrt[e]*(b*d*e - 3*b*c*f + 2*a*d*f)*Sqrt[c + d*x^2]*El
lipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*(b*c -
a*d)^2*Sqrt[f]*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))
]*Sqrt[e + f*x^2]) + (b^3*c^(3/2)*Sqrt[e + f*x^2]*EllipticPi[1 -
(b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(a*Sq
rt[d]*(b*c - a*d)^2*e*(b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*
x^2))/(e*(c + d*x^2))])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)/(d*x**2+c)**(3/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 7.18586, size = 1284, normalized size = 2.38

$$\sqrt{dx^2+c}\sqrt{fx^2+e}\left(-\frac{xd^3}{c(bc-ad)(cf-de)^2\,(dx^2+c)}-\frac{f^3x}{e(be-af)(de-cf)^2\,(fx^2+e)}\right)$$
$$-\frac{\sqrt{(dx^2+c)\,(fx^2+e)}\left(\frac{ib^2ef^2\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)c^3}{a\sqrt{\frac{d}{c}}\sqrt{(dx^2+c)(fx^2+e)}}+\frac{ibdef^2\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\right.\right.}{\sqrt{\frac{d}{c}}\sqrt{(dx^2+c)(fx^2+e)}}\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] Sqrt[c + d*x^2]*Sqrt[e + f*x^2]*(-((d^3*x)/(c*(b*c - a*d)*(-(d*e)
 + c*f)^2*(c + d*x^2))) - (f^3*x)/(e*(b*e - a*f)*(d*e - c*f)^2*(e
 + f*x^2))) - (Sqrt[(c + d*x^2)*(e + f*x^2)]*((I*b*d^3*e^3*Sqrt[1
 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(EllipticE[I*ArcSinh[Sqrt[d/c]*
x], (c*f)/(d*e)] - EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]
))/(Sqrt[d/c]*Sqrt[(c + d*x^2)*(e + f*x^2)]) - (I*a*d^3*e^2*f*Sqr
t[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(EllipticE[I*ArcSinh[Sqrt[d/
c]*x], (c*f)/(d*e)] - EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*
e)]))/(Sqrt[d/c]*Sqrt[(c + d*x^2)*(e + f*x^2)]) + (I*b*c^2*d*e*f^
2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(EllipticE[I*ArcSinh[Sq
rt[d/c]*x], (c*f)/(d*e)] - EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f
)/(d*e)]))/(Sqrt[d/c]*Sqrt[(c + d*x^2)*(e + f*x^2)]) - (I*a*c*d^2
*e*f^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(EllipticE[I*ArcSi
nh[Sqrt[d/c]*x], (c*f)/(d*e)] - EllipticF[I*ArcSinh[Sqrt[d/c]*x],
 (c*f)/(d*e)]))/(Sqrt[d/c]*Sqrt[(c + d*x^2)*(e + f*x^2)]) + (I*b*
c*d^2*e^2*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*A
rcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(Sqrt[d/c]*Sqrt[(c + d*x^2)*(e
 + f*x^2)]) + (I*b*c^2*d*e*f^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^
2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(Sqrt[d/c]*
Sqrt[(c + d*x^2)*(e + f*x^2)]) - ((2*I)*a*c*d^2*e*f^2*Sqrt[1 + (d
*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c
*f)/(d*e)])/(Sqrt[d/c]*Sqrt[(c + d*x^2)*(e + f*x^2)]) + (I*b^2*c*
d^2*e^3*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/
(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(a*Sqrt[d/c]*Sqrt[(c
 + d*x^2)*(e + f*x^2)]) - ((2*I)*b^2*c^2*d*e^2*f*Sqrt[1 + (d*x^2)
/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/
c]*x], (c*f)/(d*e)])/(a*Sqrt[d/c]*Sqrt[(c + d*x^2)*(e + f*x^2)])
+ (I*b^2*c^3*e*f^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Ellipt
icPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(a*Sqrt[d
/c]*Sqrt[(c + d*x^2)*(e + f*x^2)])))/(c*(b*c - a*d)*e*(b*e - a*f)
*(-(d*e) + c*f)^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.056, size = 956, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x)
```

```
[Out] (x^3*a^2*c*d^2*f^3*(-d/c)^(1/2)+x^3*a^2*d^3*e*f^2*(-d/c)^(1/2)-x^
3*a*b*c^2*d*f^3*(-d/c)^(1/2)-x^3*a*b*d^3*e^2*f*(-d/c)^(1/2)-Ellip
ticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c*d^2*e*f^2*((d*x^2+c)/c
)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1
/2))*a^2*d^3*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+Ellipt
icF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c*d^2*e^2*f*((d*x^2+c)/c)
^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/
2))*a*b*d^3*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-EllipticE
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c*d^2*e*f^2*((d*x^2+c)/c)^(1
/2)*((f*x^2+e)/e)^(1/2)-EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))
*a^2*d^3*e^2*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticE(
x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c^2*d*e*f^2*((d*x^2+c)/c)^(1/
2)*((f*x^2+e)/e)^(1/2)+EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*
a*b*d^3*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+EllipticPi(x*
(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c^3*e*f^2*((d
*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*EllipticPi(x*(-d/c)^(1/2),
b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c^2*d*e^2*f*((d*x^2+c)/c)^
(1/2)*((f*x^2+e)/e)^(1/2)+EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e
)^(1/2)/(-d/c)^(1/2))*b^2*c*d^2*e^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e
)/e)^(1/2)+x*a^2*c^2*d*f^3*(-d/c)^(1/2)+x*a^2*d^3*e^2*f*(-d/c)^(1
/2)-x*a*b*c^3*f^3*(-d/c)^(1/2)-x*a*b*d^3*e^3*(-d/c)^(1/2))*(f*x^2
+e)^(1/2)*(d*x^2+c)^(1/2)/e/c/(c*f-d*e)^2/a/(-d/c)^(1/2)/(a*f-b*e
)/(a*d-b*c)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)(dx^2 + c)^{\frac{3}{2}}(fx^2 + e)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)),x, algorithm="maxi
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)),x, algorithm="fric
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{\frac{3}{2}}(e+fx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)/(d*x**2+c)**(3/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*(c + d*x**2)**(3/2)*(e + f*x**2)**(3/2))
, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)),x, algorithm="giac
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

3.88 $\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=814

$$\frac{e^{3/2}\sqrt{dx^2+c}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)b^4}{ac(bc-ad)^2\sqrt{f}(be-af)^2\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}+\frac{f^{3/2}\sqrt{dx^2+c}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)b^2}{(bc-ad)^2\sqrt{e}(be-af)(de-cf)\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}$$
$$-\frac{\sqrt{e}\sqrt{f}(2bde-bcf-adf)\sqrt{dx^2+c}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)b^2}{c(bc-ad)^2(be-af)^2(de-cf)\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}$$
$$-\frac{d\sqrt{f}\left(bc\left(5d^2e^2-7cdfe-6c^2f^2\right)-ad\left(2d^2e^2-7cdfe-3c^2f^2\right)\right)\sqrt{dx^2+c}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2(bc-ad)^2\sqrt{e}(de-cf)^3\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}$$
$$+\frac{d^2\sqrt{e}\sqrt{f}(bc(7de-15cf)-ad(de-9cf))\sqrt{dx^2+c}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2(bc-ad)^2(de-cf)^3\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}$$
$$-\frac{d^2(bc(5de-9cf)-2ad(de-3cf))x}{3c^2(bc-ad)^2(de-cf)^2\sqrt{dx^2+c}\sqrt{fx^2+e}}-\frac{d^2x}{3c(bc-ad)(de-cf)\left(dx^2+c\right)^{3/2}\sqrt{fx^2+e}}$$

```
[Out] -(d^2*x)/(3*c*(b*c - a*d)*(d*e - c*f)*(c + d*x^2)^(3/2)*Sqrt[e +
f*x^2]) - (d^2*(b*c*(5*d*e - 9*c*f) - 2*a*d*(d*e - 3*c*f))*x)/(3*
c^2*(b*c - a*d)^2*(d*e - c*f)^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
+ (b^2*f^(3/2)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[
e]], 1 - (d*e)/(c*f)])/((b*c - a*d)^2*Sqrt[e]*(b*e - a*f)*(d*e -
c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (d*
Sqrt[f]*(b*c*(5*d^2*e^2 - 7*c*d*e*f - 6*c^2*f^2) - a*d*(2*d^2*e^2
 - 7*c*d*e*f - 3*c^2*f^2))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt
[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*(b*c - a*d)^2*Sqrt[e]*(
d*e - c*f)^3*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2
]) - (b^2*Sqrt[e]*Sqrt[f]*(2*b*d*e - b*c*f - a*d*f)*Sqrt[c + d*x^
2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*(b
*c - a*d)^2*(b*e - a*f)^2*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e
+ f*x^2))]*Sqrt[e + f*x^2]) + (d^2*Sqrt[e]*Sqrt[f]*(b*c*(7*d*e -
15*c*f) - a*d*(d*e - 9*c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sq
rt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*(b*c - a*d)^2*(d*e -
c*f)^3*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (
b^4*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(S
qrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)^2*Sqrt[f]*
(b*e - a*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^
2])
```

Rubi [A] time = 2.69766, antiderivative size = 814, normalized size of antiderivative = 1., number of

steps used = 11, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$\frac{e^{3/2}\sqrt{dx^2+c}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)b^4}{ac(bc-ad)^2\sqrt{f}(be-af)^2\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}+\frac{f^{3/2}\sqrt{dx^2+c}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)b^2}{(bc-ad)^2\sqrt{e}(be-af)(de-cf)\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}$$
$$-\frac{\sqrt{e}\sqrt{f}(2bde-bcf-adf)\sqrt{dx^2+c}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)b^2}{c(bc-ad)^2(be-af)^2(de-cf)\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}$$
$$-\frac{d\sqrt{f}\left(bc\left(5d^2e^2-7cdfe-6c^2f^2\right)-ad\left(2d^2e^2-7cdfe-3c^2f^2\right)\right)\sqrt{dx^2+c}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2(bc-ad)^2\sqrt{e}(de-cf)^3\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}$$
$$+\frac{d^2\sqrt{e}\sqrt{f}(bc(7de-15cf)-ad(de-9cf))\sqrt{dx^2+c}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2(bc-ad)^2(de-cf)^3\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}$$
$$-\frac{d^2(bc(5de-9cf)-2ad(de-3cf))x}{3c^2(bc-ad)^2(de-cf)^2\sqrt{dx^2+c}\sqrt{fx^2+e}}-\frac{d^2x}{3c(bc-ad)(de-cf)\left(dx^2+c\right)^{3/2}\sqrt{fx^2+e}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] -(d^2*x)/(3*c*(b*c - a*d)*(d*e - c*f)*(c + d*x^2)^(3/2)*Sqrt[e +
f*x^2]) - (d^2*(b*c*(5*d*e - 9*c*f) - 2*a*d*(d*e - 3*c*f))*x)/(3*
c^2*(b*c - a*d)^2*(d*e - c*f)^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
+ (b^2*f^(3/2)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[
e]], 1 - (d*e)/(c*f)])/((b*c - a*d)^2*Sqrt[e]*(b*e - a*f)*(d*e -
c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (d*
Sqrt[f]*(b*c*(5*d^2*e^2 - 7*c*d*e*f - 6*c^2*f^2) - a*d*(2*d^2*e^2
 - 7*c*d*e*f - 3*c^2*f^2))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt
[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*(b*c - a*d)^2*Sqrt[e]*(
d*e - c*f)^3*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2
]) - (b^2*Sqrt[e]*Sqrt[f]*(2*b*d*e - b*c*f - a*d*f)*Sqrt[c + d*x^
2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*(b
*c - a*d)^2*(b*e - a*f)^2*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e
+ f*x^2))]*Sqrt[e + f*x^2]) + (d^2*Sqrt[e]*Sqrt[f]*(b*c*(7*d*e -
15*c*f) - a*d*(d*e - 9*c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sq
rt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*(b*c - a*d)^2*(d*e -
c*f)^3*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (
b^4*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(S
qrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)^2*Sqrt[f]*
(b*e - a*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^
2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)/(d*x**2+c)**(5/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 9.02774, size = 2744, normalized size = 3.37

Result too large to show

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] Sqrt[c + d*x^2]*Sqrt[e + f*x^2]*(-(d^3*x)/(3*c*(b*c - a*d)*(-(d*e
) + c*f)^2*(c + d*x^2)^2) - (d^3*(-5*b*c*d*e + 2*a*d^2*e + 10*b*c
^2*f - 7*a*c*d*f)*x)/(3*c^2*(b*c - a*d)^2*(-(d*e) + c*f)^3*(c + d
*x^2)) + (f^4*x)/(e*(b*e - a*f)*(d*e - c*f)^3*(e + f*x^2))) + (Sq
rt[(c + d*x^2)*(e + f*x^2)]*(((5*I)*b^2*c*d^4*e^4*Sqrt[1 + (d*x^2
)/c]*Sqrt[1 + (f*x^2)/e]*(EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)
/(d*e)] - EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(Sqrt[
d/c]*Sqrt[(c + d*x^2)*(e + f*x^2)]) - ((2*I)*a*b*d^5*e^4*Sqrt[1 +
 (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(EllipticE[I*ArcSinh[Sqrt[d/c]*x]
, (c*f)/(d*e)] - EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))
/(Sqrt[d/c]*Sqrt[(c + d*x^2)*(e + f*x^2)]) - ((10*I)*b^2*c^2*d^3*
e^3*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(EllipticE[I*ArcSin
h[Sqrt[d/c]*x], (c*f)/(d*e)] - EllipticF[I*ArcSinh[Sqrt[d/c]*x],
(c*f)/(d*e)]))/(Sqrt[d/c]*Sqrt[(c + d*x^2)*(e + f*x^2)]) + ((2*I)
*a*b*c*d^4*e^3*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(Ellipti
cE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - EllipticF[I*ArcSinh[Sqr
t[d/c]*x], (c*f)/(d*e)]))/(Sqrt[d/c]*Sqrt[(c + d*x^2)*(e + f*x^2)
]) + ((2*I)*a^2*d^5*e^3*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]
*(EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - EllipticF[I*Ar
cSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(Sqrt[d/c]*Sqrt[(c + d*x^2)*(e
 + f*x^2)]) + ((10*I)*a*b*c^2*d^3*e^2*f^2*Sqrt[1 + (d*x^2)/c]*Sqr
t[1 + (f*x^2)/e]*(EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]
- EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(Sqrt[d/c]*Sqr
t[(c + d*x^2)*(e + f*x^2)]) - ((7*I)*a^2*c*d^4*e^2*f^2*Sqrt[1 + (
d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(EllipticE[I*ArcSinh[Sqrt[d/c]*x],
(c*f)/(d*e)] - EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(
Sqrt[d/c]*Sqrt[(c + d*x^2)*(e + f*x^2)]) - ((3*I)*b^2*c^4*d*e*f^3
*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(EllipticE[I*ArcSinh[Sqr
t[d/c]*x], (c*f)/(d*e)] - EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)
/(d*e)]))/(Sqrt[d/c]*Sqrt[(c + d*x^2)*(e + f*x^2)]) + ((6*I)*a*b*
c^3*d^2*e*f^3*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(EllipticE[
I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - EllipticF[I*ArcSinh[Sqrt[d
/c]*x], (c*f)/(d*e)]))/(Sqrt[d/c]*Sqrt[(c + d*x^2)*(e + f*x^2)])
- ((3*I)*a^2*c^2*d^3*e*f^3*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e
]*(EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - EllipticF[I*A
rcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(Sqrt[d/c]*Sqrt[(c + d*x^2)*(
e + f*x^2)]) + ((4*I)*b^2*c^2*d^3*e^3*f*Sqrt[1 + (d*x^2)/c]*Sqrt[
```

```
1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(S
qrt[d/c]*Sqrt[(c + d*x^2)*(e + f*x^2)]) - (I*a*b*c*d^4*e^3*f*Sqrt
[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]
*x], (c*f)/(d*e)])/(Sqrt[d/c]*Sqrt[(c + d*x^2)*(e + f*x^2)]) - ((
9*I)*b^2*c^3*d^2*e^2*f^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*
EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(Sqrt[d/c]*Sqrt[(
c + d*x^2)*(e + f*x^2)]) + ((2*I)*a*b*c^2*d^3*e^2*f^2*Sqrt[1 + (d
*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c
*f)/(d*e)])/(Sqrt[d/c]*Sqrt[(c + d*x^2)*(e + f*x^2)]) + (I*a^2*c*
d^4*e^2*f^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*A
rcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(Sqrt[d/c]*Sqrt[(c + d*x^2)*(e
 + f*x^2)]) - ((3*I)*b^2*c^4*d*e*f^3*Sqrt[1 + (d*x^2)/c]*Sqrt[1 +
 (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(Sqrt
[d/c]*Sqrt[(c + d*x^2)*(e + f*x^2)]) + ((15*I)*a*b*c^3*d^2*e*f^3*
Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[
d/c]*x], (c*f)/(d*e)])/(Sqrt[d/c]*Sqrt[(c + d*x^2)*(e + f*x^2)])
- ((9*I)*a^2*c^2*d^3*e*f^3*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e
]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(Sqrt[d/c]*Sqrt
[(c + d*x^2)*(e + f*x^2)]) + ((3*I)*b^3*c^2*d^3*e^4*Sqrt[1 + (d*x
^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt
[d/c]*x], (c*f)/(d*e)])/(a*Sqrt[d/c]*Sqrt[(c + d*x^2)*(e + f*x^2)
]) - ((9*I)*b^3*c^3*d^2*e^3*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2
)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]
)/(a*Sqrt[d/c]*Sqrt[(c + d*x^2)*(e + f*x^2)]) + ((9*I)*b^3*c^4*d*
e^2*f^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/
(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(a*Sqrt[d/c]*Sqrt[(c
 + d*x^2)*(e + f*x^2)]) - ((3*I)*b^3*c^5*e*f^3*Sqrt[1 + (d*x^2)/c
]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]
*x], (c*f)/(d*e)])/(a*Sqrt[d/c]*Sqrt[(c + d*x^2)*(e + f*x^2)])))/
(3*c^2*(b*c - a*d)^2*e*(b*e - a*f)*(-(d*e) + c*f)^3*Sqrt[c + d*x^
2]*Sqrt[e + f*x^2])
```

Maple [B] time = 0.088, size = 4115, normalized size = 5.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x)
```

```
[Out] -1/3*(-3*x*a^2*b*c*d^5*e^4*(-d/c)^(1/2)+6*x*a*b^2*c^2*d^4*e^4*(-d
/c)^(1/2)+3*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)
^(1/2))*b^3*c^6*e*f^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-3*E
llipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^3*c
^3*d^3*e^4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-8*EllipticF(x*
(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^3*c*d^5*e^2*f^2*((d*x^2+c)/c)
^(1/2)*((f*x^2+e)/e)^(1/2)+5*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(
1/2))*x^2*a*b^2*c*d^5*e^4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)
-6*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*c^4*d^2*e*f^3*
((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-10*EllipticE(x*(-d/c)^(1/
2),(c*f/d/e)^(1/2))*a^2*b*c^3*d^3*e^2*f^2*((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)-2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*
```

```
b*c^2*d^4*e^3*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+3*Ellipti
cE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c^5*d*e*f^3*((d*x^2+c)/c
)^(1/2)*((f*x^2+e)/e)^(1/2)+10*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)
^(1/2))*a*b^2*c^3*d^3*e^3*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/
2)-9*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*c^4*d^2*e*f^
3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+8*EllipticF(x*(-d/c)^(1
/2),(c*f/d/e)^(1/2))*a^2*b*c^3*d^3*e^2*f^2*((d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)+3*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2
*b*c^2*d^4*e^3*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+9*Ellipt
icF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c^4*d^2*e^2*f^2*((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-3*x^5*a^3*c^2*d^4*f^4*(-d/c)^(1/
2)+2*x^5*a^3*d^6*e^2*f^2*(-d/c)^(1/2)-6*x^3*a^3*c^3*d^3*f^4*(-d/c
)^(1/2)+2*x^3*a^3*d^6*e^3*f*(-d/c)^(1/2)-2*x^3*a^2*b*d^6*e^4*(-d/
c)^(1/2)-3*x*a^3*c^4*d^2*f^4*(-d/c)^(1/2)-3*x*a*b^2*c^6*f^4*(-d/c
)^(1/2)+2*x*a^2*b*c^2*d^4*e^3*f*(-d/c)^(1/2)-11*x*a*b^2*c^3*d^3*e
^3*f*(-d/c)^(1/2)-3*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2
)/(-d/c)^(1/2))*x^2*b^3*c^2*d^4*e^4*((d*x^2+c)/c)^(1/2)*((f*x^2+e
)/e)^(1/2)-2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^3*d^
6*e^3*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*EllipticE(x*(-d
/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^2*b*d^6*e^4*((d*x^2+c)/c)^(1/2)*
((f*x^2+e)/e)^(1/2)+2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x
^2*a^3*d^6*e^3*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*Ellipt
icF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^2*b*d^6*e^4*((d*x^2+c)/
c)^(1/2)*((f*x^2+e)/e)^(1/2)-9*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,
(-f/e)^(1/2)/(-d/c)^(1/2))*b^3*c^5*d*e^2*f^2*((d*x^2+c)/c)^(1/2)*
((f*x^2+e)/e)^(1/2)+9*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1
/2)/(-d/c)^(1/2))*b^3*c^4*d^2*e^3*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e
)/e)^(1/2)+3*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^3*c^3*d^
3*e*f^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+7*EllipticE(x*(-d
/c)^(1/2),(c*f/d/e)^(1/2))*a^3*c^2*d^4*e^2*f^2*((d*x^2+c)/c)^(1/2
)*((f*x^2+e)/e)^(1/2)-2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))
*a^3*c*d^5*e^3*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+2*Ellipt
icE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*c*d^5*e^4*((d*x^2+c)/c)
^(1/2)*((f*x^2+e)/e)^(1/2)-5*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(
1/2))*a*b^2*c^2*d^4*e^4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+6
*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^3*c^3*d^3*e*f^3*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-8*EllipticF(x*(-d/c)^(1/2),(c
*f/d/e)^(1/2))*a^3*c^2*d^4*e^2*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)
/e)^(1/2)+2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^3*c*d^5*e
^3*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-2*EllipticF(x*(-d/c)
^(1/2),(c*f/d/e)^(1/2))*a^2*b*c*d^5*e^4*((d*x^2+c)/c)^(1/2)*((f*x
^2+e)/e)^(1/2)+5*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*
c^2*d^4*e^4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+10*x^5*a^2*b*
c^2*d^4*e*f^3*(-d/c)^(1/2)+2*x^5*a^2*b*c*d^5*e^2*f^2*(-d/c)^(1/2)
-10*x^5*a*b^2*c^2*d^4*e^2*f^2*(-d/c)^(1/2)+5*x^5*a*b^2*c*d^5*e^3*
f*(-d/c)^(1/2)+11*x^3*a^2*b*c^3*d^3*e*f^3*(-d/c)^(1/2)+12*x^3*a^2
*b*c^2*d^4*e^2*f^2*(-d/c)^(1/2)-x^3*a^2*b*c*d^5*e^3*f*(-d/c)^(1/2
)-11*x^3*a*b^2*c^3*d^3*e^2*f^2*(-d/c)^(1/2)-4*x^3*a*b^2*c^2*d^4*e
^3*f*(-d/c)^(1/2)+11*x*a^2*b*c^3*d^3*e^2*f^2*(-d/c)^(1/2)-6*Ellip
ticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^2*b*c^3*d^3*e*f^3*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-10*EllipticE(x*(-d/c)^(1/2),(
c*f/d/e)^(1/2))*x^2*a^2*b*c^2*d^4*e^2*f^2*((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)-2*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*
a^2*b*c*d^5*e^3*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+3*Ellip
ticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*b^2*c^4*d^2*e*f^3*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+10*EllipticE(x*(-d/c)^(1/2),(
```

```
c*f/d/e)^(1/2))*x^2*a*b^2*c^2*d^4*e^3*f*((d*x^2+c)/c)^(1/2)*((f*x
^2+e)/e)^(1/2)-9*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^
2*b*c^3*d^3*e*f^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+8*Ellip
ticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^2*b*c^2*d^4*e^2*f^2*((
d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+3*EllipticF(x*(-d/c)^(1/2),
(c*f/d/e)^(1/2))*x^2*a^2*b*c*d^5*e^3*f*((d*x^2+c)/c)^(1/2)*((f*x^
2+e)/e)^(1/2)+9*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*b
^2*c^3*d^3*e^2*f^2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-14*Ell
ipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c^3*d^3*e^3*f*((d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+3*EllipticPi(x*(-d/c)^(1/2),b*c
/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*x^2*b^3*c^5*d*e*f^3*((d*x^2+c)/c)
^(1/2)*((f*x^2+e)/e)^(1/2)-9*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-
f/e)^(1/2)/(-d/c)^(1/2))*x^2*b^3*c^4*d^2*e^2*f^2*((d*x^2+c)/c)^(1
/2)*((f*x^2+e)/e)^(1/2)+9*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e
)^(1/2)/(-d/c)^(1/2))*x^2*b^3*c^3*d^3*e^3*f*((d*x^2+c)/c)^(1/2)*(
(f*x^2+e)/e)^(1/2)+3*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^
2*a^3*c^2*d^4*e*f^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)+7*Ell
ipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^3*c*d^5*e^2*f^2*((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-5*EllipticE(x*(-d/c)^(1/2),(c
*f/d/e)^(1/2))*x^2*a*b^2*c*d^5*e^4*((d*x^2+c)/c)^(1/2)*((f*x^2+e)
/e)^(1/2)+6*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^3*c^2
*d^4*e*f^3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)-7*x^5*a^3*c*d^
5*e*f^3*(-d/c)^(1/2)+6*x^5*a^2*b*c^3*d^3*f^4*(-d/c)^(1/2)-2*x^5*a
^2*b*d^6*e^3*f*(-d/c)^(1/2)-3*x^5*a*b^2*c^4*d^2*f^4*(-d/c)^(1/2)-
8*x^3*a^3*c^2*d^4*e*f^3*(-d/c)^(1/2)-4*x^3*a^3*c*d^5*e^2*f^2*(-d/
c)^(1/2)+12*x^3*a^2*b*c^4*d^2*f^4*(-d/c)^(1/2)-6*x^3*a*b^2*c^5*d*
f^4*(-d/c)^(1/2)+5*x^3*a*b^2*c*d^5*e^4*(-d/c)^(1/2)-8*x*a^3*c^2*d
^4*e^2*f^2*(-d/c)^(1/2)+3*x*a^3*c*d^5*e^3*f*(-d/c)^(1/2)+6*x*a^2*
b*c^5*d*f^4*(-d/c)^(1/2)-14*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1
/2))*x^2*a*b^2*c^2*d^4*e^3*f*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1
/2))/(f*x^2+e)^(1/2)/(c*f-d*e)^3/(a*d-b*c)^2/c^2/(-d/c)^(1/2)/(a*
f-b*e)/e/a/(d*x^2+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)),x, algorithm="maxi
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)),x, algorithm="fric
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)/(d*x**2+c)**(5/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)),x, algorithm="giac
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)), x)
```

3.89 $\int \frac{(1+x^2)^{3/2}\sqrt{2+x^2}}{a+bx^2}\,dx$

Optimal. Leaf size=242

$$-\frac{\sqrt{x^2+2}\,x(a-2b)}{b^2\sqrt{x^2+1}}-\frac{\sqrt{x^2+2}(3a-7b)F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{2}b^2\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}+\frac{\sqrt{2}\sqrt{x^2+2}(a-2b)E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{b^2\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}+\frac{\sqrt{x^2+2}(a-2b)(a-b)\Pi\left(1-\frac{b}{a};\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}ab^2\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}+\frac{\sqrt{x^2+1}\sqrt{x^2+2}\,x}{3b}$$

```
[Out] -(((a - 2*b)*x*Sqrt[2 + x^2])/(b^2*Sqrt[1 + x^2])) + (x*Sqrt[1 +
x^2]*Sqrt[2 + x^2])/(3*b) + (Sqrt[2]*(a - 2*b)*Sqrt[2 + x^2]*Elli
pticE[ArcTan[x], 1/2])/(b^2*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2
)]) - ((3*a - 7*b)*Sqrt[2 + x^2]*EllipticF[ArcTan[x], 1/2])/(3*Sq
rt[2]*b^2*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)]) + ((a - 2*b)*(
a - b)*Sqrt[2 + x^2]*EllipticPi[1 - b/a, ArcTan[x], 1/2])/(Sqrt[2
]*a*b^2*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)])
```

Rubi [A] time = 0.449705, antiderivative size = 239, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{x\sqrt{x^2+2}(a-2b)}{b^2\sqrt{x^2+1}}-\frac{\sqrt{2}\sqrt{x^2+2}(3a-5b)F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3b^2\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}+\frac{\sqrt{2}\sqrt{x^2+2}(a-2b)E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{b^2\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}+\frac{2\sqrt{x^2+1}(a-b)^2\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|-1\right)}{ab^2\sqrt{\frac{x^2+1}{x^2+2}}\sqrt{x^2+2}}+\frac{x\sqrt{x^2+1}\sqrt{x^2+2}}{3b}$$

Antiderivative was successfully verified.

```
[In]  Int[((1 + x^2)^(3/2)*Sqrt[2 + x^2])/(a + b*x^2),x]
```

```
[Out] -(((a - 2*b)*x*Sqrt[2 + x^2])/(b^2*Sqrt[1 + x^2])) + (x*Sqrt[1 +
x^2]*Sqrt[2 + x^2])/(3*b) + (Sqrt[2]*(a - 2*b)*Sqrt[2 + x^2]*Elli
pticE[ArcTan[x], 1/2])/(b^2*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2
)]) - (Sqrt[2]*(3*a - 5*b)*Sqrt[2 + x^2]*EllipticF[ArcTan[x], 1/2
])/(3*b^2*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)]) + (2*(a - b)^2
*Sqrt[1 + x^2]*EllipticPi[1 - (2*b)/a, ArcTan[x/Sqrt[2]], -1])/(a
*b^2*Sqrt[(1 + x^2)/(2 + x^2)]*Sqrt[2 + x^2])
```

Rubi in Sympy [A] time = 63.0948, size = 221, normalized size = 0.91

$$\frac{x\sqrt{x^2+1}\sqrt{x^2+2}}{3b}-\frac{x(a-2b)\sqrt{x^2+2}}{b^2\sqrt{x^2+1}}+\frac{\sqrt{2}(a-2b)\sqrt{x^2+2}E\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{b^2\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^2+1}}-\frac{\sqrt{2}(3a-5b)\sqrt{x^2+2}F\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{3b^2\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^2+1}}+\frac{2\sqrt{2}(a-b)^2\sqrt{x^2+1}\left(1-\frac{2b}{a};\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|-1\right)}{ab^2\sqrt{\frac{2x^2+2}{x^2+2}}\sqrt{x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)**(3/2)*(x**2+2)**(1/2)/(b*x**2+a),x)
```

```
[Out] x*sqrt(x**2 + 1)*sqrt(x**2 + 2)/(3*b) - x*(a - 2*b)*sqrt(x**2 + 2
)/(b**2*sqrt(x**2 + 1)) + sqrt(2)*(a - 2*b)*sqrt(x**2 + 2)*ellipt
ic_e(atan(x), 1/2)/(b**2*sqrt((x**2 + 2)/(x**2 + 1))*sqrt(x**2 +
1)) - sqrt(2)*(3*a - 5*b)*sqrt(x**2 + 2)*elliptic_f(atan(x), 1/2)
/(3*b**2*sqrt((x**2 + 2)/(x**2 + 1))*sqrt(x**2 + 1)) + 2*sqrt(2)*
(a - b)**2*sqrt(x**2 + 1)*elliptic_pi(1 - 2*b/a, atan(sqrt(2)*x/2
), -1)/(a*b**2*sqrt((2*x**2 + 2)/(x**2 + 2))*sqrt(x**2 + 2))
```

Mathematica [C] time = 0.383346, size = 204, normalized size = 0.84

$$\frac{3ia^3\left(\frac{2b}{a};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-ia\left(3a^2-9ab+7b^2\right)F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-12ia^2b\left(\frac{2b}{a};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-6ib^3\left(\frac{2b}{a};i\sinh^{-1}\right.}{3ab^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((1 + x^2)^(3/2)*Sqrt[2 + x^2])/(a + b*x^2),x]
```

```
[Out] (a*b^2*x*Sqrt[1 + x^2]*Sqrt[2 + x^2] + (3*I)*a*(a - 2*b)*b*Ellipt
icE[I*ArcSinh[x/Sqrt[2]], 2] - I*a*(3*a^2 - 9*a*b + 7*b^2)*Ellipt
icF[I*ArcSinh[x/Sqrt[2]], 2] + (3*I)*a^3*EllipticPi[(2*b)/a, I*Ar
cSinh[x/Sqrt[2]], 2] - (12*I)*a^2*b*EllipticPi[(2*b)/a, I*ArcSinh
[x/Sqrt[2]], 2] + (15*I)*a*b^2*EllipticPi[(2*b)/a, I*ArcSinh[x/Sq
rt[2]], 2] - (6*I)*b^3*EllipticPi[(2*b)/a, I*ArcSinh[x/Sqrt[2]],
2])/(3*a*b^3)
```

Maple [C] time = 0.052, size = 370, normalized size = 1.5

$$-\frac{1}{(3x^4+9x^2+6)b^3a}\sqrt{x^2+1}\sqrt{x^2+2}\left(-ab^2x^5+3i\sqrt{x^2+1}\sqrt{x^2+2}EllipticF\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)a^3-9i\sqrt{x^2+1}\sqrt{x^2+2}EllipticF\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)^(3/2)*(x^2+2)^(1/2)/(b*x^2+a),x)
```

```
[Out] -1/3*(x^2+1)^(1/2)*(x^2+2)^(1/2)*(-a*b^2*x^5+3*I*(x^2+1)^(1/2)*(x
^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))*a^3-9*I*(x^2+1)^(1
/2)*(x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))*a^2*b+7*I*(x
^2+1)^(1/2)*(x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(1/2),2^(1/2))*a*b^
2-3*I*(x^2+1)^(1/2)*(x^2+2)^(1/2)*EllipticE(1/2*I*x*2^(1/2),2^(1/
2))*a^2*b+6*I*(x^2+1)^(1/2)*(x^2+2)^(1/2)*EllipticE(1/2*I*x*2^(1/
2),2^(1/2))*a*b^2-3*I*(x^2+1)^(1/2)*(x^2+2)^(1/2)*EllipticPi(1/2*
I*x*2^(1/2),2*b/a,2^(1/2))*a^3+12*I*(x^2+1)^(1/2)*(x^2+2)^(1/2)*E
llipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*a^2*b-15*I*(x^2+1)^(1/2)
*(x^2+2)^(1/2)*EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*a*b^2+6*
I*(x^2+1)^(1/2)*(x^2+2)^(1/2)*EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^
(1/2))*b^3-3*a*b^2*x^3-2*a*b^2*x)/(x^4+3*x^2+2)/b^3/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+2}\,(x^2+1)^{\frac{3}{2}}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^2 + 2)*(x^2 + 1)^(3/2)/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 2)*(x^2 + 1)^(3/2)/(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^2+2}\,(x^2+1)^{\frac{3}{2}}}{bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^2 + 2)*(x^2 + 1)^(3/2)/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + 2)*(x^2 + 1)^(3/2)/(b*x^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)**(3/2)*(x**2+2)**(1/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+2}\left(x^2+1\right)^{\frac{3}{2}}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 2)*(x^2 + 1)^(3/2)/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 2)*(x^2 + 1)^(3/2)/(b*x^2 + a), x)
```

3.90 $\int \frac{\sqrt{1+x^2}\sqrt{2+x^2}}{a+bx^2}\,dx$

Optimal. Leaf size=192

$$-\frac{\sqrt{x^2+2}(a-2b)\,\Pi\left(1-\frac{b}{a};\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}ab\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}+\frac{\sqrt{x^2+2}x}{b\sqrt{x^2+1}}+\frac{\sqrt{x^2+2}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}b\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}-\frac{\sqrt{2}\sqrt{x^2+2}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{b\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}$$

```
[Out] (x*Sqrt[2 + x^2])/(b*Sqrt[1 + x^2]) - (Sqrt[2]*Sqrt[2 + x^2]*Elli
pticE[ArcTan[x], 1/2])/(b*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)]
) + (Sqrt[2 + x^2]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*b*Sqrt[1 +
 x^2]*Sqrt[(2 + x^2)/(1 + x^2)]) - ((a - 2*b)*Sqrt[2 + x^2]*Ellip
ticPi[1 - b/a, ArcTan[x], 1/2])/(Sqrt[2]*a*b*Sqrt[1 + x^2]*Sqrt[(
2 + x^2)/(1 + x^2)])
```

Rubi [A] time = 0.292636, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$-\frac{\sqrt{x^2+2}(a-2b)\,\Pi\left(1-\frac{b}{a};\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}ab\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}+\frac{\sqrt{x^2+2}x}{b\sqrt{x^2+1}}+\frac{\sqrt{x^2+2}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}b\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}-\frac{\sqrt{2}\sqrt{x^2+2}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{b\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[1 + x^2]*Sqrt[2 + x^2])/(a + b*x^2),x]
```

```
[Out] (x*Sqrt[2 + x^2])/(b*Sqrt[1 + x^2]) - (Sqrt[2]*Sqrt[2 + x^2]*Elli
pticE[ArcTan[x], 1/2])/(b*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)]
) + (Sqrt[2 + x^2]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*b*Sqrt[1 +
 x^2]*Sqrt[(2 + x^2)/(1 + x^2)]) - ((a - 2*b)*Sqrt[2 + x^2]*Ellip
ticPi[1 - b/a, ArcTan[x], 1/2])/(Sqrt[2]*a*b*Sqrt[1 + x^2]*Sqrt[(
2 + x^2)/(1 + x^2)])
```

Rubi in Sympy [A] time = 42.3078, size = 165, normalized size = 0.86

$$\frac{x\sqrt{x^2+2}}{b\sqrt{x^2+1}}-\frac{\sqrt{2}\sqrt{x^2+2}E\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{b\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^2+1}}+\frac{\sqrt{2}\sqrt{x^2+2}F\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{2b\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^2+1}}-\frac{\sqrt{2}(a-2b)\sqrt{x^2+2}\,\Pi\left(1-\frac{b}{a};\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{2ab\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+1)**(1/2)*(x**2+2)**(1/2)/(b*x**2+a),x)
```

```
[Out] x*sqrt(x**2 + 2)/(b*sqrt(x**2 + 1)) - sqrt(2)*sqrt(x**2 + 2)*elli
ptic_e(atan(x), 1/2)/(b*sqrt((x**2 + 2)/(x**2 + 1))*sqrt(x**2 + 1
)) + sqrt(2)*sqrt(x**2 + 2)*elliptic_f(atan(x), 1/2)/(2*b*sqrt((x
**2 + 2)/(x**2 + 1))*sqrt(x**2 + 1)) - sqrt(2)*(a - 2*b)*sqrt(x**
2 + 2)*elliptic_pi(1 - b/a, atan(x), 1/2)/(2*a*b*sqrt((x**2 + 2)/
(x**2 + 1))*sqrt(x**2 + 1))
```

Mathematica [C] time = 0.183848, size = 71, normalized size = 0.37

$$\frac{i\left((a-b)\left(aF\left(i\sinh^{-1}(x)|\frac{1}{2}\right)-(a-2b)\left(\frac{b}{a};i\sinh^{-1}(x)|\frac{1}{2}\right)\right)-2abE\left(i\sinh^{-1}(x)|\frac{1}{2}\right)\right)}{\sqrt{2}ab^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[1 + x^2]*Sqrt[2 + x^2])/(a + b*x^2),x]
```

```
[Out] (I*(-2*a*b*EllipticE[I*ArcSinh[x], 1/2] + (a - b)*(a*EllipticF[I*
ArcSinh[x], 1/2] - (a - 2*b)*EllipticPi[b/a, I*ArcSinh[x], 1/2]))
)/(Sqrt[2]*a*b^2)
```

Maple [C] time = 0.015, size = 121, normalized size = 0.6

$$\frac{i}{ab^2}\left(\mathit{EllipticF}\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)a^2-2\,\mathit{EllipticF}\left(i/2x\sqrt{2},\sqrt{2}\right)ba-a^2\mathit{EllipticPi}\left(\frac{i}{2}x\sqrt{2},2\,\frac{b}{a},\sqrt{2}\right)+3\,\mathit{EllipticPi}\left(i/2x\sqrt{2},2\,\frac{b}{a},\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+1)^(1/2)*(x^2+2)^(1/2)/(b*x^2+a),x)
```

```
[Out] I*(EllipticF(1/2*I*x*2^(1/2),2^(1/2))*a^2-2*EllipticF(1/2*I*x*2^(
1/2),2^(1/2))*b*a-a^2*EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))+3
*EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*b*a-2*EllipticPi(1/2*I
*x*2^(1/2),2*b/a,2^(1/2))*b^2-EllipticE(1/2*I*x*2^(1/2),2^(1/2))*
b*a)/a/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^2+2}\sqrt{x^2+1}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2+2}\sqrt{x^2+1}}{bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+1}\sqrt{x^2+2}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+1)**(1/2)*(x**2+2)**(1/2)/(b*x**2+a),x)
```

```
[Out] Integral(sqrt(x**2 + 1)*sqrt(x**2 + 2)/(a + b*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+2}\sqrt{x^2+1}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^2 + a), x)
```

3.91 $\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}(a+bx^2)}\,dx$

Optimal. Leaf size=58

$$\frac{2\sqrt{x^2+1}\,\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{a\sqrt{\frac{x^2+1}{x^2+2}}\sqrt{x^2+2}}$$

```
[Out] (2*Sqrt[1 + x^2]*EllipticPi[1 - (2*b)/a, ArcTan[x/Sqrt[2]], -1])/
(a*Sqrt[(1 + x^2)/(2 + x^2)]*Sqrt[2 + x^2])
```

Rubi [A] time = 0.0756475, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036

$$\frac{2\sqrt{x^2+1}\,\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{a\sqrt{\frac{x^2+1}{x^2+2}}\sqrt{x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2 + x^2]/(Sqrt[1 + x^2]*(a + b*x^2)),x]
```

```
[Out] (2*Sqrt[1 + x^2]*EllipticPi[1 - (2*b)/a, ArcTan[x/Sqrt[2]], -1])/
(a*Sqrt[(1 + x^2)/(2 + x^2)]*Sqrt[2 + x^2])
```

Rubi in Sympy [A] time = 13.3641, size = 58, normalized size = 1.

$$\frac{2\sqrt{2}\sqrt{x^2+1}\,\Pi\left(1-\frac{2b}{a};\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|-1\right)}{a\sqrt{\frac{2x^2+2}{x^2+2}}\sqrt{x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+2)**(1/2)/(x**2+1)**(1/2)/(b*x**2+a),x)
```

```
[Out] 2*sqrt(2)*sqrt(x**2 + 1)*elliptic_pi(1 - 2*b/a, atan(sqrt(2)*x/2)
, -1)/(a*sqrt((2*x**2 + 2)/(x**2 + 2))*sqrt(x**2 + 2))
```

Mathematica [C] time = 0.101301, size = 50, normalized size = 0.86

$$-\frac{i\left(aF\left(i\sinh^{-1}(x)\middle|\frac{1}{2}\right)-(a-2b)\left(\frac{b}{a};i\sinh^{-1}(x)\middle|\frac{1}{2}\right)\right)}{\sqrt{2}ab}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[2 + x^2]/(Sqrt[1 + x^2]*(a + b*x^2)),x]
```

```
[Out] ((-I)*(a*EllipticF[I*ArcSinh[x], 1/2] - (a - 2*b)*EllipticPi[b/a,
 I*ArcSinh[x], 1/2]))/(Sqrt[2]*a*b)
```

Maple [A] time = 0.021, size = 64, normalized size = 1.1

$$\frac{i}{ab}\left(aEllipticPi\left(\frac{i}{2}x\sqrt{2},2\frac{b}{a},\sqrt{2}\right)-2\,bEllipticPi\left(i/2x\sqrt{2},2\frac{b}{a},\sqrt{2}\right)-aEllipticF\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+2)^(1/2)/(x^2+1)^(1/2)/(b*x^2+a),x)
```

```
[Out] I*(a*EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))-2*b*EllipticPi(1/2
*I*x*2^(1/2),2*b/a,2^(1/2))-a*EllipticF(1/2*I*x*2^(1/2),2^(1/2)))
/a/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^2+2}}{(bx^2+a)\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^2 + 2)/((b*x^2 + a)*sqrt(x^2 + 1)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 2)/((b*x^2 + a)*sqrt(x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2+2}}{(bx^2+a)\sqrt{x^2+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 2)/((b*x^2 + a)*sqrt(x^2 + 1)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + 2)/((b*x^2 + a)*sqrt(x^2 + 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+2}}{(a+bx^2)\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+2)**(1/2)/(x**2+1)**(1/2)/(b*x**2+a),x)
```

```
[Out] Integral(sqrt(x**2 + 2)/((a + b*x**2)*sqrt(x**2 + 1)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+2}}{(bx^2+a)\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 2)/((b*x^2 + a)*sqrt(x^2 + 1)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 2)/((b*x^2 + a)*sqrt(x^2 + 1)), x)
```

3.92 $\int \frac{\sqrt{2+x^2}}{(1+x^2)^{3/2}(a+bx^2)}\,dx$

Optimal. Leaf size=121

$$\frac{\sqrt{2}\sqrt{x^2+2}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}(a-b)}-\frac{2b\sqrt{x^2+1}\,\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{a\sqrt{\frac{x^2+1}{x^2+2}}\sqrt{x^2+2}(a-b)}$$

```
[Out] (Sqrt[2]*Sqrt[2 + x^2]*EllipticE[ArcTan[x], 1/2])/((a - b)*Sqrt[1
 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)]) - (2*b*Sqrt[1 + x^2]*EllipticP
i[1 - (2*b)/a, ArcTan[x/Sqrt[2]], -1])/(a*(a - b)*Sqrt[(1 + x^2)/
(2 + x^2)]*Sqrt[2 + x^2])
```

Rubi [A] time = 0.195056, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107

$$\frac{\sqrt{2}\sqrt{x^2+2}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}(a-b)}-\frac{2b\sqrt{x^2+1}\,\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{a\sqrt{\frac{x^2+1}{x^2+2}}\sqrt{x^2+2}(a-b)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2 + x^2]/((1 + x^2)^(3/2)*(a + b*x^2)),x]
```

```
[Out] (Sqrt[2]*Sqrt[2 + x^2]*EllipticE[ArcTan[x], 1/2])/((a - b)*Sqrt[1
 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)]) - (2*b*Sqrt[1 + x^2]*EllipticP
i[1 - (2*b)/a, ArcTan[x/Sqrt[2]], -1])/(a*(a - b)*Sqrt[(1 + x^2)/
(2 + x^2)]*Sqrt[2 + x^2])
```

Rubi in Sympy [A] time = 28.4648, size = 109, normalized size = 0.9

$$\frac{\sqrt{2}\sqrt{x^2+2}E\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{\sqrt{\frac{x^2+2}{x^2+1}}\,(a-b)\,\sqrt{x^2+1}}-\frac{2\sqrt{2}b\sqrt{x^2+1}\,\Pi\left(1-\frac{2b}{a};\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|-1\right)}{a\sqrt{\frac{2x^2+2}{x^2+2}}\,(a-b)\,\sqrt{x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+2)**(1/2)/(x**2+1)**(3/2)/(b*x**2+a),x)
```

```
[Out] sqrt(2)*sqrt(x**2 + 2)*elliptic_e(atan(x), 1/2)/(sqrt((x**2 + 2)/
(x**2 + 1))*(a - b)*sqrt(x**2 + 1)) - 2*sqrt(2)*b*sqrt(x**2 + 1)*
elliptic_pi(1 - 2*b/a, atan(sqrt(2)*x/2), -1)/(a*sqrt((2*x**2 + 2
```

```
)/(x**2 + 2))*(a - b)*sqrt(x**2 + 2))
```

Mathematica [C] time = 0.434484, size = 122, normalized size = 1.01

$$\frac{\frac{2i\sqrt{2}b\Pi\left(\frac{b}{a};i\sinh^{-1}(x)|\frac{1}{2}\right)}{a}-i\sqrt{2}\,\Pi\left(\frac{b}{a};i\sinh^{-1}(x)|\frac{1}{2}\right)+\frac{2\sqrt{x^2+2}x}{\sqrt{x^2+1}}-i\sqrt{2}F\left(i\sinh^{-1}(x)|\frac{1}{2}\right)+2i\sqrt{2}E\left(i\sinh^{-1}(x)|\frac{1}{2}\right)}{2a-2b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[2 + x^2]/((1 + x^2)^(3/2)*(a + b*x^2)),x]
```

```
[Out] ((2*x*Sqrt[2 + x^2])/Sqrt[1 + x^2] + (2*I)*Sqrt[2]*EllipticE[I*Ar
cSinh[x], 1/2] - I*Sqrt[2]*EllipticF[I*ArcSinh[x], 1/2] - I*Sqrt[
2]*EllipticPi[b/a, I*ArcSinh[x], 1/2] + ((2*I)*Sqrt[2]*b*Elliptic
Pi[b/a, I*ArcSinh[x], 1/2])/a)/(2*a - 2*b)
```

Maple [A] time = 0.05, size = 147, normalized size = 1.2

$$\frac{1}{a\left(x^4+3x^2+2\right)(a-b)}\left(iEllipticE\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)a\sqrt{x^2+1}\sqrt{x^2+2}-iEllipticPi\left(\frac{i}{2}x\sqrt{2},2\frac{b}{a},\sqrt{2}\right)a\sqrt{x^2+1}\sqrt{x^2+2}+2\,iEllipt\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+2)^(1/2)/(x^2+1)^(3/2)/(b*x^2+a),x)
```

```
[Out] (I*EllipticE(1/2*I*x*2^(1/2),2^(1/2))*a*(x^2+1)^(1/2)*(x^2+2)^(1/
2)-I*EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*a*(x^2+1)^(1/2)*(x
^2+2)^(1/2)+2*I*EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*b*(x^2+
1)^(1/2)*(x^2+2)^(1/2)+a*x^3+2*a*x)*(x^2+1)^(1/2)*(x^2+2)^(1/2)/a
/(x^4+3*x^2+2)/(a-b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^2+2}}{(bx^2+a)(x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^2 + 2)/((b*x^2 + a)*(x^2 + 1)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 2)/((b*x^2 + a)*(x^2 + 1)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2+2}}{(bx^4+(a+b)x^2+a)\sqrt{x^2+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^2 + 2)/((b*x^2 + a)*(x^2 + 1)^(3/2)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + 2)/((b*x^4 + (a + b)*x^2 + a)*sqrt(x^2 + 1)),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2)**(1/2)/(x**2+1)**(3/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+2}}{(bx^2+a)(x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^2 + 2)/((b*x^2 + a)*(x^2 + 1)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 2)/((b*x^2 + a)*(x^2 + 1)^(3/2)), x)
```

3.93 $\int \frac{\sqrt{2+x^2}}{(1+x^2)^{5/2}(a+bx^2)}\,dx$

Optimal. Leaf size=215

$$\frac{2b^2\sqrt{x^2+1}\,\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{a\sqrt{\frac{x^2+1}{x^2+2}}\sqrt{x^2+2}(a-b)^2}+\frac{x\sqrt{x^2+2}}{3\left(x^2+1\right)^{3/2}(a-b)}-\frac{\sqrt{2}\sqrt{x^2+2}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{3\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}(a-b)}+\frac{\sqrt{2}\sqrt{x^2+2}(a-2b)E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}(a-b)^2}$$

```
[Out] (x*Sqrt[2 + x^2])/(3*(a - b)*(1 + x^2)^(3/2)) + (Sqrt[2]*(a - 2*b
)*Sqrt[2 + x^2]*EllipticE[ArcTan[x], 1/2])/((a - b)^2*Sqrt[1 + x^
2]*Sqrt[(2 + x^2)/(1 + x^2)]) - (Sqrt[2]*Sqrt[2 + x^2]*EllipticF[
ArcTan[x], 1/2])/(3*(a - b)*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2
)]) + (2*b^2*Sqrt[1 + x^2]*EllipticPi[1 - (2*b)/a, ArcTan[x/Sqrt[
2]], -1])/(a*(a - b)^2*Sqrt[(1 + x^2)/(2 + x^2)]*Sqrt[2 + x^2])
```

Rubi [A] time = 0.425159, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$\frac{2b^2\sqrt{x^2+1}\,\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{a\sqrt{\frac{x^2+1}{x^2+2}}\sqrt{x^2+2}(a-b)^2}+\frac{x\sqrt{x^2+2}}{3\left(x^2+1\right)^{3/2}(a-b)}-\frac{\sqrt{2}\sqrt{x^2+2}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{3\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}(a-b)}+\frac{\sqrt{2}\sqrt{x^2+2}(a-2b)E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}(a-b)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2 + x^2]/((1 + x^2)^(5/2)*(a + b*x^2)),x]
```

```
[Out] (x*Sqrt[2 + x^2])/(3*(a - b)*(1 + x^2)^(3/2)) + (Sqrt[2]*(a - 2*b
)*Sqrt[2 + x^2]*EllipticE[ArcTan[x], 1/2])/((a - b)^2*Sqrt[1 + x^
2]*Sqrt[(2 + x^2)/(1 + x^2)]) - (Sqrt[2]*Sqrt[2 + x^2]*EllipticF[
ArcTan[x], 1/2])/(3*(a - b)*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2
)]) + (2*b^2*Sqrt[1 + x^2]*EllipticPi[1 - (2*b)/a, ArcTan[x/Sqrt[
2]], -1])/(a*(a - b)^2*Sqrt[(1 + x^2)/(2 + x^2)]*Sqrt[2 + x^2])
```

Rubi in Sympy [A] time = 55.113, size = 190, normalized size = 0.88

$$\frac{x\sqrt{x^2+2}}{3(a-b)(x^2+1)^{\frac{3}{2}}}+\frac{\sqrt{2}(a-2b)\sqrt{x^2+2}E\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{\sqrt{\frac{x^2+2}{x^2+1}}(a-b)^2\sqrt{x^2+1}}-\frac{\sqrt{2}\sqrt{x^2+2}F\left(\operatorname{atan}(x)\middle|\frac{1}{2}\right)}{3\sqrt{\frac{x^2+2}{x^2+1}}(a-b)\sqrt{x^2+1}}+\frac{2\sqrt{2}b^2\sqrt{x^2+1}\left(1-\frac{2b}{a};\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\middle|-1\right)}{a\sqrt{\frac{2x^2+2}{x^2+2}}(a-b)^2\sqrt{x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+2)**(1/2)/(x**2+1)**(5/2)/(b*x**2+a),x)
```

```
[Out] x*sqrt(x**2 + 2)/(3*(a - b)*(x**2 + 1)**(3/2)) + sqrt(2)*(a - 2*b
)*sqrt(x**2 + 2)*elliptic_e(atan(x), 1/2)/(sqrt((x**2 + 2)/(x**2
+ 1))*(a - b)**2*sqrt(x**2 + 1)) - sqrt(2)*sqrt(x**2 + 2)*ellipti
c_f(atan(x), 1/2)/(3*sqrt((x**2 + 2)/(x**2 + 1))*(a - b)*sqrt(x**
2 + 1)) + 2*sqrt(2)*b**2*sqrt(x**2 + 1)*elliptic_pi(1 - 2*b/a, at
an(sqrt(2)*x/2), -1)/(a*sqrt((2*x**2 + 2)/(x**2 + 2))*(a - b)**2*
sqrt(x**2 + 2))
```

Mathematica [C] time = 0.47772, size = 357, normalized size = 1.66

$$8a^2\sqrt{x^2+1}\sqrt{x^2+2}x+6a^2\sqrt{x^2+1}\sqrt{x^2+2}x^3-6i\sqrt{2}b^2x^4\left(\frac{b}{a};i\sinh^{-1}(x)\middle|\frac{1}{2}\right)-12i\sqrt{2}b^2x^2\left(\frac{b}{a};i\sinh^{-1}(x)\middle|\frac{1}{2}\right)-6i\sqrt{2}b^2\left(\frac{b}{a};i\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[2 + x^2]/((1 + x^2)^(5/2)*(a + b*x^2)),x]
```

```
[Out] (8*a^2*x*Sqrt[1 + x^2]*Sqrt[2 + x^2] - 14*a*b*x*Sqrt[1 + x^2]*Sqr
t[2 + x^2] + 6*a^2*x^3*Sqrt[1 + x^2]*Sqrt[2 + x^2] - 12*a*b*x^3*S
qrt[1 + x^2]*Sqrt[2 + x^2] + (6*I)*Sqrt[2]*a*(a - 2*b)*(1 + x^2)^
2*EllipticE[I*ArcSinh[x], 1/2] - I*Sqrt[2]*a*(4*a - 7*b)*(1 + x^2
)^2*EllipticF[I*ArcSinh[x], 1/2] + (3*I)*Sqrt[2]*a*b*EllipticPi[b
/a, I*ArcSinh[x], 1/2] - (6*I)*Sqrt[2]*b^2*EllipticPi[b/a, I*ArcS
inh[x], 1/2] + (6*I)*Sqrt[2]*a*b*x^2*EllipticPi[b/a, I*ArcSinh[x]
, 1/2] - (12*I)*Sqrt[2]*b^2*x^2*EllipticPi[b/a, I*ArcSinh[x], 1/2
] + (3*I)*Sqrt[2]*a*b*x^4*EllipticPi[b/a, I*ArcSinh[x], 1/2] - (6
*I)*Sqrt[2]*b^2*x^4*EllipticPi[b/a, I*ArcSinh[x], 1/2])/(6*a*(a -
 b)^2*(1 + x^2)^2)
```

Maple [B] time = 0.051, size = 477, normalized size = 2.2

$$-\frac{1}{3(a-b)^2a}\left(-3\,iEllipticE\left(\frac{i}{2}x\sqrt{2},\sqrt{2}\right)a^2\sqrt{x^2+2}\sqrt{x^2+1}+6\,iEllipticPi\left(\frac{i}{2}x\sqrt{2},2\,\frac{b}{a},\sqrt{2}\right)x^2b^2\sqrt{x^2+2}\sqrt{x^2+1}+6\,iEllipti\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+2)^(1/2)/(x^2+1)^(5/2)/(b*x^2+a),x)
```

```
[Out] -1/3*(-3*I*EllipticE(1/2*I*x*2^(1/2),2^(1/2))*a^2*(x^2+2)^(1/2)*(
x^2+1)^(1/2)+6*I*EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*x^2*b^
2*(x^2+2)^(1/2)*(x^2+1)^(1/2)+6*I*EllipticE(1/2*I*x*2^(1/2),2^(1/
2))*x^2*a*b*(x^2+2)^(1/2)*(x^2+1)^(1/2)+6*I*EllipticPi(1/2*I*x*2^
(1/2),2*b/a,2^(1/2))*b^2*(x^2+2)^(1/2)*(x^2+1)^(1/2)-3*I*Elliptic
E(1/2*I*x*2^(1/2),2^(1/2))*x^2*a^2*(x^2+2)^(1/2)*(x^2+1)^(1/2)-I*
EllipticF(1/2*I*x*2^(1/2),2^(1/2))*a*b*(x^2+2)^(1/2)*(x^2+1)^(1/2
)-3*x^5*a^2+6*x^5*a*b+I*EllipticF(1/2*I*x*2^(1/2),2^(1/2))*a^2*(x
^2+2)^(1/2)*(x^2+1)^(1/2)+6*I*EllipticE(1/2*I*x*2^(1/2),2^(1/2))*
a*b*(x^2+2)^(1/2)*(x^2+1)^(1/2)-3*I*EllipticPi(1/2*I*x*2^(1/2),2*
b/a,2^(1/2))*x^2*a*b*(x^2+2)^(1/2)*(x^2+1)^(1/2)-3*I*EllipticPi(1
/2*I*x*2^(1/2),2*b/a,2^(1/2))*a*b*(x^2+2)^(1/2)*(x^2+1)^(1/2)-I*E
llipticF(1/2*I*x*2^(1/2),2^(1/2))*x^2*a*b*(x^2+2)^(1/2)*(x^2+1)^(
1/2)+I*EllipticF(1/2*I*x*2^(1/2),2^(1/2))*x^2*a^2*(x^2+2)^(1/2)*(
x^2+1)^(1/2)-10*x^3*a^2+19*a*b*x^3-8*x*a^2+14*a*b*x)/(x^2+2)^(1/2
)/(a-b)^2/a/(x^2+1)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+2}}{(bx^2+a)(x^2+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^2 + 2)/((b*x^2 + a)*(x^2 + 1)^(5/2)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 2)/((b*x^2 + a)*(x^2 + 1)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2+2}}{(bx^6+(a+2\,b)x^4+(2\,a+b)x^2+a)\sqrt{x^2+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^2 + 2)/((b*x^2 + a)*(x^2 + 1)^(5/2)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + 2)/((b*x^6 + (a + 2*b)*x^4 + (2*a + b)*x^2 +
a)*sqrt(x^2 + 1)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+2)**(1/2)/(x**2+1)**(5/2)/(b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+2}}{(bx^2+a)(x^2+1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^2 + 2)/((b*x^2 + a)*(x^2 + 1)^(5/2)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 2)/((b*x^2 + a)*(x^2 + 1)^(5/2)), x)
```

3.94 $\int \frac{\sqrt{2+dx^2}\sqrt{3+fx^2}}{a+bx^2}\,dx$

Optimal. Leaf size=298

$$\frac{3\sqrt{dx^2+2}(2b-ad)\left(1-\frac{3b}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{\sqrt{2}ab\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}+\frac{fx\sqrt{dx^2+2}}{b\sqrt{fx^2+3}}+\frac{3d\sqrt{dx^2+2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{\sqrt{2}b\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}-\frac{\sqrt{2}\sqrt{f}\sqrt{dx^2+2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{b\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}$$

```
[Out] (f*x*Sqrt[2 + d*x^2])/(b*Sqrt[3 + f*x^2]) - (Sqrt[2]*Sqrt[f]*Sqrt
[2 + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f
)])/(b*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2]) + (3*d*Sqrt
[2 + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f
)])/(Sqrt[2]*b*Sqrt[f]*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x
^2]) + (3*(2*b - a*d)*Sqrt[2 + d*x^2]*EllipticPi[1 - (3*b)/(a*f),
 ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(Sqrt[2]*a*b*Sqrt
[f]*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2])
```

Rubi [A] time = 0.587406, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$\frac{3\sqrt{dx^2+2}(2b-ad)\left(1-\frac{3b}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{\sqrt{2}ab\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}+\frac{fx\sqrt{dx^2+2}}{b\sqrt{fx^2+3}}+\frac{3d\sqrt{dx^2+2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{\sqrt{2}b\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}-\frac{\sqrt{2}\sqrt{f}\sqrt{dx^2+2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{b\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2])/(a + b*x^2),x]
```


```
[Out] (f*x*Sqrt[2 + d*x^2])/(b*Sqrt[3 + f*x^2]) - (Sqrt[2]*Sqrt[f]*Sqrt
[2 + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f
)])/(b*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2]) + (3*d*Sqrt
[2 + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f
)])/(Sqrt[2]*b*Sqrt[f]*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x
^2]) + (3*(2*b - a*d)*Sqrt[2 + d*x^2]*EllipticPi[1 - (3*b)/(a*f),
 ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(Sqrt[2]*a*b*Sqrt
[f]*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2])
```

Rubi in Sympy [A] time = 66.3375, size = 277, normalized size = 0.93

$$-\frac{\sqrt{2}\sqrt{d}\sqrt{fx^2+3}E\left(\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{d}x}{2}\right)\middle|1-\frac{2f}{3d}\right)}{b\sqrt{\frac{2fx^2+6}{3dx^2+6}}\sqrt{dx^2+2}}+\frac{dx\sqrt{fx^2+3}}{b\sqrt{dx^2+2}}+\frac{3\sqrt{3}d\sqrt{dx^2+2}F\left(\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{f}x}{3}\right)\middle|-\frac{3d}{2f}+1\right)}{2b\sqrt{f}\sqrt{\frac{3dx^2+6}{2fx^2+6}}\sqrt{fx^2+3}}+\frac{3\sqrt{3}(-ad+2b)\sqrt{dx^2+2}\left(1-\frac{3b}{af};\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{f}x}{3}\right)\middle|-\frac{3d}{2f}+1\right)}{2ab\sqrt{f}\sqrt{\frac{3dx^2+6}{2fx^2+6}}\sqrt{fx^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+2)**(1/2)*(f*x**2+3)**(1/2)/(b*x**2+a),x)
```

```
[Out] -sqrt(2)*sqrt(d)*sqrt(f*x**2 + 3)*elliptic_e(atan(sqrt(2)*sqrt(d)
*x/2), 1 - 2*f/(3*d))/(b*sqrt((2*f*x**2 + 6)/(3*d*x**2 + 6))*sqrt
(d*x**2 + 2)) + d*x*sqrt(f*x**2 + 3)/(b*sqrt(d*x**2 + 2)) + 3*sqr
t(3)*d*sqrt(d*x**2 + 2)*elliptic_f(atan(sqrt(3)*sqrt(f)*x/3), -3*
d/(2*f) + 1)/(2*b*sqrt(f)*sqrt((3*d*x**2 + 6)/(2*f*x**2 + 6))*sqr
t(f*x**2 + 3)) + 3*sqrt(3)*(-a*d + 2*b)*sqrt(d*x**2 + 2)*elliptic
_pi(1 - 3*b/(a*f), atan(sqrt(3)*sqrt(f)*x/3), -3*d/(2*f) + 1)/(2*
a*b*sqrt(f)*sqrt((3*d*x**2 + 6)/(2*f*x**2 + 6))*sqrt(f*x**2 + 3))
```

Mathematica [C] time = 0.284124, size = 134, normalized size = 0.45

$$\frac{i\left((ad-2b)\left((3b-af)\left(\frac{2b}{ad};i\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)+afF\left(i\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)\right)-3abdE\left(i\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)\right)}{\sqrt{3}ab^2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2])/(a + b*x^2),x]
```

```
[Out] (I*(-3*a*b*d*EllipticE[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d
)] + (-2*b + a*d)*(a*f*EllipticF[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]],
(2*f)/(3*d)] + (3*b - a*f)*EllipticPi[(2*b)/(a*d), I*ArcSinh[(Sqr
t[d]*x)/Sqrt[2]], (2*f)/(3*d)])))/(Sqrt[3]*a*b^2*Sqrt[d])
```

Maple [A] time = 0.043, size = 293, normalized size = 1.

$$-\frac{\sqrt{2}}{2\,ab^{2}}\left(EllipticF\left(\frac{x\sqrt{3}}{3}\sqrt{-f},\frac{\sqrt{3}\sqrt{2}}{2}\sqrt{\frac{d}{f}}\right)a^{2}df-a^{2}EllipticPi\left(\frac{x\sqrt{3}}{3}\sqrt{-f},3\,\frac{b}{af},\frac{\sqrt{3}\sqrt{2}}{2}\sqrt{-d}\frac{1}{\sqrt{-f}}\right)df-3\,EllipticF\left(1/3\,x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)/(b*x^2+a),x)
```

```
[Out] -1/2*(EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*3^(1/2)*2^(1/2)*(1/f
*d)^(1/2))*a^2*d*f-a^2*EllipticPi(1/3*x*3^(1/2)*(-f)^(1/2),3*b/a/
f,1/2*2^(1/2)*(-d)^(1/2)*3^(1/2)/(-f)^(1/2))*d*f-3*EllipticF(1/3*
x*3^(1/2)*(-f)^(1/2),1/2*3^(1/2)*2^(1/2)*(1/f*d)^(1/2))*d*b*a-2*f
*EllipticE(1/3*x*3^(1/2)*(-f)^(1/2),1/2*3^(1/2)*2^(1/2)*(1/f*d)^(
1/2))*b*a+3*EllipticPi(1/3*x*3^(1/2)*(-f)^(1/2),3*b/a/f,1/2*2^(1/
2)*(-d)^(1/2)*3^(1/2)/(-f)^(1/2))*d*b*a+2*EllipticPi(1/3*x*3^(1/2
)*(-f)^(1/2),3*b/a/f,1/2*2^(1/2)*(-d)^(1/2)*3^(1/2)/(-f)^(1/2))*f
*b*a-6*EllipticPi(1/3*x*3^(1/2)*(-f)^(1/2),3*b/a/f,1/2*2^(1/2)*(-
d)^(1/2)*3^(1/2)/(-f)^(1/2))*b^2)*2^(1/2)/a/(-f)^(1/2)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+2}\sqrt{fx^2+3}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)/(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)/(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^2+2}\sqrt{fx^2+3}}{bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)/(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)/(b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+2}\sqrt{fx^2+3}}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+2)**(1/2)*(f*x**2+3)**(1/2)/(b*x**2+a),x)
```

```
[Out] Integral(sqrt(d*x**2 + 2)*sqrt(f*x**2 + 3)/(a + b*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+2}\sqrt{fx^2+3}}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)/(b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)/(b*x^2 + a), x)
```

3.95 $\int \frac{\sqrt{2+dx^2}}{(a+bx^2)\sqrt{3+fx^2}}\,dx$

Optimal. Leaf size=93

$$\frac{2\sqrt{fx^2+3}\,\Pi\left(1-\frac{2b}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|1-\frac{2f}{3d}\right)}{\sqrt{3}a\sqrt{d}\sqrt{dx^2+2}\sqrt{\frac{fx^2+3}{dx^2+2}}}$$

```
[Out] (2*Sqrt[3 + f*x^2]*EllipticPi[1 - (2*b)/(a*d), ArcTan[(Sqrt[d]*x)
/Sqrt[2]], 1 - (2*f)/(3*d)])/(Sqrt[3]*a*Sqrt[d]*Sqrt[2 + d*x^2]*S
qrt[(3 + f*x^2)/(2 + d*x^2)])
```

Rubi [A] time = 0.13467, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$

$$\frac{2\sqrt{fx^2+3}\,\Pi\left(1-\frac{2b}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|1-\frac{2f}{3d}\right)}{\sqrt{3}a\sqrt{d}\sqrt{dx^2+2}\sqrt{\frac{fx^2+3}{dx^2+2}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2 + d*x^2]/((a + b*x^2)*Sqrt[3 + f*x^2]),x]
```

```
[Out] (2*Sqrt[3 + f*x^2]*EllipticPi[1 - (2*b)/(a*d), ArcTan[(Sqrt[d]*x)
/Sqrt[2]], 1 - (2*f)/(3*d)])/(Sqrt[3]*a*Sqrt[d]*Sqrt[2 + d*x^2]*S
qrt[(3 + f*x^2)/(2 + d*x^2)])
```

Rubi in Sympy [A] time = 17.2507, size = 85, normalized size = 0.91

$$\frac{2\sqrt{2}\sqrt{fx^2+3}\,\Pi\left(1-\frac{2b}{ad};\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{d}x}{2}\right)\middle|1-\frac{2f}{3d}\right)}{3a\sqrt{d}\sqrt{\frac{2fx^2+6}{3dx^2+6}}\sqrt{dx^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x**2+2)**(1/2)/(b*x**2+a)/(f*x**2+3)**(1/2),x)
```

```
[Out] 2*sqrt(2)*sqrt(f*x**2 + 3)*elliptic_pi(1 - 2*b/(a*d), atan(sqrt(2
)*sqrt(d)*x/2), 1 - 2*f/(3*d))/(3*a*sqrt(d)*sqrt((2*f*x**2 + 6)/(
3*d*x**2 + 6))*sqrt(d*x**2 + 2))
```

Mathematica [C] time = 0.110367, size = 94, normalized size = 1.01

$$-\frac{i\left((2b-ad)\left(\frac{2b}{ad};i\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)+adF\left(i\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)\right)}{\sqrt{3}ab\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[2 + d*x^2]/((a + b*x^2)*Sqrt[3 + f*x^2]),x]
```

```
[Out] ((-I)*(a*d*EllipticF[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)]
 + (2*b - a*d)*EllipticPi[(2*b)/(a*d), I*ArcSinh[(Sqrt[d]*x)/Sqrt
[2]], (2*f)/(3*d)]))/(Sqrt[3]*a*b*Sqrt[d])
```

Maple [A] time = 0.032, size = 133, normalized size = 1.4

$$\frac{\sqrt{2}}{2\,ab}\left(EllipticF\left(\frac{x\sqrt{3}}{3}\sqrt{-f},\frac{\sqrt{3}\sqrt{2}}{2}\sqrt{\frac{d}{f}}\right)ad-EllipticPi\left(\frac{x\sqrt{3}}{3}\sqrt{-f},3\,\frac{b}{af},\frac{\sqrt{3}\sqrt{2}}{2}\sqrt{-d}\frac{1}{\sqrt{-f}}\right)ad+2\,EllipticPi\left(1/3\,x\sqrt{3}\sqrt{-f}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+2)^(1/2)/(b*x^2+a)/(f*x^2+3)^(1/2),x)
```

```
[Out] 1/2*2^(1/2)*(EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*3^(1/2)*2^(1/
2)*(1/f*d)^(1/2))*a*d-EllipticPi(1/3*x*3^(1/2)*(-f)^(1/2),3*b/a/f
,1/2*2^(1/2)*(-d)^(1/2)*3^(1/2)/(-f)^(1/2))*a*d+2*EllipticPi(1/3*
x*3^(1/2)*(-f)^(1/2),3*b/a/f,1/2*2^(1/2)*(-d)^(1/2)*3^(1/2)/(-f)^
(1/2))*b)/(-f)^(1/2)/a/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+2}}{(bx^2+a)\sqrt{fx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + 2)/((b*x^2 + a)*sqrt(f*x^2 + 3)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + 2)/((b*x^2 + a)*sqrt(f*x^2 + 3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + 2)/((b*x^2 + a)*sqrt(f*x^2 + 3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+2}}{(a+bx^2)\sqrt{fx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+2)**(1/2)/(b*x**2+a)/(f*x**2+3)**(1/2),x)
```

```
[Out] Integral(sqrt(d*x**2 + 2)/((a + b*x**2)*sqrt(f*x**2 + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+2}}{(bx^2+a)\sqrt{fx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + 2)/((b*x^2 + a)*sqrt(f*x^2 + 3)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + 2)/((b*x^2 + a)*sqrt(f*x^2 + 3)), x)
```

3.96 $\int \frac{1}{(a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx$

Optimal. Leaf size=49

$$\frac{\left(\frac{2b}{ad};\sin^{-1}\left(\frac{\sqrt{-d}x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)}{\sqrt{3}a\sqrt{-d}}$$

```
[Out] EllipticPi[(2*b)/(a*d), ArcSin[(Sqrt[-d]*x)/Sqrt[2]], (2*f)/(3*d)
]/(Sqrt[3]*a*Sqrt[-d])
```

Rubi [A] time = 0.146693, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031

$$\frac{\left(\frac{2b}{ad};\sin^{-1}\left(\frac{\sqrt{-d}x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)}{\sqrt{3}a\sqrt{-d}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2]),x]
```

```
[Out] EllipticPi[(2*b)/(a*d), ArcSin[(Sqrt[-d]*x)/Sqrt[2]], (2*f)/(3*d)
]/(Sqrt[3]*a*Sqrt[-d])
```

Rubi in Sympy [A] time = 41.6285, size = 177, normalized size = 3.61

$$-\frac{\sqrt{3}\sqrt{f}\sqrt{dx^2+2}F\left(\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{f}x}{3}\right)\middle|-\frac{3d}{2f}+1\right)}{2\sqrt{\frac{3dx^2+6}{2fx^2+6}}\left(-af+3b\right)\sqrt{fx^2+3}}+\frac{3\sqrt{3}b\sqrt{dx^2+2}\left(1-\frac{3b}{af};\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{f}x}{3}\right)\middle|-\frac{3d}{2f}+1\right)}{2a\sqrt{f}\sqrt{\frac{3dx^2+6}{2fx^2+6}}\left(-af+3b\right)\sqrt{fx^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)/(d*x**2+2)**(1/2)/(f*x**2+3)**(1/2),x)
```

```
[Out] -sqrt(3)*sqrt(f)*sqrt(d*x**2 + 2)*elliptic_f(atan(sqrt(3)*sqrt(f)
*x/3), -3*d/(2*f) + 1)/(2*sqrt((3*d*x**2 + 6)/(2*f*x**2 + 6))*(-a
*f + 3*b)*sqrt(f*x**2 + 3)) + 3*sqrt(3)*b*sqrt(d*x**2 + 2)*ellipt
ic_pi(1 - 3*b/(a*f), atan(sqrt(3)*sqrt(f)*x/3), -3*d/(2*f) + 1)/(
2*a*sqrt(f)*sqrt((3*d*x**2 + 6)/(2*f*x**2 + 6))*(-a*f + 3*b)*sqrt
(f*x**2 + 3))
```

Mathematica [C] time = 0.0988031, size = 52, normalized size = 1.06

$$-\frac{i\left(\frac{2b}{ad};i\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)}{\sqrt{3}a\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2]),x]
```

```
[Out] ((-I)*EllipticPi[(2*b)/(a*d), I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*
f)/(3*d)])/(Sqrt[3]*a*Sqrt[d])
```

Maple [A] time = 0.032, size = 53, normalized size = 1.1

$$\frac{\sqrt{2}}{2\,a}\mathit{EllipticPi}\left(\frac{x\sqrt{3}}{3}\sqrt{-f},3\,\frac{b}{af},\frac{\sqrt{3}\sqrt{2}}{2}\sqrt{-d}\frac{1}{\sqrt{-f}}\right)\frac{1}{\sqrt{-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x)
```

```
[Out] 1/2*2^(1/2)*EllipticPi(1/3*x*3^(1/2)*(-f)^(1/2),3*b/a/f,1/2*2^(1/
2)*(-d)^(1/2)*3^(1/2)/(-f)^(1/2))/(-f)^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)\sqrt{dx^2+2}\sqrt{fx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^2)\sqrt{dx^2+2}\sqrt{fx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**2+2)**(1/2)/(f*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*sqrt(d*x**2 + 2)*sqrt(f*x**2 + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)\sqrt{dx^2+2}\sqrt{fx^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)), x)
```

3.97 $\int \frac{\sqrt{1-x^2}}{(-1+x^2)\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=36

$$-\frac{\sqrt{\frac{bx^2}{a}+1}F\left(\sin^{-1}(x)|-\frac{b}{a}\right)}{\sqrt{a+bx^2}}$$

```
[Out] -((Sqrt[1 + (b*x^2)/a]*EllipticF[ArcSin[x], -(b/a)])/Sqrt[a + b*x
^2])
```

Rubi [A] time = 0.0888049, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{\sqrt{\frac{bx^2}{a}+1}F\left(\sin^{-1}(x)|-\frac{b}{a}\right)}{\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - x^2]/((-1 + x^2)*Sqrt[a + b*x^2]),x]
```

```
[Out] -((Sqrt[1 + (b*x^2)/a]*EllipticF[ArcSin[x], -(b/a)])/Sqrt[a + b*x
^2])
```

Rubi in Sympy [A] time = 19.6536, size = 31, normalized size = 0.86

$$-\frac{\sqrt{1+\frac{bx^2}{a}}F\left(\operatorname{asin}(x)|-\frac{b}{a}\right)}{\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+1)**(1/2)/(x**2-1)/(b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(1 + b*x**2/a)*elliptic_f(asin(x), -b/a)/sqrt(a + b*x**2)
```

Mathematica [A] time = 0.0699204, size = 37, normalized size = 1.03

$$-\frac{\sqrt{\frac{a+bx^2}{a}}F\left(\sin^{-1}(x)|-\frac{b}{a}\right)}{\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - x^2]/((-1 + x^2)*Sqrt[a + b*x^2]),x]
```

```
[Out] -((Sqrt[(a + b*x^2)/a]*EllipticF[ArcSin[x], -(b/a)])/Sqrt[a + b*x
^2])
```

Maple [A] time = 0.042, size = 35, normalized size = 1.

$$-1\sqrt{\frac{bx^2+a}{a}}\mathit{EllipticF}\left(x,\sqrt{-\frac{b}{a}}\right)\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+1)^(1/2)/(x^2-1)/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/(b*x^2+a)^(1/2)*((b*x^2+a)/a)^(1/2)*EllipticF(x,(-b/a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}}{\sqrt{bx^2+a}(x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^2 + 1)/(sqrt(b*x^2 + a)*(x^2 - 1)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/(sqrt(b*x^2 + a)*(x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^2+1}}{\sqrt{bx^2+a}(x^2-1)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-x^2 + 1)/(sqrt(b*x^2 + a)*(x^2 - 1)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 + 1)/(sqrt(b*x^2 + a)*(x^2 - 1)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{a+bx^2}(x-1)(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+1)**(1/2)/(x**2-1)/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/(sqrt(a + b*x**2)*(x - 1)*(x + 1)
), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}}{\sqrt{bx^2+a}(x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 1)/(sqrt(b*x^2 + a)*(x^2 - 1)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + 1)/(sqrt(b*x^2 + a)*(x^2 - 1)), x)
```

3.98 $\int \frac{a+bx^2}{\sqrt{c+dx^2}(e+fx^2)^2}\,dx$

Optimal. Leaf size=113

$$\frac{x\sqrt{c+dx^2}(be-af)}{2e\left(e+fx^2\right)(de-cf)}-\frac{(acf-2ade+bce)\tanh^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{e}\sqrt{c+dx^2}}\right)}{2e^{3/2}(de-cf)^{3/2}}$$

```
[Out] ((b*e - a*f)*x*Sqrt[c + d*x^2])/(2*e*(d*e - c*f)*(e + f*x^2)) - (
(b*c*e - 2*a*d*e + a*c*f)*ArcTanh[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sq
rt[c + d*x^2])])/(2*e^(3/2)*(d*e - c*f)^(3/2))
```

Rubi [A] time = 0.336599, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{x\sqrt{c+dx^2}(be-af)}{2e\left(e+fx^2\right)(de-cf)}-\frac{(acf-2ade+bce)\tanh^{-1}\left(\frac{x\sqrt{de-cf}}{\sqrt{e}\sqrt{c+dx^2}}\right)}{2e^{3/2}(de-cf)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2)/(Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

```
[Out] ((b*e - a*f)*x*Sqrt[c + d*x^2])/(2*e*(d*e - c*f)*(e + f*x^2)) - (
(b*c*e - 2*a*d*e + a*c*f)*ArcTanh[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sq
rt[c + d*x^2])])/(2*e^(3/2)*(d*e - c*f)^(3/2))
```

Rubi in Sympy [A] time = 34.0952, size = 97, normalized size = 0.86

$$\frac{x\sqrt{c+dx^2}\,(af-be)}{2e\left(e+fx^2\right)(cf-de)}+\frac{(acf-2ade+bce)\,\mathrm{atan}\left(\frac{x\sqrt{cf-de}}{\sqrt{e}\sqrt{c+dx^2}}\right)}{2e^{\frac{3}{2}}\,(cf-de)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)/(f*x**2+e)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] x*sqrt(c + d*x**2)*(a*f - b*e)/(2*e*(e + f*x**2)*(c*f - d*e)) + (
a*c*f - 2*a*d*e + b*c*e)*atan(x*sqrt(c*f - d*e)/(sqrt(e)*sqrt(c +
 d*x**2)))/(2*e**(3/2)*(c*f - d*e)**(3/2))
```

Mathematica [A] time = 0.236834, size = 114, normalized size = 1.01

$$\frac{\frac{\sqrt{e}x\sqrt{c+dx^2}(be-af)}{e+fx^2}-\frac{(acf-2ade+bce)\tan^{-1}\left(\frac{x\sqrt{cf-de}}{\sqrt{e}\sqrt{c+dx^2}}\right)}{\sqrt{cf-de}}}{2e^{3/2}(de-cf)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2)/(Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

```
[Out] ((Sqrt[e]*(b*e - a*f)*x*Sqrt[c + d*x^2])/(e + f*x^2) - ((b*c*e -
2*a*d*e + a*c*f)*ArcTan[(Sqrt[-(d*e) + c*f]*x)/(Sqrt[e]*Sqrt[c +
d*x^2])])/Sqrt[-(d*e) + c*f])/(2*e^(3/2)*(d*e - c*f))
```

Maple [B] time = 0.049, size = 1622, normalized size = 14.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)/(f*x^2+e)^2/(d*x^2+c)^(1/2),x)
```

```
[Out] 1/4/e/(c*f-d*e)/(x-(-e*f)^(1/2)/f)*((x-(-e*f)^(1/2)/f)^2*d+2*d*(-
e*f)^(1/2)/f*(x-(-e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2)*a-1/4/f/(c*f-d
*e)/(x-(-e*f)^(1/2)/f)*((x-(-e*f)^(1/2)/f)^2*d+2*d*(-e*f)^(1/2)/f
*(x-(-e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2)*b-1/4/e/f*d*(-e*f)^(1/2)/(
c*f-d*e)/((c*f-d*e)/f)^(1/2)*ln((2*(c*f-d*e)/f+2*d*(-e*f)^(1/2)/f
*(x-(-e*f)^(1/2)/f)+2*((c*f-d*e)/f)^(1/2)*((x-(-e*f)^(1/2)/f)^2*d
+2*d*(-e*f)^(1/2)/f*(x-(-e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2))/(x-(-e
*f)^(1/2)/f))*a+1/4/f^2*d*(-e*f)^(1/2)/(c*f-d*e)/((c*f-d*e)/f)^(1
/2)*ln((2*(c*f-d*e)/f+2*d*(-e*f)^(1/2)/f*(x-(-e*f)^(1/2)/f)+2*((c
*f-d*e)/f)^(1/2)*((x-(-e*f)^(1/2)/f)^2*d+2*d*(-e*f)^(1/2)/f*(x-(-
e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2))/(x-(-e*f)^(1/2)/f))*b+1/4/e/(c*
f-d*e)/(x+(-e*f)^(1/2)/f)*((x+(-e*f)^(1/2)/f)^2*d-2*d*(-e*f)^(1/2
)/f*(x+(-e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2)*a-1/4/f/(c*f-d*e)/(x+(-
e*f)^(1/2)/f)*((x+(-e*f)^(1/2)/f)^2*d-2*d*(-e*f)^(1/2)/f*(x+(-e*f
)^(1/2)/f)+(c*f-d*e)/f)^(1/2)*b+1/4/e/f*d*(-e*f)^(1/2)/(c*f-d*e)/
((c*f-d*e)/f)^(1/2)*ln((2*(c*f-d*e)/f-2*d*(-e*f)^(1/2)/f*(x+(-e*f
)^(1/2)/f)+2*((c*f-d*e)/f)^(1/2)*((x+(-e*f)^(1/2)/f)^2*d-2*d*(-e*
f)^(1/2)/f*(x+(-e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2))/(x+(-e*f)^(1/2)
/f))*a-1/4/f^2*d*(-e*f)^(1/2)/(c*f-d*e)/((c*f-d*e)/f)^(1/2)*ln((2
*(c*f-d*e)/f-2*d*(-e*f)^(1/2)/f*(x+(-e*f)^(1/2)/f)+2*((c*f-d*e)/f
)^(1/2)*((x+(-e*f)^(1/2)/f)^2*d-2*d*(-e*f)^(1/2)/f*(x+(-e*f)^(1/2
)/f)+(c*f-d*e)/f)^(1/2))/(x+(-e*f)^(1/2)/f))*b+1/4/e/(-e*f)^(1/2)
/((c*f-d*e)/f)^(1/2)*ln((2*(c*f-d*e)/f-2*d*(-e*f)^(1/2)/f*(x+(-e*
f)^(1/2)/f)+2*((c*f-d*e)/f)^(1/2)*((x+(-e*f)^(1/2)/f)^2*d-2*d*(-e
*f)^(1/2)/f*(x+(-e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2))/(x+(-e*f)^(1/2
)/f))*a+1/4/(-e*f)^(1/2)/f/((c*f-d*e)/f)^(1/2)*ln((2*(c*f-d*e)/f-
2*d*(-e*f)^(1/2)/f*(x+(-e*f)^(1/2)/f)+2*((c*f-d*e)/f)^(1/2)*((x+(
```

```
-e*f)^(1/2)/f)^2*d-2*d*(-e*f)^(1/2)/f*(x+(-e*f)^(1/2)/f)+(c*f-d*e
)/f)^(1/2))/(x+(-e*f)^(1/2)/f))*b-1/4/e/(-e*f)^(1/2)/((c*f-d*e)/f
)^(1/2)*ln((2*(c*f-d*e)/f+2*d*(-e*f)^(1/2)/f*(x-(-e*f)^(1/2)/f)+2
*((c*f-d*e)/f)^(1/2)*((x-(-e*f)^(1/2)/f)^2*d+2*d*(-e*f)^(1/2)/f*(
x-(-e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2))/(x-(-e*f)^(1/2)/f))*a-1/4/(
-e*f)^(1/2)/f/((c*f-d*e)/f)^(1/2)*ln((2*(c*f-d*e)/f+2*d*(-e*f)^(1
/2)/f*(x-(-e*f)^(1/2)/f)+2*((c*f-d*e)/f)^(1/2)*((x-(-e*f)^(1/2)/f
)^2*d+2*d*(-e*f)^(1/2)/f*(x-(-e*f)^(1/2)/f)+(c*f-d*e)/f)^(1/2))/(
x-(-e*f)^(1/2)/f))*b
```

<u>Maxima</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^2 + a}{\sqrt{dx^2 + c}(fx^2 + e)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)
```

<u>Fricas</u> [A] time = 1.6132, size = 1, normalized size = 0.01

$$\left[\frac{4\sqrt{de^2 - cef}\sqrt{dx^2 + c}(be - af)x - \left(acef + (bc - 2\,ad)e^2 + \left(acf^2 + (bc - 2\,ad)ef\right)x^2\right)\log\left(\frac{\left((8\,d^2e^2 - 8\,cdef + c^2f^2\right)x^4 + c^2e^2 + 2\,(4}{}\right.}{8\,(de^3 - ce^2f + (de^2f - cef^2)x^2)\sqrt{de^2 - cef}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(d*e^2 - c*e*f)*sqrt(d*x^2 + c)*(b*e - a*f)*x - (a*c*
e*f + (b*c - 2*a*d)*e^2 + (a*c*f^2 + (b*c - 2*a*d)*e*f)*x^2)*log(
(((8*d^2*e^2 - 8*c*d*e*f + c^2*f^2)*x^4 + c^2*e^2 + 2*(4*c*d*e^2
- 3*c^2*e*f)*x^2)*sqrt(d*e^2 - c*e*f) + 4*((2*d^2*e^3 - 3*c*d*e^2
*f + c^2*e*f^2)*x^3 + (c*d*e^3 - c^2*e^2*f)*x)*sqrt(d*x^2 + c))/(
f^2*x^4 + 2*e*f*x^2 + e^2)))/((d*e^3 - c*e^2*f + (d*e^2*f - c*e*f
^2)*x^2)*sqrt(d*e^2 - c*e*f)), 1/4*(2*sqrt(-d*e^2 + c*e*f)*sqrt(d
*x^2 + c)*(b*e - a*f)*x - (a*c*e*f + (b*c - 2*a*d)*e^2 + (a*c*f^2
 + (b*c - 2*a*d)*e*f)*x^2)*arctan(1/2*sqrt(-d*e^2 + c*e*f)*((2*d*
e - c*f)*x^2 + c*e)/((d*e^2 - c*e*f)*sqrt(d*x^2 + c)*x)))/((d*e^3
 - c*e^2*f + (d*e^2*f - c*e*f^2)*x^2)*sqrt(-d*e^2 + c*e*f))]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)/(f*x**2+e)**2/(d*x**2+c)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

GIAC/XCAS [A] time = 1.63653, size = 4, normalized size = 0.04

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.99 $\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=359

$$\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}\left(a^2df+b^2ce\right)\left(-\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2a^2b^2\sqrt{d}\sqrt{c-dx^2}\sqrt{e+fx^2}}-\frac{\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(af+be)F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2ab^2\sqrt{c-dx^2}\sqrt{e+fx^2}}+\frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)}+\frac{\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2ab\sqrt{c-dx^2}\sqrt{\frac{fx^2}{e}+1}}$$

```
[Out] (x*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])/(2*a*(a + b*x^2)) + (Sqrt[c]*
Sqrt[d]*Sqrt[1 - (d*x^2)/c]*Sqrt[e + f*x^2]*EllipticE[ArcSin[(Sqr
t[d]*x)/Sqrt[c]], -((c*f)/(d*e))])/(2*a*b*Sqrt[c - d*x^2]*Sqrt[1
+ (f*x^2)/e]) - (Sqrt[c]*Sqrt[d]*(b*e + a*f)*Sqrt[1 - (d*x^2)/c]*
Sqrt[1 + (f*x^2)/e]*EllipticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((c*f
)/(d*e))])/(2*a*b^2*Sqrt[c - d*x^2]*Sqrt[e + f*x^2]) + (Sqrt[c]*(
b^2*c*e + a^2*d*f)*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Ellipt
icPi[-((b*c)/(a*d)), ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((c*f)/(d*e))]
)/(2*a^2*b^2*Sqrt[d]*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])
```

Rubi [A] time = 1.17052, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}\left(a^2df+b^2ce\right)\left(-\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2a^2b^2\sqrt{d}\sqrt{c-dx^2}\sqrt{e+fx^2}}-\frac{\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(af+be)F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2ab^2\sqrt{c-dx^2}\sqrt{e+fx^2}}+\frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)}+\frac{\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2ab\sqrt{c-dx^2}\sqrt{\frac{fx^2}{e}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[c - d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2)^2,x]
```

```
[Out] (x*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])/(2*a*(a + b*x^2)) + (Sqrt[c]*
Sqrt[d]*Sqrt[1 - (d*x^2)/c]*Sqrt[e + f*x^2]*EllipticE[ArcSin[(Sqr
t[d]*x)/Sqrt[c]], -((c*f)/(d*e))])/(2*a*b*Sqrt[c - d*x^2]*Sqrt[1
+ (f*x^2)/e]) - (Sqrt[c]*Sqrt[d]*(b*e + a*f)*Sqrt[1 - (d*x^2)/c]*
```

```
Sqrt[1 + (f*x^2)/e]*EllipticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((c*f
)/(d*e))])/(2*a*b^2*Sqrt[c - d*x^2]*Sqrt[e + f*x^2]) + (Sqrt[c]*(
b^2*c*e + a^2*d*f)*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Ellipt
icPi[-((b*c)/(a*d)), ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((c*f)/(d*e))]
)/(2*a^2*b^2*Sqrt[d]*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])
```

Rubi in Sympy [A] time = 170.75, size = 308, normalized size = 0.86

$$\frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)}+\frac{\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}E\left(\operatorname{asin}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}\right)}{2ab\sqrt{1+\frac{fx^2}{e}}\sqrt{c-dx^2}}-\frac{\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\left(af+be\right)F\left(\operatorname{asin}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}\right)}{2ab^2\sqrt{c-dx^2}\sqrt{e+fx^2}}+\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\left(a^2df+b^2ce\right)\left(-\frac{bc}{ad};\operatorname{asin}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}\right)}{2a^2b^2\sqrt{d}\sqrt{c-dx^2}\sqrt{e+fx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((-d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] x*sqrt(c - d*x**2)*sqrt(e + f*x**2)/(2*a*(a + b*x**2)) + sqrt(c)*
sqrt(d)*sqrt(1 - d*x**2/c)*sqrt(e + f*x**2)*elliptic_e(asin(sqrt(
d)*x/sqrt(c)), -c*f/(d*e))/(2*a*b*sqrt(1 + f*x**2/e)*sqrt(c - d*x
**2)) - sqrt(c)*sqrt(d)*sqrt(1 - d*x**2/c)*sqrt(1 + f*x**2/e)*(a*
f + b*e)*elliptic_f(asin(sqrt(d)*x/sqrt(c)), -c*f/(d*e))/(2*a*b**
2*sqrt(c - d*x**2)*sqrt(e + f*x**2)) + sqrt(c)*sqrt(1 - d*x**2/c)
*sqrt(1 + f*x**2/e)*(a**2*d*f + b**2*c*e)*elliptic_pi(-b*c/(a*d),
 asin(sqrt(d)*x/sqrt(c)), -c*f/(d*e))/(2*a**2*b**2*sqrt(d)*sqrt(c
 - d*x**2)*sqrt(e + f*x**2))
```

Mathematica [C] time = 4.55728, size = 422, normalized size = 1.18

$$\frac{-\frac{ic\sqrt{-\frac{d}{c}}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(af+be)F\left(i\sinh^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|-\frac{cf}{de}\right)}{b^2}+\frac{iacf\sqrt{-\frac{d}{c}}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}\left(-\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|-\frac{cf}{de}\right)}{b^2}+\frac{ide\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}\left(-\frac{bc}{ad};i \cdots\right.}{a\left(-\frac{d}{c}\right)^{\cdots}}\cdots}{2a\sqrt{c-dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[c - d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2)^2,x]
```

```
[Out] ((c*e*x)/(a + b*x^2) - (d*e*x^3)/(a + b*x^2) + (c*f*x^3)/(a + b*x
^2) - (d*f*x^5)/(a + b*x^2) + (I*c*Sqrt[-(d/c)]*e*Sqrt[1 - (d*x^2
```

```
)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[-(d/c)]*x], -((
c*f)/(d*e))])/b - (I*c*Sqrt[-(d/c)]*(b*e + a*f)*Sqrt[1 - (d*x^2)/
c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[-(d/c)]*x], -((c*
f)/(d*e))])/b^2 + (I*d*e*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*
EllipticPi[-((b*c)/(a*d)), I*ArcSinh[Sqrt[-(d/c)]*x], -((c*f)/(d*
e))])/(a*(-(d/c))^(3/2)) + (I*a*c*Sqrt[-(d/c)]*f*Sqrt[1 - (d*x^2)
/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*ArcSinh[Sqrt
[-(d/c)]*x], -((c*f)/(d*e))])/b^2)/(2*a*Sqrt[c - d*x^2]*Sqrt[e +
f*x^2])
```

Maple [B] time = 0.069, size = 793, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*(-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*((d/c)^(1/2)*x^5*a*b^2*d*f+(
-(d*x^2-c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(d/c)^(1/2),(
-c*f/d/e)^(1/2))*x^2*a^2*b*d*f+(-(d*x^2-c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)*EllipticF(x*(d/c)^(1/2),(-c*f/d/e)^(1/2))*x^2*a*b^2*d*e-(-
(d*x^2-c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(d/c)^(1/2),(-
c*f/d/e)^(1/2))*x^2*a*b^2*d*e-(-(d*x^2-c)/c)^(1/2)*((f*x^2+e)/e)^
(1/2)*EllipticPi(x*(d/c)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(d/c)^(1/2))
*x^2*a^2*b*d*f-(-(d*x^2-c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticP
i(x*(d/c)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(d/c)^(1/2))*x^2*b^3*c*e-(d
/c)^(1/2)*x^3*a*b^2*c*f+(d/c)^(1/2)*x^3*a*b^2*d*e+(-(d*x^2-c)/c)^
(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(d/c)^(1/2),(-c*f/d/e)^(1/2
))*a^3*d*f+(-(d*x^2-c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(
d/c)^(1/2),(-c*f/d/e)^(1/2))*a^2*b*d*e-(-(d*x^2-c)/c)^(1/2)*((f*x
^2+e)/e)^(1/2)*EllipticE(x*(d/c)^(1/2),(-c*f/d/e)^(1/2))*a^2*b*d*
e-(-(d*x^2-c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(d/c)^(1/
2),-b*c/a/d,(-f/e)^(1/2)/(d/c)^(1/2))*a^3*d*f-(-(d*x^2-c)/c)^(1/2
)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(d/c)^(1/2),-b*c/a/d,(-f/e)^(1
/2)/(d/c)^(1/2))*a*b^2*c*e-(d/c)^(1/2)*x*a*b^2*c*e)/(d*f*x^4-c*f*
x^2+d*e*x^2-c*e)/a^2/(b*x^2+a)/b^2/(d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2 + c}\sqrt{fx^2 + e}}{(bx^2 + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-dx^2 + c}\sqrt{fx^2 + e}}{(bx^2 + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^2, x)
```

3.100 $\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=381

$$\frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\,(b^2ce-a^2df)\,\left(\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{2a^2b^2\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\,(a+bx^2)}-\frac{fx\sqrt{c+dx^2}}{2ab\sqrt{e+fx^2}}+\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{2ab\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{d\sqrt{e}\sqrt{f}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{2b^2c\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

```
[Out] -(f*x*Sqrt[c + d*x^2])/(2*a*b*Sqrt[e + f*x^2]) + (x*Sqrt[c + d*x^
2]*Sqrt[e + f*x^2])/(2*a*(a + b*x^2)) + (Sqrt[e]*Sqrt[f]*Sqrt[c +
 d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/
(2*a*b*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (
d*Sqrt[e]*Sqrt[f]*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sq
rt[e]], 1 - (d*e)/(c*f)])/(2*b^2*c*Sqrt[(e*(c + d*x^2))/(c*(e + f
*x^2))]*Sqrt[e + f*x^2]) + (Sqrt[-c]*(b^2*c*e - a^2*d*f)*Sqrt[1 +
 (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), ArcSin[(S
qrt[d]*x)/Sqrt[-c]], (c*f)/(d*e)])/(2*a^2*b^2*Sqrt[d]*Sqrt[c + d*
x^2]*Sqrt[e + f*x^2])
```

Rubi [A] time = 1.07625, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219

$$\frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\,(b^2ce-a^2df)\,\left(\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{2a^2b^2\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\,(a+bx^2)}-\frac{fx\sqrt{c+dx^2}}{2ab\sqrt{e+fx^2}}+\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{2ab\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{d\sqrt{e}\sqrt{f}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{2b^2c\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2)^2,x]
```

```
[Out] -(f*x*Sqrt[c + d*x^2])/(2*a*b*Sqrt[e + f*x^2]) + (x*Sqrt[c + d*x^
2]*Sqrt[e + f*x^2])/(2*a*(a + b*x^2)) + (Sqrt[e]*Sqrt[f]*Sqrt[c +
 d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/
(2*a*b*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (
d*Sqrt[e]*Sqrt[f]*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sq
rt[e]], 1 - (d*e)/(c*f)])/(2*b^2*c*Sqrt[(e*(c + d*x^2))/(c*(e + f
*x^2))]*Sqrt[e + f*x^2]) + (Sqrt[-c]*(b^2*c*e - a^2*d*f)*Sqrt[1 +
 (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), ArcSin[(S
qrt[d]*x)/Sqrt[-c]], (c*f)/(d*e)])/(2*a^2*b^2*Sqrt[d]*Sqrt[c + d*
```

```
x^2]*Sqrt[e + f*x^2])
```

Rubi in Sympy [A] time = 119.931, size = 432, normalized size = 1.13

$$\frac{\sqrt{c}\sqrt{d}f\sqrt{e+fx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{2b^2e\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)}$$
$$+\frac{\sqrt{c}\sqrt{d}\sqrt{e+fx^2}E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{2ab\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}}-\frac{dx\sqrt{e+fx^2}}{2ab\sqrt{c+dx^2}}$$
$$-\frac{\sqrt{c}f\sqrt{e+fx^2}\left(a^2df-b^2ce\right)F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{2ab^2\sqrt{d}e\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}\left(af-be\right)}$$
$$+\frac{e^{\frac{3}{2}}\sqrt{c+dx^2}\left(a^2df-b^2ce\right)\left(1-\frac{be}{af};\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{2a^2bc\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}\left(af-be\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/(b*x**2+a)**2,x)
```

```
[Out] sqrt(c)*sqrt(d)*f*sqrt(e + f*x**2)*elliptic_f(atan(sqrt(d)*x/sqrt
(c)), -c*f/(d*e) + 1)/(2*b**2*e*sqrt(c*(e + f*x**2)/(e*(c + d*x**
2)))*sqrt(c + d*x**2)) + x*sqrt(c + d*x**2)*sqrt(e + f*x**2)/(2*a
*(a + b*x**2)) + sqrt(c)*sqrt(d)*sqrt(e + f*x**2)*elliptic_e(atan
(sqrt(d)*x/sqrt(c)), -c*f/(d*e) + 1)/(2*a*b*sqrt(c*(e + f*x**2)/(
e*(c + d*x**2)))*sqrt(c + d*x**2)) - d*x*sqrt(e + f*x**2)/(2*a*b*
sqrt(c + d*x**2)) - sqrt(c)*f*sqrt(e + f*x**2)*(a**2*d*f - b**2*c
*e)*elliptic_f(atan(sqrt(d)*x/sqrt(c)), -c*f/(d*e) + 1)/(2*a*b**2
*sqrt(d)*e*sqrt(c*(e + f*x**2)/(e*(c + d*x**2)))*sqrt(c + d*x**2)
*(a*f - b*e)) + e**(3/2)*sqrt(c + d*x**2)*(a**2*d*f - b**2*c*e)*e
lliptic_pi(1 - b*e/(a*f), atan(sqrt(f)*x/sqrt(e)), 1 - d*e/(c*f))
/(2*a**2*b*c*sqrt(f)*sqrt(e*(c + d*x**2)/(c*(e + f*x**2)))*sqrt(e
 + f*x**2)*(a*f - b*e))
```

Mathematica [C] time = 5.21343, size = 401, normalized size = 1.05

$$\frac{-\frac{ic\sqrt{\frac{d}{c}}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(af+be)F\left(i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)\middle|\frac{cf}{de}\right)}{b^2}+\frac{iacf\sqrt{\frac{d}{c}}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)\middle|\frac{cf}{de}\right)}{b^2}-\frac{ice\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x \right.\right.}{a\sqrt{\frac{d}{c}}}}{2a\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2)^2,x]
```

```
[Out] ((c*e*x)/(a + b*x^2) + (d*e*x^3)/(a + b*x^2) + (c*f*x^3)/(a + b*x
^2) + (d*f*x^5)/(a + b*x^2) + (I*c*Sqrt[d/c]*e*Sqrt[1 + (d*x^2)/c
]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*
e)])/b - (I*c*Sqrt[d/c]*(b*e + a*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 +
(f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/b^2 -
(I*c*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(
a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(a*Sqrt[d/c]) + (I*a*
c*Sqrt[d/c]*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[
(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/b^2)/(2*a*Sqrt
[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] time = 0.059, size = 765, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x)
```

```
[Out] 1/2*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)*((-d/c)^(1/2)*x^5*a*b^2*d*f+(
(d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(
c*f/d/e)^(1/2))*x^2*a^2*b*d*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(
1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*b^2*d*e-((d*
x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f
/d/e)^(1/2))*x^2*a*b^2*d*e-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2
)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*x^
2*a^2*b*d*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*
(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*x^2*b^3*c*e+(-d/c
)^(1/2)*x^3*a*b^2*c*f+(-d/c)^(1/2)*x^3*a*b^2*d*e+((d*x^2+c)/c)^(1
/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))
*a^3*d*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/
c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*d*e-((d*x^2+c)/c)^(1/2)*((f*x^2+e
)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*d*e-((
d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b
*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^3*d*f+((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(
-d/c)^(1/2))*a*b^2*c*e+(-d/c)^(1/2)*x*a*b^2*c*e)/(d*f*x^4+c*f*x^2
+d*e*x^2+c*e)/a^2/(b*x^2+a)/b^2/(-d/c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + c}\sqrt{fx^2 + e}}{(bx^2 + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^2,x, algorithm="maxima")

[Out] integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^2,x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/(b*x**2+a)**2,x)

[Out] Integral(sqrt(c + d*x**2)*sqrt(e + f*x**2)/(a + b*x**2)**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}\sqrt{fx^2+e}}{(bx^2+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^2,x, algorithm="giac")

[Out] integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^2, x)
```

3.101 $\int \frac{1}{(a+bx^2)^2\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=426

$$\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}\left(-3a^2df+ab(2de-2cf)+b^2ce\right)\Pi\left(-\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2a^2\sqrt{d}\sqrt{c-dx^2}\sqrt{e+fx^2}(ad+bc)(be-af)}+\frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(ad+bc)(be-af)}-\frac{\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2a\sqrt{c-dx^2}\sqrt{e+fx^2}(ad+bc)}+\frac{b\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2a\sqrt{c-dx^2}\sqrt{\frac{fx^2}{e}+1}(ad+bc)(be-af)}$$

```
[Out] (b^2*x*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])/(2*a*(b*c + a*d)*(b*e - a
*f)*(a + b*x^2)) + (b*Sqrt[c]*Sqrt[d]*Sqrt[1 - (d*x^2)/c]*Sqrt[e
+ f*x^2]*EllipticE[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((c*f)/(d*e))])/
(2*a*(b*c + a*d)*(b*e - a*f)*Sqrt[c - d*x^2]*Sqrt[1 + (f*x^2)/e])
 - (Sqrt[c]*Sqrt[d]*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Ellip
ticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((c*f)/(d*e))])/(2*a*(b*c + a*
d)*Sqrt[c - d*x^2]*Sqrt[e + f*x^2]) + (Sqrt[c]*(b^2*c*e - 3*a^2*d
*f + a*b*(2*d*e - 2*c*f))*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]
*EllipticPi[-((b*c)/(a*d)), ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((c*f)/
(d*e))])/(2*a^2*Sqrt[d]*(b*c + a*d)*(b*e - a*f)*Sqrt[c - d*x^2]*S
qrt[e + f*x^2])
```

Rubi [A] time = 1.33234, antiderivative size = 426, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}\left(-3a^2df+2ab(de-cf)+b^2ce\right)\Pi\left(-\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2a^2\sqrt{d}\sqrt{c-dx^2}\sqrt{e+fx^2}(ad+bc)(be-af)}+\frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(ad+bc)(be-af)}-\frac{\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2a\sqrt{c-dx^2}\sqrt{e+fx^2}(ad+bc)}+\frac{b\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2a\sqrt{c-dx^2}\sqrt{\frac{fx^2}{e}+1}(ad+bc)(be-af)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^2*Sqrt[c - d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (b^2*x*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])/(2*a*(b*c + a*d)*(b*e - a
*f)*(a + b*x^2)) + (b*Sqrt[c]*Sqrt[d]*Sqrt[1 - (d*x^2)/c]*Sqrt[e
```

```
+ f*x^2]*EllipticE[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((c*f)/(d*e))])/
(2*a*(b*c + a*d)*(b*e - a*f)*Sqrt[c - d*x^2]*Sqrt[1 + (f*x^2)/e])
 - (Sqrt[c]*Sqrt[d]*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Ellip
ticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((c*f)/(d*e))])/(2*a*(b*c + a*
d)*Sqrt[c - d*x^2]*Sqrt[e + f*x^2]) + (Sqrt[c]*(b^2*c*e - 3*a^2*d
*f + 2*a*b*(d*e - c*f))*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*E
llipticPi[-((b*c)/(a*d)), ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((c*f)/(d
*e))])/(2*a^2*Sqrt[d]*(b*c + a*d)*(b*e - a*f)*Sqrt[c - d*x^2]*Sqr
t[e + f*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(b*x**2+a)**2/(-d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 6.64912, size = 773, normalized size = 1.81

$$-\frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(ad+bc)(af-be)}+\frac{\sqrt{\left(c-dx^2\right)\left(e+fx^2\right)}\left(\frac{ib^2ce\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}\left(-\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|-\frac{cf}{de}\right)}{a\sqrt{-\frac{d}{c}}\sqrt{(c-dx^2)(e+fx^2)}}+\frac{2ibde\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}\left(-\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|-\frac{cf}{de}\right)}{\sqrt{-\frac{d}{c}}\sqrt{(c-dx^2)(e+fx^2)}}-\frac{2ibcf\sqrt{1-}}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)^2*Sqrt[c - d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] -(b^2*x*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])/(2*a*(b*c + a*d)*(-(b*e)
 + a*f)*(a + b*x^2)) + (Sqrt[(c - d*x^2)*(e + f*x^2)]*((I*b*d*e*S
qrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(EllipticE[I*ArcSinh[Sqrt[
-(d/c)]*x], -((c*f)/(d*e))] - EllipticF[I*ArcSinh[Sqrt[-(d/c)]*x]
, -((c*f)/(d*e))]))/(Sqrt[-(d/c)]*Sqrt[(c - d*x^2)*(e + f*x^2)])
+ (I*a*d*f*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*Ar
cSinh[Sqrt[-(d/c)]*x], -((c*f)/(d*e))])/(Sqrt[-(d/c)]*Sqrt[(c - d
*x^2)*(e + f*x^2)]) + (I*b^2*c*e*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*
x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*ArcSinh[Sqrt[-(d/c)]*x], -((
c*f)/(d*e))])/(a*Sqrt[-(d/c)]*Sqrt[(c - d*x^2)*(e + f*x^2)]) + ((
2*I)*b*d*e*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[-((
b*c)/(a*d)), I*ArcSinh[Sqrt[-(d/c)]*x], -((c*f)/(d*e))])/(Sqrt[-(
d/c)]*Sqrt[(c - d*x^2)*(e + f*x^2)]) - ((2*I)*b*c*f*Sqrt[1 - (d*x
```

```
^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*ArcSinh[S
qrt[-(d/c)]*x], -((c*f)/(d*e))])/(Sqrt[-(d/c)]*Sqrt[(c - d*x^2)*(
e + f*x^2)]) - ((3*I)*a*d*f*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/
e]*EllipticPi[-((b*c)/(a*d)), I*ArcSinh[Sqrt[-(d/c)]*x], -((c*f)/
(d*e))])/(Sqrt[-(d/c)]*Sqrt[(c - d*x^2)*(e + f*x^2)])))/(2*a*(b*c
 + a*d)*(-(b*e) + a*f)*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])
```

Maple [B] time = 0.073, size = 1105, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^2/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] 1/2*(-(d/c)^(1/2)*x^5*a*b^2*d*f+(-(d*x^2-c)/c)^(1/2)*((f*x^2+e)/e
)^(1/2)*EllipticF(x*(d/c)^(1/2),(-c*f/d/e)^(1/2))*x^2*a^2*b*d*f-(
-(d*x^2-c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(d/c)^(1/2),(
-c*f/d/e)^(1/2))*x^2*a*b^2*d*e+(-(d*x^2-c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)*EllipticE(x*(d/c)^(1/2),(-c*f/d/e)^(1/2))*x^2*a*b^2*d*e-3*
(-(d*x^2-c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(d/c)^(1/2)
,-b*c/a/d,(-f/e)^(1/2)/(d/c)^(1/2))*x^2*a^2*b*d*f-2*(-(d*x^2-c)/c
)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(d/c)^(1/2),-b*c/a/d,(-f
/e)^(1/2)/(d/c)^(1/2))*x^2*a*b^2*c*f+2*(-(d*x^2-c)/c)^(1/2)*((f*x
^2+e)/e)^(1/2)*EllipticPi(x*(d/c)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(d/
c)^(1/2))*x^2*a*b^2*d*e+(-(d*x^2-c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*
EllipticPi(x*(d/c)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(d/c)^(1/2))*x^2*b
^3*c*e+(d/c)^(1/2)*x^3*a*b^2*c*f-(d/c)^(1/2)*x^3*a*b^2*d*e+(-(d*x
^2-c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(d/c)^(1/2),(-c*f/
d/e)^(1/2))*a^3*d*f-(-(d*x^2-c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elli
pticF(x*(d/c)^(1/2),(-c*f/d/e)^(1/2))*a^2*b*d*e+(-(d*x^2-c)/c)^(1
/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(d/c)^(1/2),(-c*f/d/e)^(1/2))
*a^2*b*d*e-3*(-(d*x^2-c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(
x*(d/c)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(d/c)^(1/2))*a^3*d*f-2*(-(d*x
^2-c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(d/c)^(1/2),-b*c/
a/d,(-f/e)^(1/2)/(d/c)^(1/2))*a^2*b*c*f+2*(-(d*x^2-c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)*EllipticPi(x*(d/c)^(1/2),-b*c/a/d,(-f/e)^(1/2)/
(d/c)^(1/2))*a^2*b*d*e+(-(d*x^2-c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*E
llipticPi(x*(d/c)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(d/c)^(1/2))*a*b^2*
c*e+(d/c)^(1/2)*x*a*b^2*c*e)*(f*x^2+e)^(1/2)*(-d*x^2+c)^(1/2)/(d/
c)^(1/2)/(b*x^2+a)/a^2/(a*f-b*e)/(a*d+b*c)/(d*f*x^4-c*f*x^2+d*e*x
^2-c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^2\sqrt{-dx^2+c}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="maxima
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="frica
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**2/(-d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2 + a)^2 \sqrt{-dx^2 + c} \sqrt{fx^2 + e}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="giac"
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.102 $\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=485

$$\frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(3a^2df-2ab(cf+de)+b^2ce\right)\Pi\left(\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{2a^2\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}(bc-ad)(be-af)}+\frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(bc-ad)(be-af)}-\frac{bfx\sqrt{c+dx^2}}{2a\sqrt{e+fx^2}(bc-ad)(be-af)}-\frac{d\sqrt{e}\sqrt{f}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{2c\sqrt{e+fx^2}(bc-ad)(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{b\sqrt{e}\sqrt{f}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{2a\sqrt{e+fx^2}(bc-ad)(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

```
[Out] -(b*f*x*Sqrt[c + d*x^2])/(2*a*(b*c - a*d)*(b*e - a*f)*Sqrt[e + f*
x^2]) + (b^2*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(2*a*(b*c - a*d)*
(b*e - a*f)*(a + b*x^2)) + (b*Sqrt[e]*Sqrt[f]*Sqrt[c + d*x^2]*Ell
ipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(2*a*(b*c -
 a*d)*(b*e - a*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e +
f*x^2]) - (d*Sqrt[e]*Sqrt[f]*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sq
rt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(2*c*(b*c - a*d)*(b*e - a*f)
*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (Sqrt[-
c]*(b^2*c*e + 3*a^2*d*f - 2*a*b*(d*e + c*f))*Sqrt[1 + (d*x^2)/c]*
Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), ArcSin[(Sqrt[d]*x)/Sq
rt[-c]], (c*f)/(d*e)])/(2*a^2*Sqrt[d]*(b*c - a*d)*(b*e - a*f)*Sqr
t[c + d*x^2]*Sqrt[e + f*x^2])
```

Rubi [A] time = 1.22597, antiderivative size = 485, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$

$$\frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(3a^2df-2ab(cf+de)+b^2ce\right)\Pi\left(\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{2a^2\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}(bc-ad)(be-af)}+\frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)(bc-ad)(be-af)}-\frac{bfx\sqrt{c+dx^2}}{2a\sqrt{e+fx^2}(bc-ad)(be-af)}-\frac{d\sqrt{e}\sqrt{f}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{2c\sqrt{e+fx^2}(bc-ad)(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{b\sqrt{e}\sqrt{f}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{2a\sqrt{e+fx^2}(bc-ad)(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((a + b*x^2)^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] -(b*f*x*Sqrt[c + d*x^2])/(2*a*(b*c - a*d)*(b*e - a*f)*Sqrt[e + f*
x^2]) + (b^2*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(2*a*(b*c - a*d)*
```

```
(b*e - a*f)*(a + b*x^2)) + (b*Sqrt[e]*Sqrt[f]*Sqrt[c + d*x^2]*Ell
ipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(2*a*(b*c -
 a*d)*(b*e - a*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e +
f*x^2]) - (d*Sqrt[e]*Sqrt[f]*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sq
rt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(2*c*(b*c - a*d)*(b*e - a*f)
*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (Sqrt[-
c]*(b^2*c*e + 3*a^2*d*f - 2*a*b*(d*e + c*f))*Sqrt[1 + (d*x^2)/c]*
Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), ArcSin[(Sqrt[d]*x)/Sq
rt[-c]], (c*f)/(d*e)])/(2*a^2*Sqrt[d]*(b*c - a*d)*(b*e - a*f)*Sqr
t[c + d*x^2]*Sqrt[e + f*x^2])
```

Rubi in Sympy [A] time = 160.607, size = 527, normalized size = 1.09

$$-\frac{\sqrt{c}\sqrt{d}f\sqrt{e+fx^2}F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{2e\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}\,(ad-bc)\,(af-be)}+\frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\,(a+bx^2)\,(ad-bc)\,(af-be)}+\frac{b\sqrt{c}\sqrt{d}\sqrt{e+fx^2}E\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{2a\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}\,(ad-bc)\,(af-be)}-\frac{bdx\sqrt{e+fx^2}}{2a\sqrt{c+dx^2}\,(ad-bc)\,(af-be)}-\frac{\sqrt{c}f\sqrt{e+fx^2}\left(-3a^2df+2ab\,(cf+de)-b^2ce\right)F\left(\operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{cf}{de}+1\right)}{2a\sqrt{d}e\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{c+dx^2}\,(ad-bc)\,(af-be)^2}+\frac{be^{\frac{3}{2}}\sqrt{c+dx^2}\left(-3a^2df+2ab\,(cf+de)-b^2ce\right)\left(1-\frac{be}{af};\operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\middle|1-\frac{de}{cf}\right)}{2a^2c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}\,(ad-bc)\,(af-be)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(b*x**2+a)**2/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] -sqrt(c)*sqrt(d)*f*sqrt(e + f*x**2)*elliptic_f(atan(sqrt(d)*x/sqr
t(c)), -c*f/(d*e) + 1)/(2*e*sqrt(c*(e + f*x**2)/(e*(c + d*x**2)))
*sqrt(c + d*x**2)*(a*d - b*c)*(a*f - b*e)) + b**2*x*sqrt(c + d*x*
*2)*sqrt(e + f*x**2)/(2*a*(a + b*x**2)*(a*d - b*c)*(a*f - b*e)) +
 b*sqrt(c)*sqrt(d)*sqrt(e + f*x**2)*elliptic_e(atan(sqrt(d)*x/sqr
t(c)), -c*f/(d*e) + 1)/(2*a*sqrt(c*(e + f*x**2)/(e*(c + d*x**2)))
*sqrt(c + d*x**2)*(a*d - b*c)*(a*f - b*e)) - b*d*x*sqrt(e + f*x**
2)/(2*a*sqrt(c + d*x**2)*(a*d - b*c)*(a*f - b*e)) - sqrt(c)*f*sqr
t(e + f*x**2)*(-3*a**2*d*f + 2*a*b*(c*f + d*e) - b**2*c*e)*ellipt
ic_f(atan(sqrt(d)*x/sqrt(c)), -c*f/(d*e) + 1)/(2*a*sqrt(d)*e*sqrt
(c*(e + f*x**2)/(e*(c + d*x**2)))*sqrt(c + d*x**2)*(a*d - b*c)*(a
*f - b*e)**2) + b*e**(3/2)*sqrt(c + d*x**2)*(-3*a**2*d*f + 2*a*b*
(c*f + d*e) - b**2*c*e)*elliptic_pi(1 - b*e/(a*f), atan(sqrt(f)*x
/sqrt(e)), 1 - d*e/(c*f))/(2*a**2*c*sqrt(f)*sqrt(e*(c + d*x**2)/(
c*(e + f*x**2)))*sqrt(e + f*x**2)*(a*d - b*c)*(a*f - b*e)**2)
```

Mathematica [C] time = 6.16083, size = 587, normalized size = 1.21

$$-\frac{ib^2ce\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\left(\frac{bc}{ad};i\sinh^{-1}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)}{a\sqrt{\frac{d}{c}}}+\frac{b^2cex}{a+bx^2}+\frac{b^2cfx^3}{a+bx^2}+\frac{b^2dex^3}{a+bx^2}+\frac{b^2dfx^5}{a+bx^2}-ic\sqrt{\frac{d}{c}}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(be-af)F\left(i\sinh^{-}\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((a + b*x^2)^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] ((b^2*c*e*x)/(a + b*x^2) + (b^2*d*e*x^3)/(a + b*x^2) + (b^2*c*f*x
^3)/(a + b*x^2) + (b^2*d*f*x^5)/(a + b*x^2) + I*b*c*Sqrt[d/c]*e*S
qrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d
/c]*x], (c*f)/(d*e)] - I*c*Sqrt[d/c]*(b*e - a*f)*Sqrt[1 + (d*x^2)
/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(
d*e)] - (I*b^2*c*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Ellipt
icPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(a*Sqrt[d
/c]) + (2*I)*b*c*Sqrt[d/c]*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)
/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]
+ ((2*I)*b*c*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi
[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/Sqrt[d/c] - (
3*I)*a*c*Sqrt[d/c]*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Elli
pticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(2*a*(-
(b*c) + a*d)*(-(b*e) + a*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [B] time = 0.043, size = 1078, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] -1/2*(-(-d/c)^(1/2)*x^5*a*b^2*d*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/
e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a^2*b*d*f-
((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),
(c*f/d/e)^(1/2))*x^2*a*b^2*d*e+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^
(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*x^2*a*b^2*d*e-3*(
(d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),
b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*x^2*a^2*b*d*f+2*((d*x^2+c)/c)^
(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e
)^(1/2)/(-d/c)^(1/2))*x^2*a*b^2*c*f+2*((d*x^2+c)/c)^(1/2)*((f*x^2
+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c
)^(1/2))*x^2*a*b^2*d*e-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*El
lipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*x^2*b^
3*c*e-(-d/c)^(1/2)*x^3*a*b^2*c*f-(-d/c)^(1/2)*x^3*a*b^2*d*e+((d*x
^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/
d/e)^(1/2))*a^3*d*f-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellip
```

```
ticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*d*e+((d*x^2+c)/c)^(1/2
)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a
^2*b*d*e-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(
-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^3*d*f+2*((d*x^2+
c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d
,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*b*c*f+2*((d*x^2+c)/c)^(1/2)*((f*x
^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d
/c)^(1/2))*a^2*b*d*e-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elli
pticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b^2*c*
e-(-d/c)^(1/2)*x*a*b^2*c*e)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/(-d/c
)^(1/2)/(b*x^2+a)/a^2/(a*d-b*c)/(a*f-b*e)/(d*f*x^4+c*f*x^2+d*e*x^
2+c*e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^2\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="maxima
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="fricas
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**2/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^2+a\right)^2 \sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="giac")

[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.103 $\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=37

$$\text{Int}\left(\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}},x\right)$$

```
[Out] Unintegrable[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],
 x]
```

Rubi [A] time = 0.164821, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$\text{Int}\left(\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}},x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

```
[Out] Defer[Int][((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2], x
]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.23438, size = 0, normalized size = 0.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]

[Out] Integrate[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2], x]
```

Maple [A] time = 0.104, size = 0, normalized size = 0.

$$\int 1 \left(bx^2 + a\right)^{\frac{3}{2}} \sqrt{dx^2 + c} \frac{1}{\sqrt{fx^2 + e}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)

[Out] int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2 + a\right)^{\frac{3}{2}} \sqrt{dx^2 + c}}{\sqrt{fx^2 + e}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e),x, algorithm="maxima

[Out] integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2 + a\right)^{\frac{3}{2}} \sqrt{dx^2 + c}}{\sqrt{fx^2 + e}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e),x, algorithm="fricas

[Out] integral((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)*sqrt(c + d*x**2)/sqrt(e + f*x**2), x
)
```

GIAC/XCAS [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}{\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

3.104 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=545

$$\frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}}+\frac{b\sqrt{e}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}F\left(\sin^{-1}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{2df\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{e}\sqrt{a+bx^2}\sqrt{de-cf}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\sin^{-1}\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{2f\sqrt{e+fx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{c\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}(-adf-bcf+bde)\Pi\left(\frac{de}{de-cf};\sin^{-1}\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{2adf\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (d*x*Sqrt[a + b*x^2]*Sqrt[e + f*x^2])/(2*f*Sqrt[c + d*x^2]) - (Sq
rt[e]*Sqrt[d*e - c*f]*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c
+ d*x^2))]*EllipticE[ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c +
 d*x^2])], -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/(2*f*Sqrt[(c*(a +
 b*x^2))/(a*(c + d*x^2))]*Sqrt[e + f*x^2]) + (b*Sqrt[e]*(d*e - c*
f)*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*Elliptic
F[ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((b*c -
a*d)*e)/(c*(b*e - a*f))])/(2*d*f*Sqrt[b*e - a*f]*Sqrt[(a*(c + d*x
^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2]) - (c*Sqrt[e]*(b*d*e - b*c*
f - a*d*f)*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*
EllipticPi[(d*e)/(d*e - c*f), ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]
*Sqrt[c + d*x^2])], -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/(2*a*d*f
*Sqrt[d*e - c*f]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[e + f
*x^2])
```

Rubi [A] time = 1.64071, antiderivative size = 545, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$

$$\frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}}+\frac{b\sqrt{e}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}F\left(\sin^{-1}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{2df\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{\sqrt{e}\sqrt{a+bx^2}\sqrt{de-cf}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\sin^{-1}\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{2f\sqrt{e+fx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{c\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}(-adf-bcf+bde)\Pi\left(\frac{de}{de-cf};\sin^{-1}\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{2adf\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

```
[Out] (d*x*Sqrt[a + b*x^2]*Sqrt[e + f*x^2])/(2*f*Sqrt[c + d*x^2]) - (Sq
rt[e]*Sqrt[d*e - c*f]*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c
+ d*x^2))]*EllipticE[ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c +
 d*x^2])], -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/(2*f*Sqrt[(c*(a +
 b*x^2))/(a*(c + d*x^2))]*Sqrt[e + f*x^2]) + (b*Sqrt[e]*(d*e - c*
f)*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*Elliptic
F[ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((b*c -
a*d)*e)/(c*(b*e - a*f))])/(2*d*f*Sqrt[b*e - a*f]*Sqrt[(a*(c + d*x
^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2]) - (c*Sqrt[e]*(b*d*e - b*c*
f - a*d*f)*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*
EllipticPi[(d*e)/(d*e - c*f), ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]
*Sqrt[c + d*x^2])], -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/(2*a*d*f
*Sqrt[d*e - c*f]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[e + f
*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.120747, size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

```
[Out] Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/Sqrt[e + f*x^2], x]
```

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int 1\sqrt{bx^2+a}\sqrt{dx^2+c}\frac{1}{\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/sqrt(e + f*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}\sqrt{dx^2 + c}}{\sqrt{fx^2 + e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

3.105 $\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=163

$$\frac{c\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\Pi\left(\frac{de}{de-cf};\sin^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{e}\sqrt{dx^2+c}}\right)\middle|-\frac{(bc-ad)e}{a(de-cf)}\right)}{a\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] (c*Sqrt[e]*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*
EllipticPi[(d*e)/(d*e - c*f), ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]
*Sqrt[c + d*x^2])], -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/(a*Sqrt[
d*e - c*f]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.549734, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{c\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\Pi\left(\frac{de}{de-cf};\sin^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{e}\sqrt{dx^2+c}}\right)\middle|-\frac{(bc-ad)e}{a(de-cf)}\right)}{a\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (c*Sqrt[e]*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*
EllipticPi[(d*e)/(d*e - c*f), ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]
*Sqrt[c + d*x^2])], -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/(a*Sqrt[
d*e - c*f]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[e + f*x^2])
```

Rubi in Sympy [A] time = 90.671, size = 131, normalized size = 0.8

$$\frac{c\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\sqrt{a+bx^2}\,\Pi\left(\frac{ad}{ad-bc};\operatorname{asin}\left(\frac{x\sqrt{ad-bc}}{\sqrt{a}\sqrt{c+dx^2}}\right)\middle|\frac{a(-cf+de)}{e(ad-bc)}\right)}{\sqrt{a}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}\sqrt{ad-bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] c*sqrt(c*(e + f*x**2)/(e*(c + d*x**2)))*sqrt(a + b*x**2)*elliptic
_pi(a*d/(a*d - b*c), asin(x*sqrt(a*d - b*c)/(sqrt(a)*sqrt(c + d*x
```

```
**2))), a*(-c*f + d*e)/(e*(a*d - b*c)))/(sqrt(a)*sqrt(c*(a + b*x*
*2)/(a*(c + d*x**2)))*sqrt(e + f*x**2)*sqrt(a*d - b*c))
```

Mathematica [A] time = 0.133737, size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\sqrt{e+fx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*Sqrt[e + f*x^2]),x]

[Out] Integrate[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*Sqrt[e + f*x^2]), x]
```

Maple [F] time = 0.087, size = 0, normalized size = 0.

$$\int 1\sqrt{dx^2+c}\frac{1}{\sqrt{bx^2+a}}\frac{1}{\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(1/2),x)

[Out] int((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/(sqrt(b*x^2 + a)*sqrt(f*x^2 + e)),x, algorithm="maxima

[Out] integrate(sqrt(d*x^2 + c)/(sqrt(b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/(sqrt(b*x^2 + a)*sqrt(f*x^2 + e)),x, algorithm="fricas
```

```
[Out] Timed out
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + dx^2}}{\sqrt{a + bx^2}\sqrt{e + fx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(sqrt(a + b*x**2)*sqrt(e + f*x**2)), x)
```

GIAC/XCAS [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + c}}{\sqrt{bx^2 + a}\sqrt{fx^2 + e}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/(sqrt(b*x^2 + a)*sqrt(f*x^2 + e)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(sqrt(b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

3.106 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\sin^{-1}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{a\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

```
[Out] (Sqrt[e]*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*El
lipticE[ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((
b*c - a*d)*e)/(c*(b*e - a*f))])/(a*Sqrt[b*e - a*f]*Sqrt[(a*(c + d
*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.313504, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\sin^{-1}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{a\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c + d*x^2]/((a + b*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (Sqrt[e]*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*El
lipticE[ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((
b*c - a*d)*e)/(c*(b*e - a*f))])/(a*Sqrt[b*e - a*f]*Sqrt[(a*(c + d
*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2])
```

Rubi in Sympy [A] time = 165.39, size = 580, normalized size = 3.92

$$\frac{\sqrt{e}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\sqrt{1-\frac{x^2(-ad+bc)}{c(a+bx^2)}}\sqrt{c+dx^2}F\left(\operatorname{atan}\left(\frac{x\sqrt{af-be}}{\sqrt{e}\sqrt{a+bx^2}}\right)\middle|\frac{a(cf-de)}{c(af-be)}\right)}{a\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{\frac{e\left(c+\frac{x^2(ad-bc)}{a+bx^2}\right)}{c\left(e+\frac{x^2(af-be)}{a+bx^2}\right)}}\sqrt{1-\frac{x^2(-af+be)}{e(a+bx^2)}}\sqrt{e+fx^2}\sqrt{af-be}} + \frac{ex\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\sqrt{1-\frac{x^2(-af+be)}{e(a+bx^2)}}\sqrt{c+dx^2}\,(ad-bc)}{ac\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{1-\frac{x^2(-ad+bc)}{c(a+bx^2)}}\sqrt{a+bx^2}\sqrt{e+fx^2}\,(af-be)} - \frac{e\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\sqrt{1-\frac{x^2(-af+be)}{e(a+bx^2)}}\sqrt{c+dx^2}\sqrt{ad-bc}E\left(\operatorname{atan}\left(\frac{x\sqrt{ad-bc}}{\sqrt{c}\sqrt{a+bx^2}}\right)\middle|-\frac{a(cf-de)}{e(ad-bc)}\right)}{a\sqrt{c}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{\frac{c\left(e+\frac{x^2(af-be)}{a+bx^2}\right)}{e\left(c+\frac{x^2(ad-bc)}{a+bx^2}\right)}}\sqrt{1-\frac{x^2(-ad+bc)}{c(a+bx^2)}}\sqrt{e+fx^2}\,(af-be)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(3/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] sqrt(e)*sqrt(a*(e + f*x**2)/(e*(a + b*x**2)))*sqrt(1 - x**2*(-a*d
 + b*c)/(c*(a + b*x**2)))*sqrt(c + d*x**2)*elliptic_f(atan(x*sqrt
(a*f - b*e)/(sqrt(e)*sqrt(a + b*x**2))), a*(c*f - d*e)/(c*(a*f -
b*e)))/(a*sqrt(a*(c + d*x**2)/(c*(a + b*x**2)))*sqrt(e*(c + x**2*
(a*d - b*c)/(a + b*x**2))/(c*(e + x**2*(a*f - b*e)/(a + b*x**2)))
)*sqrt(1 - x**2*(-a*f + b*e)/(e*(a + b*x**2)))*sqrt(e + f*x**2)*s
qrt(a*f - b*e)) + e*x*sqrt(a*(e + f*x**2)/(e*(a + b*x**2)))*sqrt(
1 - x**2*(-a*f + b*e)/(e*(a + b*x**2)))*sqrt(c + d*x**2)*(a*d - b
*c)/(a*c*sqrt(a*(c + d*x**2)/(c*(a + b*x**2)))*sqrt(1 - x**2*(-a*
d + b*c)/(c*(a + b*x**2)))*sqrt(a + b*x**2)*sqrt(e + f*x**2)*(a*f
 - b*e)) - e*sqrt(a*(e + f*x**2)/(e*(a + b*x**2)))*sqrt(1 - x**2*
(-a*f + b*e)/(e*(a + b*x**2)))*sqrt(c + d*x**2)*sqrt(a*d - b*c)*e
lliptic_e(atan(x*sqrt(a*d - b*c)/(sqrt(c)*sqrt(a + b*x**2))), -a*
(c*f - d*e)/(e*(a*d - b*c)))/(a*sqrt(c)*sqrt(a*(c + d*x**2)/(c*(a
 + b*x**2)))*sqrt(c*(e + x**2*(a*f - b*e)/(a + b*x**2))/(e*(c + x
**2*(a*d - b*c)/(a + b*x**2))))*sqrt(1 - x**2*(-a*d + b*c)/(c*(a
+ b*x**2)))*sqrt(e + f*x**2)*(a*f - b*e))
```

Mathematica [A] time = 1.19262, size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}\sqrt{e+fx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[Sqrt[c + d*x^2]/((a + b*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] Integrate[Sqrt[c + d*x^2]/((a + b*x^2)^(3/2)*Sqrt[e + f*x^2]), x]
```

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int 1\sqrt{dx^2+c}\left(bx^2+a\right)^{-\frac{3}{2}}\frac{1}{\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] int((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^(3/2)*sqrt(f*x^2 + e)),x, algorithm="maxi
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^(3/2)*sqrt(f*x^2 + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{3}{2}}\sqrt{fx^2+e}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^(3/2)*sqrt(f*x^2 + e)),x, algorithm="fric
```

```
[Out] integral(sqrt(d*x^2 + c)/((b*x^2 + a)^(3/2)*sqrt(f*x^2 + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{\frac{3}{2}}\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(3/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/((a + b*x**2)**(3/2)*sqrt(e + f*x**2)),
 x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^(3/2)*sqrt(f*x^2 + e)),x, algorithm="giac
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^(3/2)*sqrt(f*x^2 + e)), x)
```

3.107 $\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=37

$$\text{Int}\left(\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}},x\right)$$

```
[Out] Unintegrable[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2
), x]
```

Rubi [A] time = 0.158442, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\text{Int}\left(\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}},x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

```
[Out] Defer[Int][((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),
 x]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.3344, size = 0, normalized size = 0.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

```
[Out] Integrate[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),
x]
```

Maple [A] time = 0.09, size = 0, normalized size = 0.

$$\int 1 \left(bx^2+a\right)^{\frac{3}{2}} \sqrt{dx^2+c}\left(fx^2+e\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

```
[Out] int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{dx^2+c}}{\left(fx^2+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2),x, algorithm="maxi
```

```
[Out] integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{dx^2+c}}{\left(fx^2+e\right)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2),x, algorithm="fric
```

```
[Out] integral((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{dx^2+c}}{\left(fx^2+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2),x, algorithm="giac
```

```
[Out] integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

3.108 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=484

$$-\frac{c^{3/2}\sqrt{a+bx^2}(be-af)F\left(\tan^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\sqrt{fx^2+e}}\right)\middle|-\frac{(bc-ad)e}{a(de-cf)}\right)}{aef\sqrt{c+dx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{x\sqrt{a+bx^2}(de-cf)}{ef\sqrt{c+dx^2}\sqrt{e+fx^2}}+\frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{de-cf}\,E\left(\tan^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\sqrt{fx^2+e}}\right)\middle|-\frac{(bc-ad)e}{a(de-cf)}\right)}{ef\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{bc\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\Pi\left(\frac{de}{de-cf};\sin^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{e}\sqrt{dx^2+c}}\right)\middle|-\frac{(bc-ad)e}{a(de-cf)}\right)}{af\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] -(((d*e - c*f)*x*Sqrt[a + b*x^2])/(e*f*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2])) + (Sqrt[c]*Sqrt[d*e - c*f]*Sqrt[a + b*x^2]*EllipticE[ArcT
an[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])], -(((b*c - a*d)
*e)/(a*(d*e - c*f)))])/(e*f*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]
*Sqrt[c + d*x^2]) - (c^(3/2)*(b*e - a*f)*Sqrt[a + b*x^2]*Elliptic
F[ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])], -(((b*c
- a*d)*e)/(a*(d*e - c*f)))])/(a*e*f*Sqrt[d*e - c*f]*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (b*c*Sqrt[e]*Sqrt[a +
 b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*EllipticPi[(d*e)/(d
*e - c*f), ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])],
 -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/(a*f*Sqrt[d*e - c*f]*Sqrt[(
c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 2.22447, antiderivative size = 484, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$-\frac{c^{3/2}\sqrt{a+bx^2}(be-af)F\left(\tan^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\sqrt{fx^2+e}}\right)\middle|-\frac{(bc-ad)e}{a(de-cf)}\right)}{aef\sqrt{c+dx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{x\sqrt{a+bx^2}(de-cf)}{ef\sqrt{c+dx^2}\sqrt{e+fx^2}}+\frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{de-cf}\,E\left(\tan^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\sqrt{fx^2+e}}\right)\middle|-\frac{(bc-ad)e}{a(de-cf)}\right)}{ef\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{bc\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\Pi\left(\frac{de}{de-cf};\sin^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{e}\sqrt{dx^2+c}}\right)\middle|-\frac{(bc-ad)e}{a(de-cf)}\right)}{af\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

```
[Out] -(((d*e - c*f)*x*Sqrt[a + b*x^2])/(e*f*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2])) + (Sqrt[c]*Sqrt[d*e - c*f]*Sqrt[a + b*x^2]*EllipticE[ArcT
an[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])], -(((b*c - a*d)
*e)/(a*(d*e - c*f)))])/(e*f*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]
*Sqrt[c + d*x^2]) - (c^(3/2)*(b*e - a*f)*Sqrt[a + b*x^2]*Elliptic
F[ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])], -(((b*c
- a*d)*e)/(a*(d*e - c*f)))])/(a*e*f*Sqrt[d*e - c*f]*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (b*c*Sqrt[e]*Sqrt[a +
 b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*EllipticPi[(d*e)/(d
*e - c*f), ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])],
 -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/(a*f*Sqrt[d*e - c*f]*Sqrt[(
c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[e + f*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.856013, size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

```
[Out] Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2), x]
```

Maple [F] time = 0.082, size = 0, normalized size = 0.

$$\int 1\sqrt{bx^2+a}\sqrt{dx^2+c}\left(fx^2+e\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

```
[Out] int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2),x, algorithm="maxima
```

```
[Out] integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2),x, algorithm="fricas
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2)**(3/2), x
)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2 + a}\sqrt{dx^2 + c}}{(fx^2 + e)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

3.109 $\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=319

$$\frac{c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{fx^2+e}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{ae\sqrt{c+dx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}(de-cf)}{e\sqrt{c+dx^2}\sqrt{e+fx^2}(be-af)}-\frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{de-cf}E\left(\tan^{-1}\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{fx^2+e}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{e\sqrt{c+dx^2}(be-af)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] ((d*e - c*f)*x*Sqrt[a + b*x^2])/(e*(b*e - a*f)*Sqrt[c + d*x^2]*Sq
rt[e + f*x^2]) - (Sqrt[c]*Sqrt[d*e - c*f]*Sqrt[a + b*x^2]*Ellipti
cE[ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])], -(((b*c
 - a*d)*e)/(a*(d*e - c*f)))])/(e*(b*e - a*f)*Sqrt[(c*(a + b*x^2))
/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (c^(3/2)*Sqrt[a + b*x^2]*Ell
ipticF[ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])], -((
(b*c - a*d)*e)/(a*(d*e - c*f)))])/(a*e*Sqrt[d*e - c*f]*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi [A] time = 1.35714, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$

$$\frac{c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{fx^2+e}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{ae\sqrt{c+dx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}(de-cf)}{e\sqrt{c+dx^2}\sqrt{e+fx^2}(be-af)}-\frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{de-cf}E\left(\tan^{-1}\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{fx^2+e}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{e\sqrt{c+dx^2}(be-af)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*(e + f*x^2)^(3/2)),x]
```


```
[Out] ((d*e - c*f)*x*Sqrt[a + b*x^2])/(e*(b*e - a*f)*Sqrt[c + d*x^2]*Sq
rt[e + f*x^2]) - (Sqrt[c]*Sqrt[d*e - c*f]*Sqrt[a + b*x^2]*Ellipti
cE[ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])], -(((b*c
 - a*d)*e)/(a*(d*e - c*f)))])/(e*(b*e - a*f)*Sqrt[(c*(a + b*x^2))
/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (c^(3/2)*Sqrt[a + b*x^2]*Ell
ipticF[ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])], -((
(b*c - a*d)*e)/(a*(d*e - c*f)))])/(a*e*Sqrt[d*e - c*f]*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rubi in Sympy [A] time = 51.6074, size = 122, normalized size = 0.38

$$\frac{\sqrt{a}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\sqrt{c+dx^2}E\left(\operatorname{asin}\left(\frac{x\sqrt{af-be}}{\sqrt{a}\sqrt{e+fx^2}}\right)\middle|\frac{a(cf-de)}{c(af-be)}\right)}{e\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{a+bx^2}\sqrt{af-be}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(1/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] sqrt(a)*sqrt(e*(a + b*x**2)/(a*(e + f*x**2)))*sqrt(c + d*x**2)*el
liptic_e(asin(x*sqrt(a*f - b*e)/(sqrt(a)*sqrt(e + f*x**2))), a*(c
*f - d*e)/(c*(a*f - b*e)))/(e*sqrt(e*(c + d*x**2)/(c*(e + f*x**2)
))*sqrt(a + b*x**2)*sqrt(a*f - b*e))
```

Mathematica [A] time = 1.20629, size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\,(e+fx^2)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*(e + f*x^2)^(3/2)),x]
```

```
[Out] Integrate[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*(e + f*x^2)^(3/2)), x]
```

Maple [F] time = 0.104, size = 0, normalized size = 0.

$$\int 1\sqrt{dx^2+c}\frac{1}{\sqrt{bx^2+a}}\left(fx^2+e\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(3/2),x)
```

```
[Out] int((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)/(sqrt(b*x^2 + a)*(f*x^2 + e)^(3/2)),x, algorithm="maxi
```

```
[Out] integrate(sqrt(d*x^2 + c)/(sqrt(b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)/(sqrt(b*x^2 + a)*(f*x^2 + e)^(3/2)),x, algorithm="fric
```

```
[Out] integral(sqrt(d*x^2 + c)/(sqrt(b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(1/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(sqrt(a + b*x**2)*(e + f*x**2)**(3/2)),
 x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)/(sqrt(b*x^2 + a)*(f*x^2 + e)^(3/2)),x, algorithm="giac

[Out] integrate(sqrt(d*x^2 + c)/(sqrt(b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

3.110 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}\,dx$

Optimal. Leaf size=37

$$\text{Int}\left(\frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}\,(e+fx^2)^{3/2}},x\right)$$

```
[Out] Unintegrable[Sqrt[c + d*x^2]/((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2)
), x]
```

Rubi [A] time = 0.163491, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\text{Int}\left(\frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}\,(e+fx^2)^{3/2}},x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[Sqrt[c + d*x^2]/((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] Defer[Int][Sqrt[c + d*x^2]/((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2)),
 x]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(3/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.68724, size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}\,(e+fx^2)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[Sqrt[c + d*x^2]/((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]

[Out] Integrate[Sqrt[c + d*x^2]/((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2)),
x]
```

Maple [A] time = 0.104, size = 0, normalized size = 0.

$$\int 1\sqrt{dx^2+c}\left(bx^2+a\right)^{-\frac{3}{2}}\left(fx^2+e\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(3/2),x)

[Out] int((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^(3/2)*(f*x^2 + e)^(3/2)),x, algorithm="ma

[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^(3/2)*(f*x^2 + e)^(3/2)),
x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^2+c}}{(bfx^4+(be+af)x^2+ae)\sqrt{bx^2+a}\sqrt{fx^2+e}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^(3/2)*(f*x^2 + e)^(3/2)),x, algorithm="fr

[Out] integral(sqrt(d*x^2 + c)/((b*f*x^4 + (b*e + a*f)*x^2 + a*e)*sqrt(
b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(3/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^(3/2)*(f*x^2 + e)^(3/2)),x, algorithm="gi
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^(3/2)*(f*x^2 + e)^(3/2)),
x)
```

3.111 $\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=541

$$\frac{\sqrt{e}\sqrt{c+dx^2}(bc-ad)(2be-af)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}F\left(\sin^{-1}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)\,|\,\frac{(bc-ad)e}{c(be-af)}\right)}{2b^2c\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}(adf-b(cf+de))\left(\frac{bc}{bc-ad};\sin^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)\,|\,\frac{c(be-af)}{(bc-ad)e}\right)}{2b^2\sqrt{c}\sqrt{e+fx^2}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2\sqrt{a+bx^2}}-\frac{\sqrt{c}\sqrt{e+fx^2}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}E\left(\sin^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)\,|\,\frac{c(be-af)}{(bc-ad)e}\right)}{2b\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}}$$

```
[Out] (x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(2*Sqrt[a + b*x^2]) - (Sqrt[c
]*Sqrt[b*c - a*d]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e +
f*x^2]*EllipticE[ArcSin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x
^2])], (c*(b*e - a*f))/((b*c - a*d)*e)])/(2*b*Sqrt[c + d*x^2]*Sqr
t[(a*(e + f*x^2))/(e*(a + b*x^2))]) + ((b*c - a*d)*Sqrt[e]*(2*b*e
 - a*f)*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*Ell
ipticF[ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((b
*c - a*d)*e)/(c*(b*e - a*f))])/(2*b^2*c*Sqrt[b*e - a*f]*Sqrt[(a*(
c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2]) - (a*(a*d*f - b*(d*
e + c*f))*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*E
llipticPi[(b*c)/(b*c - a*d), ArcSin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*
Sqrt[a + b*x^2])], (c*(b*e - a*f))/((b*c - a*d)*e)])/(2*b^2*Sqrt[
c]*Sqrt[b*c - a*d]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e +
 f*x^2])
```

Rubi [A] time = 1.58281, antiderivative size = 541, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206

$$\frac{\sqrt{e}\sqrt{c+dx^2}(bc-ad)(2be-af)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}F\left(\sin^{-1}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{bx^2+a}}\right)\,|\,\frac{(bc-ad)e}{c(be-af)}\right)}{2b^2c\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}(adf-b(cf+de))\left(\frac{bc}{bc-ad};\sin^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)\,|\,\frac{c(be-af)}{(bc-ad)e}\right)}{2b^2\sqrt{c}\sqrt{e+fx^2}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2\sqrt{a+bx^2}}-\frac{\sqrt{c}\sqrt{e+fx^2}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}E\left(\sin^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)\,|\,\frac{c(be-af)}{(bc-ad)e}\right)}{2b\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(2*Sqrt[a + b*x^2]) - (Sqrt[c
]*Sqrt[b*c - a*d]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e +
f*x^2]*EllipticE[ArcSin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x
^2])], (c*(b*e - a*f))/((b*c - a*d)*e)])/(2*b*Sqrt[c + d*x^2]*Sqr
t[(a*(e + f*x^2))/(e*(a + b*x^2))]) + ((b*c - a*d)*Sqrt[e]*(2*b*e
 - a*f)*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*Ell
ipticF[ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((b
*c - a*d)*e)/(c*(b*e - a*f))])/(2*b^2*c*Sqrt[b*e - a*f]*Sqrt[(a*(
c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2]) - (a*(a*d*f - b*(d*
e + c*f))*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*E
llipticPi[(b*c)/(b*c - a*d), ArcSin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*
Sqrt[a + b*x^2])], (c*(b*e - a*f))/((b*c - a*d)*e)])/(2*b^2*Sqrt[
c]*Sqrt[b*c - a*d]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e +
 f*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.125541, size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/Sqrt[a + b*x^2],x]
```

```
[Out] Integrate[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/Sqrt[a + b*x^2], x]
```

Maple [F] time = 0.075, size = 0, normalized size = 0.

$$\int 1\sqrt{dx^2+c}\sqrt{fx^2+e}\frac{1}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}\sqrt{fx^2+e}}{\sqrt{bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/sqrt(b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx^2+c}\sqrt{fx^2+e}}{\sqrt{bx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/sqrt(b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt(e + f*x**2)/sqrt(a + b*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + c}\sqrt{fx^2 + e}}{\sqrt{bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/sqrt(b*x^2 + a), x)
```

3.112 $\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=37

$$\text{Int}\left(\frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}},x\right)$$

```
[Out] Unintegrable[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),
 x]
```

Rubi [A] time = 0.167954, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\text{Int}\left(\frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}},x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] Defer[Int][(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x
]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.202538, size = 0, normalized size = 0.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] Integrate[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x]
```

Maple [A] time = 0.09, size = 0, normalized size = 0.

$$\int 1\left(bx^2+a\right)^{\frac{3}{2}}\frac{1}{\sqrt{dx^2+c}}\frac{1}{\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="maxi
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="fric
```

```
[Out] Timed out
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2\right)^{\frac{3}{2}}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/(sqrt(c + d*x**2)*sqrt(e + f*x**2)),
 x)
```

GIAC/XCAS [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^2+a\right)^{\frac{3}{2}}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="giac
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.113 $\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=159

$$\frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\Pi\left(\frac{bc}{bc-ad};\sin^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{\sqrt{c}\sqrt{e+fx^2}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

```
[Out] (a*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*Elliptic
Pi[(b*c)/(b*c - a*d), ArcSin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a
+ b*x^2])], (c*(b*e - a*f))/((b*c - a*d)*e)])/(Sqrt[c]*Sqrt[b*c -
 a*d]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.571441, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\Pi\left(\frac{bc}{bc-ad};\sin^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{bx^2+a}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{\sqrt{c}\sqrt{e+fx^2}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2]/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```


```
[Out] (a*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*Elliptic
Pi[(b*c)/(b*c - a*d), ArcSin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a
+ b*x^2])], (c*(b*e - a*f))/((b*c - a*d)*e)])/(Sqrt[c]*Sqrt[b*c -
 a*d]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.121682, size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[Sqrt[a + b*x^2]/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] Integrate[Sqrt[a + b*x^2]/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x]
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int 1\sqrt{bx^2+a}\frac{1}{\sqrt{dx^2+c}}\frac{1}{\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="maxima
```

```
[Out] integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="fricas
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/(sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.114 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}F\left(\sin^{-1}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{c\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

```
[Out] (Sqrt[e]*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*El
lipticF[ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((
b*c - a*d)*e)/(c*(b*e - a*f))])/(c*Sqrt[b*e - a*f]*Sqrt[(a*(c + d
*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2])
```

Rubi [A] time = 0.314799, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}F\left(\sin^{-1}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{c\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (Sqrt[e]*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*El
lipticF[ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((
b*c - a*d)*e)/(c*(b*e - a*f))])/(c*Sqrt[b*e - a*f]*Sqrt[(a*(c + d
*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2])
```

Rubi in Sympy [A] time = 67.0866, size = 214, normalized size = 1.45

$$\frac{\sqrt{e}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\sqrt{1-\frac{x^2(-ad+bc)}{c(a+bx^2)}}\sqrt{c+dx^2}F\left(\operatorname{atan}\left(\frac{x\sqrt{af-be}}{\sqrt{e}\sqrt{a+bx^2}}\right)\middle|\frac{a(cf-de)}{c(af-be)}\right)}{c\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{\frac{e\left(c+\frac{x^2(ad-bc)}{a+bx^2}\right)}{c\left(e+\frac{x^2(af-be)}{a+bx^2}\right)}}\sqrt{1-\frac{x^2(-af+be)}{e(a+bx^2)}}\sqrt{e+fx^2}\sqrt{af-be}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] sqrt(e)*sqrt(a*(e + f*x**2)/(e*(a + b*x**2)))*sqrt(1 - x**2*(-a*d
 + b*c)/(c*(a + b*x**2)))*sqrt(c + d*x**2)*elliptic_f(atan(x*sqrt
(a*f - b*e)/(sqrt(e)*sqrt(a + b*x**2))), a*(c*f - d*e)/(c*(a*f -
b*e)))/(c*sqrt(a*(c + d*x**2)/(c*(a + b*x**2)))*sqrt(e*(c + x**2*
(a*d - b*c)/(a + b*x**2))/(c*(e + x**2*(a*f - b*e)/(a + b*x**2)))
)*sqrt(1 - x**2*(-a*f + b*e)/(e*(a + b*x**2)))*sqrt(e + f*x**2)*s
qrt(a*f - b*e))
```

Mathematica [A] time = 0.506026, size = 152, normalized size = 1.03

$$\frac{x\sqrt{e+fx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}F\left(\sin^{-1}\left(\sqrt{\frac{(be-af)x^2}{e(bx^2+a)}}\right)|\frac{bce-ade}{bce-acf}\right)}{e\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{\frac{ax^2(e+fx^2)(be-af)}{e^2(a+bx^2)^2}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (x*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2]*Elliptic
F[ArcSin[Sqrt[((b*e - a*f)*x^2)/(e*(a + b*x^2))]], (b*c*e - a*d*e
)/(b*c*e - a*c*f)])/(e*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[(a*(b
*e - a*f)*x^2*(e + f*x^2))/(e^2*(a + b*x^2)^2)])
```

Maple [F] time = 0.093, size = 0, normalized size = 0.

$$\int 1\frac{1}{\sqrt{bx^2+a}}\frac{1}{\sqrt{dx^2+c}}\frac{1}{\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="maxi
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="fric
```

```
[Out] integral(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2)),
x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="giac
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

3.115 $\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=37

$$\text{Int}\left(\frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}},x\right)$$

```
[Out] Unintegrable[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]
), x]
```

Rubi [A] time = 0.172056, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\text{Int}\left(\frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}},x\right)$$

Verification is Not applicable to the result.

```
[In]  Int[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]

[Out] Defer[Int][1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),
 x]
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)

[Out] Timed out
```

Mathematica [A] time = 1.11097, size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] Integrate[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),
x]
```

Maple [A] time = 0.101, size = 0, normalized size = 0.

$$\int 1\left(bx^2+a\right)^{-\frac{3}{2}}\frac{1}{\sqrt{dx^2+c}}\frac{1}{\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="ma
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)),
x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}\sqrt{fx^2+e}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="fr
```

```
[Out] integral(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x
)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)*sqrt(e + f*x**2)
), x)
```

GIAC/XCAS [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)),x, algorithm="gia
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)),
x)
```

4 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Mathematica/Rubi followed by one for Maple. The following are links to the source code.

The following are the listing of the above functions.

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)
(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
  If[AtomQ[expn], 1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]
HypergeometricFunctionQ[func_] := MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
AppellFunctionQ[func_] := MemberQ[{AppellF1},func]
```

```
# File: GradeAntiderivative.mpl Original version thanks to Albert Rich emailed on 03/21/2017
#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);
  #This check below actually is not needed, since I only call this grading only for
  #passed integrals. i.e. I check for "F" before calling this.

  if not type(result,freeof('int')) then
     return "F";
  end if;

  if ExpnType_result<=ExpnType_optimal then
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
         if leaf_count_result<=2*leaf_count_optimal then
            return "A";
         else
            return "B";
         end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      return "C";
  end if
end proc:
```

```
# is_contains_complex(result) takes expressions and returns true if it contains "I"
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1 else
    max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1 else
      max(2,ExpnType(op(1,expn))) end if else
    max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' or op(0,expn)='integrate' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:
```

```
ElementaryFunctionQ := proc(func)
  member(func,[exp,log,ln, sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[erf,erfc,erfi,FresnelS,FresnelC,Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u) else
  apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```